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September 18, 2019

12.3 Homework Exercises # 1, 13, 21, 29, 31, 57, 63

12.4 Homework Exercises: 1, 5, 13, 21, 25, 27, 39, 41, 43, 45

① $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle$

$= 4+6+5 = 15$

⑬ $\langle 1,1,1\rangle \quad \langle 1,-2,-2\rangle$

$\cos\theta = \frac{A \cdot B}{(\|A\|\|B\|)} = \frac{-3}{\sqrt{3}\cdot 3} = -\frac{1}{\sqrt{3}}$

$\theta = 125.26°$ ∴ angle is obtuse

㉑ $i+j, \; j+2k$

$\langle 1,1,0\rangle \cdot \langle 0,1,2\rangle = 0+1+0 = 1$

$\cos\theta = \frac{A\cdot B}{(\|A\|\|B\|)} = \frac{1}{\sqrt{2}\cdot\sqrt{5}} = \frac{1}{\sqrt{10}} = 71.57°$

㉙ a. $\langle b,3,2\rangle$ and $\langle 1,b,1\rangle \rightarrow$ orthogonal

$= b + 3b + 2 = 0$ * 2 vectors are orthogonal when their dot product is 0.

$4b = -2$

$b = -\frac{1}{2}$

b. $\langle 4,-2,7\rangle \cdot \langle b^2, b, 0\rangle$

$= 4b^2 - 2b + 0 = 0$

$4b^2 - 2b = 0 \rightarrow b(4b-2) = 0$

$b = 0$ or $\frac{1}{2}$

㉛ 2 vectors orthogonal to $\langle 2,0,-3\rangle$

$v_1 = \langle 3,0,2\rangle$

$v_2 = \langle 0,1,0\rangle$

57 Finding the projection of u along v:

$U = 5i + 7j - 4k \quad V = K$

$\langle 5,7,-4\rangle \quad \langle 0,0,1\rangle$

* The vector projection of u onto v:

$\frac{\vec{U}\cdot\vec{V}}{\|\vec{V}\|}\cdot\vec{V} \longrightarrow = \frac{0+0-4}{1} = -4 \times \langle 0,0,1\rangle$

$= -4k$

?

63 Length of $\overline{OP}$:

CAH

$U = \langle 3,5\rangle$ and $V = \langle 8,2\rangle$

$V+U = \langle 11,7\rangle \quad |V| = 13.04$

$\cos\theta = \frac{9}{H} \longrightarrow \cos 45 = \frac{9}{13.04}$

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① $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 3 - 8 = -5$

⑤ ? $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix}$

$= 1\cdot\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2\cdot\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 1\cdot\begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$

$= -3 - 8 + (-3) = -8$

⑬ $(i+j)\times k$

$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = i\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} - j\begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} + k\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix}$

$= i - j$

㉕ $-u$

$(v \times w = -w \times v)$

?

㉑ $u \times v = \langle 1,1,0\rangle$

$u \times w = \langle 0,3,1\rangle$

$v \times w = \langle 2,-1,1\rangle$

Calculating the 2 assuming

$(u-2v)\times(u+2v) = \langle 4,4,0\rangle$

㉗ $v = \langle 3,0,0\rangle \quad w = \langle 0,1,-1\rangle$

$i \times j = k \quad j \times k = i \quad k \times i = j$

$k = 3 \quad 0 = i \quad 3 = j$

$\langle 0,3,3\rangle$

39 V of a parallelepiped

Volume $= \|a\times b\|\|c\||\cos\theta|$

$= |(a\times b)\cdot(c)|$

$U = \langle 1,0,0\rangle$

$V = \langle 0,2,0\rangle$

$W = \langle 1,1,2\rangle$

$U\times V = \begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 2 & 0 \end{vmatrix} = i\begin{vmatrix} 0 & 0 \\ 2 & 0 \end{vmatrix} - j\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} + k\begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix}$

$= 0 - 0 + 2k$

$\langle 0,0,2\rangle \cdot \langle 1,1,2\rangle = 4$ ∴ The volume is 4.

