Jinquan Lin 12.3

1. $(1,2,1)\cdot(4,3,5)$

$= 1\times4 + 2\times3 + 1\times5$

$= 4+6+5$

$= 15$

13. $(1,1,1)\cdot(1,-2,-2)$

$= 1\times1 + 1\times(-2) + 1\times(-2)$

$= -3$ not orthogonal

$\theta = \cos^{-1}\left(\frac{ab}{|a||b|}\right) = \cos^{-1}\left(\frac{-3}{\sqrt{1^2+1^2+1^2}\sqrt{1^2+(-2)^2+(-2)^2}}\right)$

$= 125.27°$ The angle is obtuse.

21. $i+j,\ j+2k \Rightarrow (1,1,0)\cdot(0,1,2) = 0+1+0 = 1$

$\theta = \cos^{-1}\left(\frac{1}{\sqrt{2}\sqrt{5}}\right) = 71.57° \Rightarrow \cos 71.57° = \frac{1}{\sqrt{10}}$

29. a. $(b,3,2)\cdot(1,b,1)$

$= b+3b+2 = 0$

$4b = -2$

$b = -\frac{1}{2}$

b. $(4,-2,7)\cdot(b^2,b,0)$

$= 4b^2-2b = 0$

$b(4b-2) = 0$

$b=0$ or $b=\frac{1}{2}$

31. $(2,0,-3)\cdot(a,b,c)$ $(a,b,c) = (3,1,2)$ or $(3,0,2)$

$= 2a+0-3c = 0$

$2a = 3c$

57. $u = 5i+7j-4k,\ v=k$

$\text{Proj}_u(v) = \frac{u\cdot v}{|u|^2}\cdot\vec{a} = \frac{-4}{1^2}(0,0,1) = (0,0,-4)$

63. $u=(3,5)$ $v=(8,2)$

$OP = \text{Proj}_v u = \frac{34}{(2\sqrt{17})^2}(8,2) = (4,1)$

12.4

1. $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$= 1\cdot 4 - 2\cdot 3 = -2$

