12.3 9/20/20

1. $\langle 1, 2, 1\rangle \cdot \langle 4,3,5\rangle = 1\cdot 4 + 2\cdot 3 + 1\cdot 5 = 15$

13. $\dfrac{\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle}{|\sqrt{1^2+1^2+1^2}|\,|\sqrt{1^2+(-2)^2+(-2)^2}|} = \dfrac{-3}{\sqrt{3}\cdot\sqrt{9}} = \dfrac{-1}{\sqrt{3}}$ not orthogonal

$\cos\theta = \frac{-1}{\sqrt{3}}$

$\theta = 125.26°$ obtuse

21. $\dfrac{\langle 1,0,1\rangle \cdot \langle 0,1,2\rangle}{|\sqrt{1^2+0^2+1^2}|\,|\sqrt{0^2+1^2+2^2}|} = \dfrac{2}{\sqrt{2}\cdot\sqrt{5}} = \cos\theta$

$\cos\theta = \frac{2}{\sqrt{10}}\cdot\frac{\sqrt{10}}{\sqrt{10}} = \frac{2\sqrt{10}}{10}$

29. a. $\langle b,3,2\rangle \cdot \langle 1,b,1\rangle = 0$

$b + 3b + 2 = 0$

$4b = -2$

$b = -\frac{1}{2}$

b. $\langle 4,-2,7\rangle \cdot \langle b^2, b, 0\rangle = 0$

$4b^2 + (-2b) = 0$

$2b(2b-1) = 0$

$2b = 0$ $\quad 2b-1=0$

$b=0$ $\quad 2b = 1$

$b = \frac{1}{2}$

31. $\langle a,b,c\rangle \cdot \langle 2,0,-3\rangle = 0$

$2a - 3c = 0$

$2a = -3c$

$a = \frac{-3}{2}c$

$\langle \frac{3}{2}, 0, 1\rangle \quad \langle 3, 12, 2\rangle$

57. $V = \dfrac{A\cdot B}{|B^2|}\cdot B$

$V = \dfrac{\langle 5,7,-4\rangle \cdot \langle 0,0,1\rangle}{|1^2|}\cdot 1 = -4$

63. $8^2 + 2^2 = c^2$

$c = \sqrt{20}$

$\overline{OP} = 8.246$

12.4 9/20/20

1. $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 1\cdot 3 - 2\cdot 4 = 3-8 = -5$

5. $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1(-3\cdot 1 - 0\cdot 0) - 2(4\cdot 1 - 1\cdot 0) + 1(-3\cdot 1 - 4\cdot 0)$
$= -3 - 8 + 3 = -14$

13. $(i+j) \times k = i \times k + j \times k$

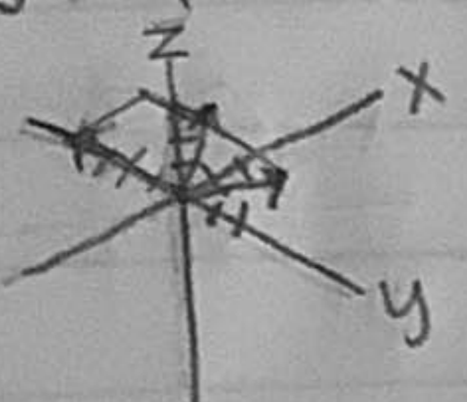
21. $(u-2v) \times (u+2v)$
$(u-2v) \times u + (u-2v) \times 2v$
$u \times u - 2v \times u + 2u \times v - 4v \times v$
$0 + 4u \times v + 0 = \langle 4,4,0 \rangle$

25. $-u$

27. $u = v \times w$
$u = |v||w| \sin\theta$
$u = |\sqrt{3^2+0^2+0^2}||\sqrt{0^2+1^2+1^2}| \sin\frac{\pi}{2}$
$u = |\sqrt{3}||\sqrt{2}|(1)$
$u = \sqrt{6}$

$\cos\theta = \frac{v \cdot w}{|v||w|}$
$\cos\theta = \frac{\langle 3,0,0 \rangle \cdot \langle 0,1,-1 \rangle}{|\sqrt{3^2+0^2+0^2}||\sqrt{0^2+1^2+1^2}|}$
$\cos\theta = 0$
$\theta = \frac{\pi}{2}$

39. $u = \langle 2,2,1 \rangle \quad v = \langle 1,0,3 \rangle \quad w = \langle 0,-4,0 \rangle$



$|u \cdot (v \times w)| = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 0 & -4 & 0 \end{vmatrix} = (0+12)i - (0)j + (-4)k$
$= 12i - 4k$
$\langle 2,2,1 \rangle \cdot \langle 12,0,-4 \rangle$
$24 - 4 = 20$