41 $U = \langle 1,0,3\rangle$, $V = \langle 2,1,1\rangle \rightarrow$ Area of a parallelogram: $\|a\|\|b\|\sin\theta$

$\|U\| = \sqrt{10}$, $\|V\| = \sqrt{6}$ $= \sqrt{60}\sin\theta$

To find the θ = $\cos\theta = \frac{5}{\sqrt{60}} = 49.8$

$\Rightarrow \sqrt{60}\sin 49.8 = 5.92 \approx \sqrt{35}$

43 $O = (0,0,0)$, $P = (3,3,0)$, $Q = (0,3,3)$

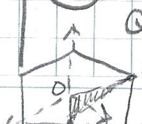
$OP = \langle 3,3,0\rangle$, $OQ = \langle 0,3,3\rangle \rightarrow \times$

$\begin{vmatrix} 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix} = 9i - 9j + 9k$

$A = \frac{1}{2}\sqrt{9^2+9^2+9^2}$

$A = 7.79$

$\approx \frac{9\sqrt{3}}{2}$



Area of △: $\frac{1}{2}\|a\times b\|$

(45) A(1,2) (A) $\vec{AB} = (2,2)$
B(3,4) (B) $\vec{AC} = (-3,0)$ $\rightarrow \times \rightarrow \begin{vmatrix} 2 & 2 \\ -3 & 0 \end{vmatrix} = -6$
C(-2,2)

$\hookrightarrow$ Area $= \frac{1}{2} \|a \times b\|$
$= \frac{1}{2} \sqrt{-6^2}$
$= \frac{1}{2}(6) = 3$

$\therefore$ The area of the Δ is 3 units^2.

Extra Notes:

1. planes through the origin must have standard equations of the form $ax + by + cz = 0$ since the values $x=0, y=0, z=0$ must satisfy the eq.
2. The x-y plane passes through the origin and has normal vector k $(z=0)$
   y-z plane $\longrightarrow x=0$
   x-z plane $\longrightarrow y=0$
3. A plane parallel to the x-y plane must have a standard eq. $z=d$ since it has a normal vector k.
   A plane parallel to the y-z plane has the eq. $x=d$
   A plane parallel to the x-z plane has the eq. $y=d$

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12.5 Homework Exercises: 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

12.5

(1) $n = \langle 1,3,2 \rangle$ $(4,-1,1)$

$ax + by + cz = d$
$x + 3y + 2z = d$
$4 + 3(-1) + 2(1) = d$
$3 = d$

$\therefore$ The equation of the plane w/ normal vector n passing through the given point in scalar form is $x + 3y + 2z = 3$.

(5) $n = i$ $(3,1,-9)$
$\hookrightarrow d = 3$
$\hookrightarrow x = d$ so $x = 3$

(9) The equation of any plane through the origin:
$\Rightarrow d = 0$ so $x + y + z = 0$

(11) A plane that is parallel to the yz-plane:
~ $n = \langle 1,0,0 \rangle$ is a normal vector
~ The equation has the form $x = d$

(13) $9x - 4y - 11z = 2$ Vector normal to this plane
$\hookrightarrow \langle 9,-4,-11 \rangle$

(15) A vector normal to this plane:
$3(x-4) - 8(y-1) + 11z = 0$
$3x - 12 - 8y + 9 + 11z = 0$
$3x - 8y + 11z = 3 \longrightarrow \langle 3,-8,11 \rangle$

(17) An eq. of the plane passing through these points:
$P = (2,-1,4)$ $\quad PQ = \langle -1,2,-3 \rangle$
$Q = (1,1,1)$ $\quad PR = \langle 1,2,-6 \rangle$
$R = (3,1,-2)$

$$\text{x product} = \begin{vmatrix} -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix} = i\begin{vmatrix} 2 & -3 \\ 2 & -6 \end{vmatrix} - j\begin{vmatrix} -1 & -3 \\ 1 & -6 \end{vmatrix} + k\begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix}$$

$= -18i - 9j - 4k = \langle -6,-9,-4 \rangle = \langle a,b,c \rangle$

$\therefore \Rightarrow P = (2,-1,4)$
$-6(x-2) + -9(y+1) - 4(z-4) = 0$
$-6x + 12 - 9y - 9 - 4z + 16 = 0$ (just dividing the whole thing by -1)
$6x + 9y + 4z = 19$

(19) $P = (1,0,0)$ $\quad PQ = \langle -1,1,1 \rangle$
$Q = (0,1,1)$ $\quad PR = \langle 1,0,1 \rangle$
$R = (2,0,1)$

$$\text{x Product} = \begin{vmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = i + 2j + k = \langle 1,2,1 \rangle = \langle a,b,c \rangle$$

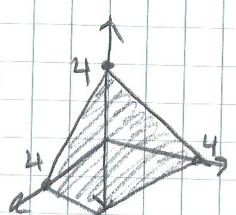
*watch your cross product results!

