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Calc 251
Dr. Z
HW# 2

12.3: 1, 13, 21, 29, 31, 57, 63

#1 $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = 4+6+5 = \boxed{15}$

#5 $\langle 3,1\rangle \cdot \langle 4,-7\rangle = 12-7 = \boxed{5}$

#13 $\langle 1,1,1\rangle \quad \langle 1,-2,-2\rangle \Rightarrow 1-2-2 = -3 < 0 \quad \therefore$ obtuse

#21 $i+j, \quad j+2K \qquad i+j = \langle 1,1,0\rangle \quad j+2K = \langle 0,1,2\rangle$

$\cos\theta = \dfrac{V\cdot W}{\|V\|\cdot\|W\|} \qquad \|V\| = \sqrt{1^2+1^2+0^2} = \sqrt{2} \qquad \|W\| = \sqrt{0^2+1^2+2^2} = \sqrt{5}$

$\langle 1,1,0\rangle \cdot \langle 0,1,2\rangle = 0+1+0 = 1$

$\cos\theta = \dfrac{1}{\sqrt{2\cdot 5}} = \boxed{\dfrac{1}{\sqrt{10}}}$

#29 a) $\langle b,3,2\rangle, \langle 1,b,1\rangle$

$b + 3b + 2 = 0$
$4b = -2$
$\boxed{b = -\frac{1}{2}}$

b) $\langle 4,-2,7\rangle \quad \langle b^2,b,0\rangle$

$4b^2 - 2b + 0 = 0$
$2b(2b-1) = 0$
$\boxed{b = 0, \ b = \frac{1}{2}}$

#31 $\langle 2,0,-3\rangle$

$2x + 0y - 3z = 0$
$2x - 3z = 0$
$2x = 3z$
$x = 1.5 \quad z = 1$

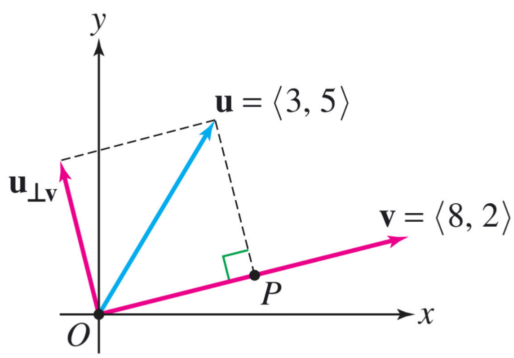
$$\boxed{\begin{array}{l} 1.\ \langle \frac{3}{2}, 12, 1\rangle \\ 2.\ \langle \frac{3}{2}, \frac{17}{13}, 1\rangle \end{array}}$$

#57 $u: 5i + 7j - 4K, \quad v = K \qquad u = \langle 5,7,-4\rangle \qquad v = \langle 0,0,1\rangle$

$u_{\parallel v} = \left(\dfrac{u\cdot v}{v\cdot v}\right)v = \dfrac{(5\cdot 0)+(7\cdot 0)+(-4\cdot 1)}{(0\cdot 0)+(0\cdot 0)+(1\cdot 1)}v = \dfrac{0+0-4}{0+0+1}v = -4v$

$\boxed{u_{\parallel v} = -4K}$

#63 length of $\overline{OP}$



12.4: 1, 5, 13, 21, 25, 27, 39, 41, 43, 45

#1 $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 1\cdot 3 - 2\cdot 4 = 3-8 = \boxed{-5}$

#5 $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 3\begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$

$= 1(-3-0) - 2(4-0) + 1(0-(-3))$

$= -3-8+3 = \boxed{-8}$

#13 $\underset{v}{(i+j)} \cdot \underset{u}{k}$ $\quad \vec{v} = \langle 1,1,0\rangle$ $\quad \vec{u} = \langle 0,0,1\rangle$

$\begin{vmatrix} a & b & c \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = a\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - b\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + c\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$

$= a(1-0) - b(1-0) + c(0-0)$

$= a-b$

$\boxed{(i+j)\cdot k = i-j}$

#21 $u\times v = \langle 1,1,0\rangle$ $\quad u\times w = \langle 0,3,1\rangle$ $\quad v\times w = \langle 2,-1,1\rangle$