41. $u \times v = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \langle 0-3, 6-1, 1-0 \rangle = \langle -3, 5, 1 \rangle$
$\sqrt{(-3)^2+5^2+1^2} = \sqrt{35}$

43. $\overline{OP} = \langle 3,3,0 \rangle \quad \overline{OQ} = \langle 0,3,3 \rangle$

$\overline{OP} \times \overline{OQ} = \langle 3\cdot 3 - 3\cdot 0, 0\cdot 0 - 3\cdot 3, 3\cdot 3 - 0\cdot 3 \rangle = \langle 9,-9,9 \rangle$

43. cont $\sqrt{9^2+9^2+9^2} = \sqrt{243} \rightarrow$ area of parallelogram

$\frac{\sqrt{243}}{2}$

45. $\overset{P}{(1,2)}, \overset{Q}{(3,4)}, \overset{R}{(-2,2)}$

$PQ = (3,4) - (1,2) = \langle 2,2 \rangle$

$PR = (-2,2) - (1,2) = \langle -3,0 \rangle$

$PQ \times PR = \begin{vmatrix} 2 & 2 \\ -3 & 0 \end{vmatrix} = 0 - (-6) = 6$

$\frac{6}{2} = 3 \rightarrow$ area of Δ

12.5. 9/20/20

1. $n = \langle 1, 3, 2 \rangle \quad (4, -1, 1)$

$1x + 3y + 2z = 1 \cdot 4 + 3 \cdot (-1) + 2 \cdot 1$

$x + 3y + 2z = 3$

5. $n = \langle 1, 0, 0 \rangle \quad (3, 1, -9)$

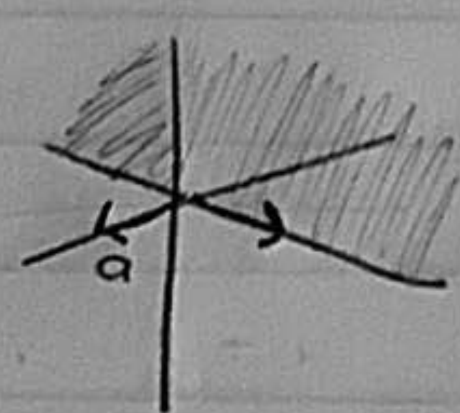
$x = 3 + 0 + 0$

$x = 3$

9. $n = \langle 0, 0, 1 \rangle \quad (0, 0, 0)$

$z = 0$

11. $\parallel$ to y-z plane



a. true

b. false

c. $n = \langle 0, 0, 1 \rangle \quad (0, 0, 0)$

$z = 0$

false

d. true $\quad z = 0, \; x = 0$

13. $\langle 9, -4, -11 \rangle$

15. $\langle 3, -8, 11 \rangle$

17. $P = (2, -1, 4) \quad Q = (1, 1, 1) \quad R = (3, 1, -2)$

$PQ = (1, 1, 1) - (2, -1, 4) = (-1, 0, -3)$

$PR = (3, 1, -2) - (2, -1, 4) = (1, 2, -6)$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ -1 & 0 & -3 \\ 1 & 2 & -6 \end{vmatrix} = \langle 0 + 6, -3 - 6, -2 - 0 \rangle =$$

$n = \langle 6, -9, -2 \rangle$

$6(x - 2) - 9(y + 1) - 2(z - 4) = 0$

$6x - 9y - 2z = 13$

19. $P=(1,0,0)$ $\quad Q=(0,1,1)$ $\quad R=(2,0,1)$

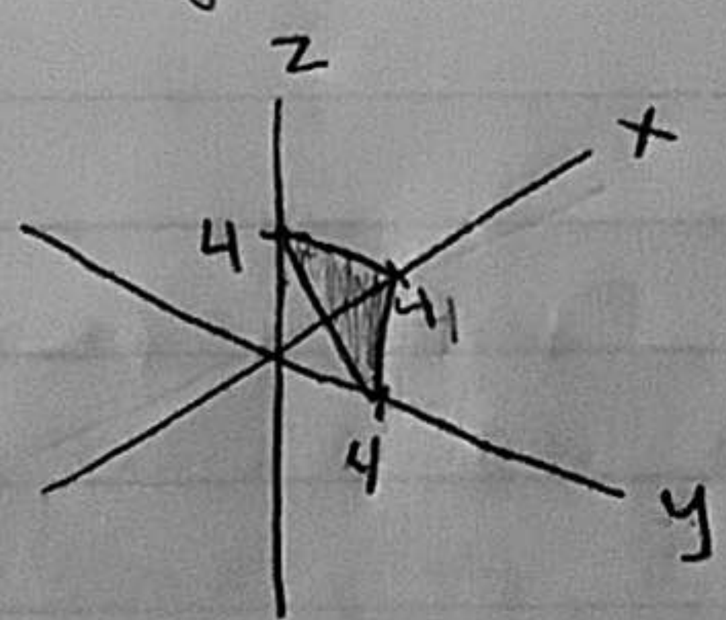
$PQ = (0,1,1)-(1,0,0) = \langle -1,1,1\rangle$
$PR = (2,0,1)-(1,0,0) = \langle 1,0,1\rangle$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \langle 1-0,\ 1+1,\ 0-1\rangle = \langle 1,2,-1\rangle$$