$(x-1) + 2(y-0) + z = 0$
$x + 2y - z = 1$

(25) Equation of the plane with the given:
Passes through $(-2,-3,5)$ and has normal vector $i + k$

$\vec{v} = \langle 1,0,1 \rangle$ and $P = (-2,-3,5)$
$(0,0,0)$ $\therefore = 1(x+2) + 0(y+3) + 1(z-5) = 0$
$x + z = 3$

(31) Drawing the plane: $x + y + z = 4$



(53) Find all planes in $\mathbb{R}^3$ whose intersection with the xz plane is the line $3x + 2z = 5$
?
$\downarrow y = 0$?
$\hookrightarrow (3\lambda)x + by + (2\lambda)z = 5\lambda$
where $\lambda \neq 0$

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13.1 Homework Exercises: 5, 17
13.2 Homework Exercises: 3, 5, 7, 15, 31, 33, 41, 49

(5) Finding a vector parametrization of the line through $P=(3,-5,7)$ and direction $v=\langle 3,0,1\rangle$.

$v=\langle 3,0,1\rangle \quad \langle a,b,c\rangle$
$P=(3,-5,7) \quad \langle x_0,y_0,z_0\rangle$
$r(t)=3(3+3t)i-5j+(7+t)k$

$y_0 \quad t \quad a$

★ $r(t)=a(x_0+at)i+b(y_0+bt)+c(z_0+ct)$

(17) $r(t)=(9\cos t)i+(9\sin t)j$
$x=9\cos t \quad$ radius $=9$
$y=9\sin t \quad$ center $=$ origin
(nothing is being added to x and y).

* Lies in the xy plane since $k=0$

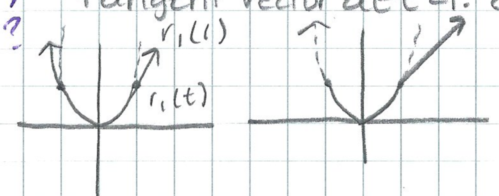
13.2 Homework Exercises:

(3) $\lim_{t\to 0} e^{2t}i+\ln(t+1)j+4k$
$= i+\ln 1 j+4k$
$= i+0j+4k = i+4k$

(5) $\lim_{h\to 0}\dfrac{r(t+h)-r(t)}{h}$ for $r(t)=\langle t^{-1},\sin t,4\rangle$
$=\langle -\frac{1}{t^2},\cos t,0\rangle$
* just find the derivatives

(7) $r(t)=\langle t,t^2,t^3\rangle$
$r'(t)=\langle 1,2t,3t^2\rangle$

(15) Sketching the curve parametrized by $r_1(t)=\langle t,t^2\rangle$ together with its tangent vector at $t=1$. and $r_2(t)=\langle t^3,t^6\rangle$.
?



(31) $r(t)=\langle 1-t^2,5t,2t^3\rangle,\ t=2$
?
$r(2)=\langle -3,10,16\rangle$
$r'(t)=\langle 2t,5,6t^2\rangle$
$r'(2)=\langle 4,5,24\rangle$
$\ell(t)=\langle 1-t^2,5t,2t^3\rangle+(t-2)(4i+5j+24k)$
$\ell(t)=\langle -3-4t,10+5t,16+24t\rangle+($

$r(2)=2r\frac{1}{3}k$
(33) $r(s)=4s^{-1}i-\frac{8}{3}s^{-3}k,\ s=2$
?
$r'(s)=-4s^{-2}i+8s^{-4}k$
$r'(2)=-1i+k$
$\ell(t)=(2i-\frac{1}{3}k)+t\langle -1,0,k\rangle$
$=(2i-\frac{1}{3}k)+\langle -t,0,tk\rangle$
$\ell(t)=\langle 2-t,0,-\frac{1}{3}+\frac{1}{2}t\rangle$

$\int_a^a=0$
(41) $\int_{-2}^{2}(u^3i+u^5j)\,du=0\to\langle 0,0\rangle$
$=\frac{u^4}{4}+\frac{u^6}{6}\Big|_{-2}^{2}=\left(\frac{2^4}{4}+\frac{2^2}{6}\right)-\left(^{-2}\right.$

(49) $r'(t)=t^2i+5tj+k$
$r(1)=j+2k \Rightarrow\langle 0,1,2\rangle$
Finding the general sol'n of the differential equation and the sol'n with the given initial condition.
(1) $\int r'(t)=\frac{1}{3}t^3i+\frac{5t^2}{2}j+tk+c$
Integrate $r(t)$
with initial conditions:
$r(t)=(\frac{1}{3}t^3-\frac{1}{3})i+(\frac{5}{2}t^2-\frac{3}{2})j+(t+1)k$