$(u-2v)\times(u+2v)$

$\begin{vmatrix} a & b & c \\ 1 & -2 & 0 \\ 1 & 2 & 0 \end{vmatrix} = a\begin{vmatrix} -2 & 0 \\ 2 & 0 \end{vmatrix} - b\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} + c\begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix}$

$= a(0-0) - b(0-0) + c(2-(-2))$

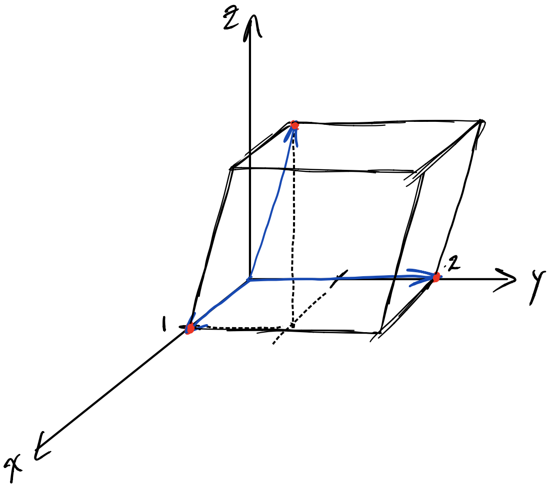
$= 4c \rightarrow \langle 0,0,4\rangle$

#25. $-u$

#27 $v = \langle 3,0,0\rangle$ $\quad w = \langle 0,1,-1\rangle$

$u = v\times w$

#39 $u = \langle 1,0,0\rangle$ $\quad v = \langle 0,2,0\rangle$ $\quad w = \langle 1,1,2\rangle$



#41 $u = \langle 1,0,3\rangle$ $\quad v = \langle 2,1,1\rangle$

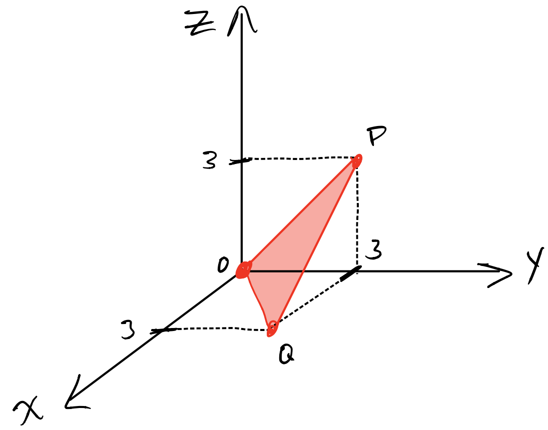
$u\times v = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = i\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix}$

$= i(0-3) - j(1-6) + k(1-0)$

$= -3i - j(-5) + k \rightarrow \langle -3,5,1\rangle$

$A = \sqrt{(-3)^2+5^2+1^2} = \sqrt{9+25+1} = \boxed{\sqrt{35}}$

43 O, $P(3,3,0)$, $Q(0,3,3)$



$$P \times Q = \begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix} = i\begin{vmatrix} 3 & 0 \\ 3 & 3 \end{vmatrix} - j\begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} + k\begin{vmatrix} 3 & 3 \\ 0 & 3 \end{vmatrix}$$

$$= i(9-0) - j(9-0) + k(9-0)$$

$$= 9i - 9j + 9k$$

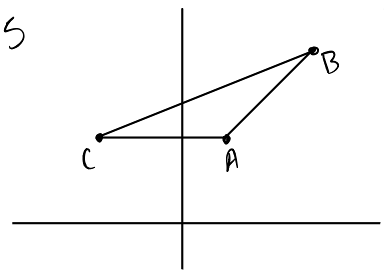
$$= \langle 9, -9, 9 \rangle$$

$$A = \frac{1}{2} \cdot \sqrt{9^2 + (-9)^2 + 9^2} = \sqrt{81+81+81} = \sqrt{243}$$

$$= \boxed{\frac{9\sqrt{3}}{2}}$$

45

$A(1,2)$
$B(3,4)$
$C(-2,2)$



$$\vec{AB} = \langle 3-1, 4-2, 0 \rangle = \langle 2, 2, 0 \rangle$$

$$\vec{AC} = \langle -2-1, 2-2, 0 \rangle = \langle -3, 0, 0 \rangle$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} i & j & k \\ 2 & 2 & 0 \\ -3 & 0 & 0 \end{vmatrix} = i\begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} - j\begin{vmatrix} 2 & 0 \\ -3 & 0 \end{vmatrix} + k\begin{vmatrix} 2 & 2 \\ -3 & 0 \end{vmatrix}$$

$$= i(0-0) - j(0-0) + k(0-(-6))$$

$$= \langle 0, 0, 6 \rangle$$

$$A = \frac{1}{2} \cdot \left(\sqrt{0^2 + 0^2 + 6^2}\right. = \frac{\sqrt{36}}{2} = \boxed{3}$$