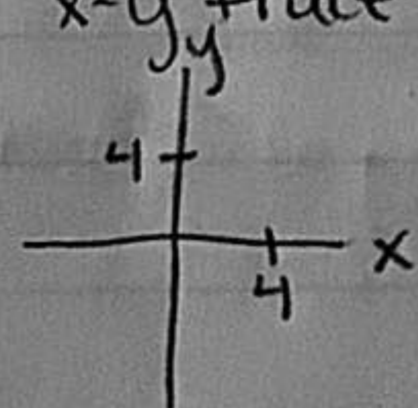
$1(x-1) + 2(y-0) - 1(z-0) = 0 \qquad x+2y-z=1$

25. $n=\langle 1,0,1\rangle \qquad (-2,-3,5)$
$x+z = -2+0+5$
$x+z=3$

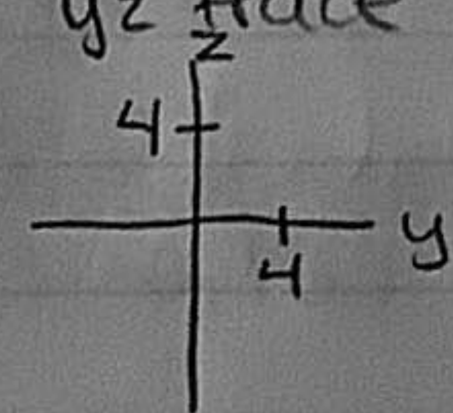
31. $x+y+z=4$



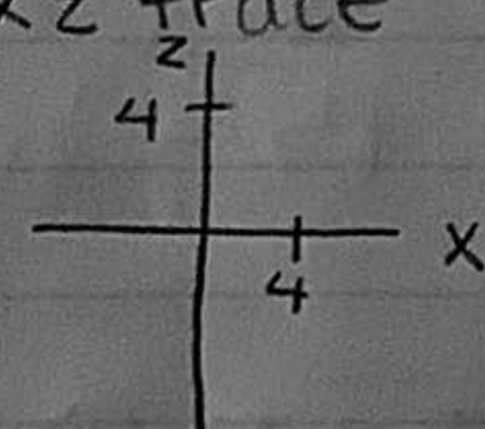
$z=0$
$x+y=4$
x-y trace



$x=0$
$y+z=4$
yz trace



$y=0$
$x+z=4$
xz trace



53. all planes in R^3 whose intersection with xy plane is in line with $3x+2z=5$

when $z=0$ need to be in line with $3x+2z=5$

intersection of plane and xy is at $ax+by=d$

$3x+by+2z=5$, b is anything

13.1 9/20/20

5. vector parametrization $P = (3, -5, 7)$ $v = \langle 3, 0, 1 \rangle$

$\langle 3, -5, 7 \rangle + t\langle 3, 0, 1 \rangle = \langle 3t+3, -5, 7+t \rangle$

17. $r(t) = (9\cos t)i + (9\sin t)j$, find r, center, plane

$\langle 0, 0, 0 \rangle + 9\langle \cos t, \sin t, 0 \rangle$

center $= \langle 0, 0, 0 \rangle$ radius $= 9$ plane $z = 0$

13.2 9/20/20

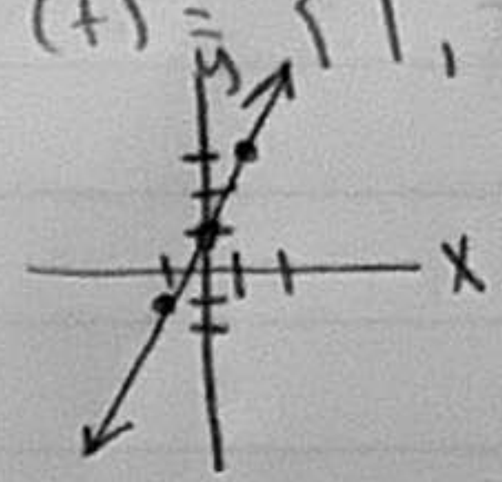
3. $\lim_{t \to 0} e^{2t} i + \ln(t+1) j + 4k = e^0 i + \ln(1) j + 4k$
$= i + 4k$