5. $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix}$

$= 1\cdot \begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$

$= 1\cdot(-3) - 2\cdot 4 + 1\cdot 3$

$= -8$

13. $(i+j)\times k$

$= i\times k + j\times k = j + i = i - j$

21. $(u-2v)\times(u+2v)$

$= u\times u + u\times 2v - 2v\times u - 2v\times 2v$

$= 2(u\times u) - 2(v\times u)$

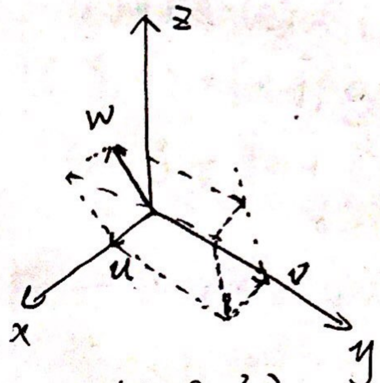
$= 4(u\times v) = (4, 4, 0)$

25. $v\times w = -u$

27. $v = (3, 0, 0)$, $w = (0, 1, -1)$

$v\times w = |v||w|\sin\theta = (0, 3, 3)$

39. $u = (1, 0, 0)$ $v = (0, 2, 0)$ $w = (1, 1, 2)$



$V = |a\cdot(b\times c)|$

$= 4$

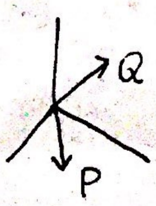
41. $u = (1, 0, 3)$ $v = (2, 1, 1)$

$u\times v = |u||v|\sin\theta = \sqrt{10}\cdot\sqrt{6}\sin\theta$

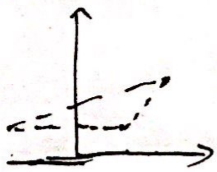
$= \sqrt{35}$

43. $P = (3, 3, 0)$, $Q = (0, 3, 3)$

$A = \frac{9\sqrt{3}}{2}$



45. $(1, 2)$, $(3, 4)$, $(-2, 2)$



$\text{Area} = 3\times 2\cdot\frac{1}{2}$

$= 3$

12.5

1. $n = (1,3,2)$. $(4,-1,1)$

$n \cdot (R-P) = 0$

$(1,3,2)\cdot(x-4,\ y+1,\ z-1) = 0$

$x-4+3y+3+2z-2=0$

5. $n = i$ $\quad x+3y+2z=3$

$(3,1,-9)$

$(1,0,0)(x-3,\ y-1,\ z+9)=0$

$x-3=0$

$x=3$

9. $x=0$

11. (a). not parallel (b). parallel (c). not parallel (d). parallel

13. $9x-4y-11z=2$

$(9,-4,-11)$

15. $3(x-4)-8(y-1)+11z=0$

$(3,-8,11)$

17. $P=(2,-1,4)$, $Q=(1,1,1)$, $R(3,1,-2)$

$P-Q=(1,-2,3)$, $R-Q=(2,0,-3)$

$(P-Q)\times(R-Q)=(6,9,4)$

$(6,9,4)\times(x-1,\ y+2,\ z-3)=0$

$6x-6+9y+18+4z-12=0$

$6x+9y+4z=0$

19. $P=(1,0,0)$ $Q=(0,1,1)$ $R=(2,0,1)$

$P-Q=(1,-1,-1)$ $R-Q=(2,-1,0)$

$(P-Q)\times(R-Q)=(-1,-2,1)$

$\Longleftrightarrow (-1,-2,1)\cdot(x-1,\ y+1,\ z+1)=0$

$-x+1-2y-2+z+1=0$

$-x-2y+z=0$

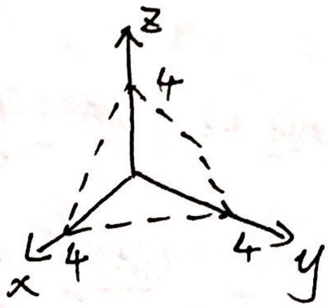
25.

$(1,1,0)\cdot(x+2, y+3, z-5)=0$

$x+2+y+3=0$

$x+y=-5$

31. $x+y+z=4$



53. $3x+ky+2z=5$

13. 1

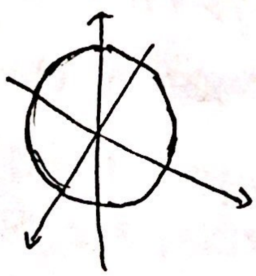
5. $P=(3,-5,7)$ $v=(3,0,1)$

$r(t)=(3,-5,7)+t(3,0,1)$

$=(3+3t, -5, 7+t)$

$r(t)=(3+3t)i, -5j, +(7+t)k$

17. $r(t)=(9\cos t)i+(9\sin t)j$



the circle has radius of 9 at the center of origin lying in the xy-plane.

13.2

3. $\lim_{t\to 0} e^{2t} i + \ln(t+1) j + 4k$

$= 1i + 0j + 4k = i + 4k$

5. $\lim_{h\to 0} \frac{r(t+h) - r(t)}{h}$ $\quad r(t) = (t^{-1}, \sin t, 4)$

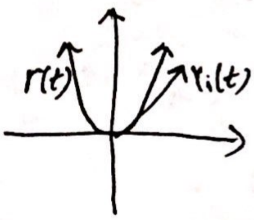
$= \lim_{h\to 0} \frac{((t+h)^{-1} i + \sin(t+h) j + 5k) - (t^{-1} i + \sin t j + k)}{h}$

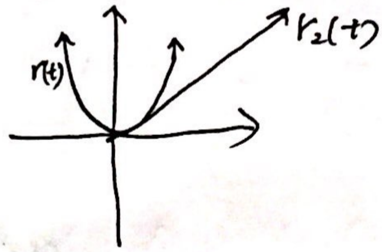
$= \lim_{h\to 0} \frac{((t+h)^{-1} - t^{-1}) i}{h} + \lim_{h\to 0} \frac{\sin(h+t) - \sin t}{h}$

$= (t^{-2}, \cos t, 0)$

7. $r(t) = (t, t^2, t^3)$
$r'(t) = (1, 2t, 3t^2)$

15. $r_1(t) = (t, t^2)$ $\quad r_2(t) = (t^3, t^6)$





17. $\frac{d}{dt}(r_1(t) \times r_2(t))$ $\quad r_1(t) = (t^2, t^3, t)$ $\quad r_2(t) = (e^{3t}, e^{2t}, e^{t})$

$= r_1'(t) r_2(t) + r_1(t) \cdot r_2'(t)$

$= (2t, 3t^2, 1)(e^{3t}, e^{2t}, e^{t}) + (t^3, t^2, t) \cdot (3e^{3t}, 2e^{2t}, e^{t})$