12.5: 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 63

#1 $n = \langle 1,3,2 \rangle$ $(4,-1,1)$

$(x-4)+3(y+1)+2(z-1)=0$

$x-4+3y+3+2z-2=0$

$\boxed{x+3y+2z=3}$

#5 $n=i$ $(3,1,-9)$

$\downarrow$

$\langle 1,0,0 \rangle$

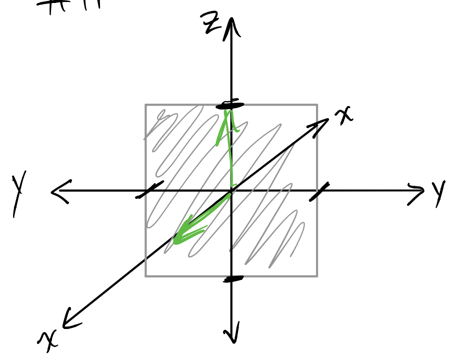
$(x-3)+0(y-1)+0(z+9)=0$

$x-3=0$

$\boxed{x=3}$

#9 $\boxed{x=0}$

#11



a) no

b) yes

c) no

d) yes

#13 $9x-4y-11z=2$

$n=\langle 9,-4,-11 \rangle$

#15 $3(x-4)-8(y-1)+11z=0$

$n=\langle 3,-8,11 \rangle$

#17 $P=(2,-1,4)$ $Q=(1,1,1)$ $R=(3,1,-2)$

$\vec{PQ}=\langle 1-2, 1+1, 1-4 \rangle = \langle -1, 2, -3 \rangle$

$\vec{PR}=\langle 3-2, 1+1, -2-4 \rangle = \langle 1, 2, -6 \rangle$

$$\vec{PQ}\times\vec{PR} = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix} = i\begin{vmatrix} 2 & -3 \\ 2 & -6 \end{vmatrix} - j\begin{vmatrix} -1 & -3 \\ 1 & -6 \end{vmatrix} + k\begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix}$$

$$= i(-12+6)-j(6+3)+k(-2-2)$$

$$= -6i-9j-4k = \langle -6,-9,-4 \rangle$$

$-6x-9y-4z=d$

$d=\langle -6,-9,-4 \rangle \cdot (1,1,1)$
$= -6-9-4 = -19$

$\boxed{-6x-9y-4z=-19}$

#19 $P(1,0,0)$ $Q(0,1,1)$ $R(2,0,1)$

$\overrightarrow{PQ} = \langle 0-1, 1-0, 1-0 \rangle = \langle -1, 1, 1 \rangle$

$\overrightarrow{PR} = \langle 2-1, 0-0, 1-0 \rangle = \langle 1, 0, 1 \rangle$

$$\overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = i \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} - j \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} + k \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}$$

$= i(1-0) - j(-1-1) + k(0-1)$

$= i + 2j - k \rightarrow \langle 1, 2, -1 \rangle$

$x + 2y - z = d$ $\quad d = \langle 1, 2, -1 \rangle \cdot (1, 0, 0)$

$= 1$

$\boxed{x + 2y - z = 1}$

#25 $n = i + k \rightarrow \langle 1, 0, 1 \rangle$ $\quad (-2, -3, 5)$

$x + z = d$ $\quad d = \langle 1, 0, 1 \rangle \cdot (-2, -3, 5)$

$= -2 + 5 = 3$

$\boxed{x + z = 3}$

#31 $x + y + z = 4$

$0 + 0 + z = 4$ $\quad (0, 0, 4)$

$0 + y + 0 = 4$ $\quad (0, 4, 0)$

$x + 0 + 0 = 4$ $\quad (4, 0, 0)$