$= (3t^2 + 2t) e^{3t} + (2t^3 + 3t^2) e^{2t} + (t+1) e^{t}$

31. $r(t) = (1 - t^2, 5t, 2t^3)$, $t = 2$
$r'(t) = (-2t, 5, 6t^2)$
$r'(2) = (-4, 5, 24)$ $\quad$ line $= r(2) + t r'(2)$

33. $r(s) = 4s^{-1} i - \frac{8}{3} s^{-3} k$, $s = 2$
$r'(s) = -4s^{-2} i + 8 s^{-4} k$
$r'(2) = -1 i + \frac{1}{2} k$ $\quad$ line $= r(2) + s r'(2)$

13.2

3. $\lim_{t\to 0} e^{2t} i + \ln(t+1) j + 4k$

$= 1i + 0j + 4k = i + 4k$

5. $\lim_{h\to 0} \frac{r(t+h) - r(t)}{h}$ $\quad r(t) = (t^{-1}, \sin t, 4)$

$= \lim_{h\to 0} \frac{((t+h)^{-1} i + \sin(t+h) j + 5k) - (t^{-1} i + \sin t j + k)}{h}$

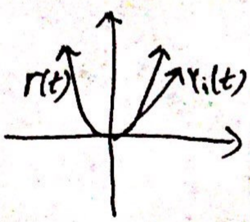
$= \lim_{h\to 0} \frac{((t+h)^{-1} - t^{-1}) i}{h} + \lim_{h\to 0} \frac{\sin(h+t) - \sin t}{h}$

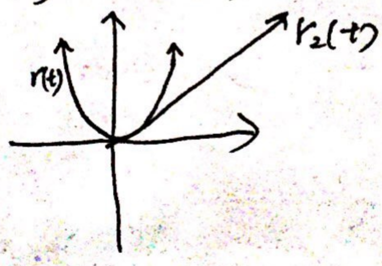
$= (t^{-2}, \cos t, 0)$

7. $r(t) = (t, t^2, t^3)$

$r'(t) = (1, 2t, 3t^2)$

15. $r_1(t) = (t, t^2)$ $\quad r_2(t) = (t^3, t^6)$





17. $\frac{d}{dt}(r_1(t) \times r_2(t))$ $\quad r_1(t) = (t^2, t^3, t)$ $\quad r_2(t) = (e^{3t}, e^{2t}, e^{t})$

$= r_1'(t) r_2(t) + r_1(t) . r_2'(t)$

$= (2t, 3t^2, 1)(e^{3t}, e^{2t}, e^{t}) + (t^3, t^2, t) \cdot (3e^{3t}, 2e^{2t}, e^{t})$

$= (3t^2 + 2t)e^{3t} + (2t^3 + 3t^2)e^{2t} + (t+1)e^{t}$

31. $r(t) = (1 - t^2, 5t, 2t^3)$, $t = 2$

$r'(t) = (-2t, 5, 6t^2)$

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33. $r(s) = 4s^{-1} i + \frac{8}{3} s^{-3} k$, $s = 2$

$r'(s) = -4s^{-2} i + 8s^{-4} k$

$r'(2) = -1 i + \frac{1}{2} k$ $\quad$ line $= r(2) + s r'(2)$

49. $r'(t) = t^2 i + 5t j + k \quad r(1) = j + 2k$

$$\int r'(t)\,dt$$
$$= \frac{1}{3}t^3 i + \frac{5}{2}t^2 j + tk + C$$
$$r(1) = \frac{1}{3}i + \frac{5}{2}j + k + C = j + 2k$$
$$C = -\frac{1}{3}i - \frac{3}{2}j + k$$
$$r(t) = \left(\frac{1}{3}t^3 - \frac{1}{3}\right)i + \left(\frac{5}{2}t^2 - \frac{3}{2}\right)j + (t+1)k$$

51. $r''(t) = 16k \quad , r(0) = (1,0,0), \; r'(0) = (0,1,0)$

$$\int r''(t)\,dt$$
$$= 16tk + C$$
$$\cancel{r'(t)} \; r'(0) = C = j$$
$$r'(t) = 16tk + j$$
$$\int r'(t)\,dt$$
$$= 8t^2 k + tj + C$$
$$r(0) = C = i$$
$$r(t) = i + tj + 8t^2 k$$