5. $\lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$, $r(t) = \langle t^{-1}, \sin t, 4 \rangle$

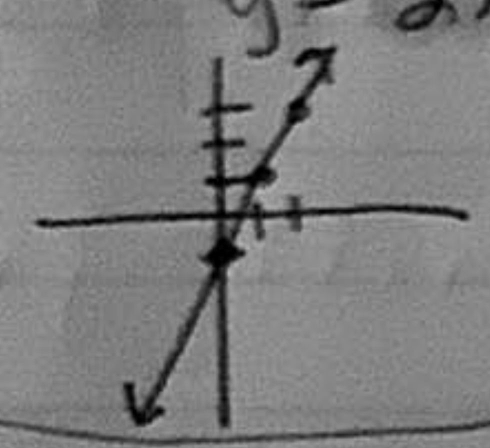
$r'(t) = \langle -t^{-2}, \cos t, 0 \rangle$

7. $r(t) = \langle t, t^2, t^3 \rangle$
$r'(t) = \langle 1, 2t, 3t^2 \rangle$

15. $r_1(t) = \langle t, t^2 \rangle$ tangent $t=1$, $r_2(t) = \langle t^3, t^6 \rangle$
$r_1'(t) = \langle 1, 2t \rangle$
$r'(1) = \langle 1, 2 \rangle$
$r(1) = \langle 1, 1 \rangle$
$\langle 1,1 \rangle + t\langle 1, 2 \rangle = \langle t+1, 2t+1 \rangle$
$t = x-1$ $y = 2(x-1)+1$
$y = 2x+1$



$r_2'(t) = \langle 3t^2, 6t^5 \rangle$
$r_2'(1) = \langle 3, 6 \rangle$
$r_2(1) = \langle 1, 1 \rangle$
$\langle 1,1 \rangle + t\langle 3,6 \rangle = \langle 3t+1, 6t+1 \rangle$
$t = \frac{x-1}{3}$ $y = 6\left(\frac{x-1}{3}\right)+1$
$y = 2x-1$



31. $r(t) = \langle 1-t^2, 5t, 2t^3 \rangle$, $t=2$
$r'(t) = \langle -2t, 5, 6t^2 \rangle$ $\langle -3, 10, 16 \rangle$
$r'(2) = \langle -4, 5, 24 \rangle$

$\langle -3, 10, 16 \rangle + t\langle -4, 5, 24 \rangle = \langle -3-4t, 10+5t, 16+24t \rangle$

$x(t) = -4t-3$ $y(t) = 5t+10$ $z(t) = 24t+16$

33. $r(s) = 4s^{-1} i - \frac{8}{3} s^{-3} k$ $s=2$
$r(s) = \langle 4s^{-1}, 0, -\frac{8}{3}s^{-3} \rangle$ $r(2) = \langle 2, 0, -\frac{1}{3} \rangle$
$r'(s) = \langle -4s^{-2}, 0, 8s^{-4} \rangle$ $r'(2) = \langle -1, 0, \frac{1}{2} \rangle$

$\langle 2, 0, -\frac{1}{3} \rangle + t\langle -1, 0, \frac{1}{2} \rangle = \langle 2-t, 0, -\frac{1}{3} + \frac{t}{2} \rangle$

$x(t) = 2-t$ $y(t) = 0$ $z(t) = \frac{t}{2} - \frac{1}{3}$

41. $\int_{-2}^{2} (u^3 i + u^5 j)\, du$

$\frac{u^4}{4} i + \frac{u^6}{6} j \Big|_{-2}^{2}$

$\left(\frac{2^4}{4} i + \frac{2^6}{6} j\right) - \left(\frac{-2^4}{4} i + \frac{-2^6}{6} j\right) =$

$\left(4i + \frac{32}{3} j\right) - \left(-4i + \frac{-32}{3} j\right) = 8i + \frac{64}{3} j$

49. $r'(t) = t^2 i + 5t j + k$ $\qquad r(1) = j + 2k$

$r(t) = \int t^2 i + 5t j + k$

$r(t) = \frac{t^3}{3} i + \frac{5t^2}{2} j + tk + C$

$j + 2k = \frac{1^3}{3} i + \frac{5(1)^2}{2} j + 1k + C$

$C = -\frac{1}{3} i - \frac{3}{2} j + k$

$r(t) = \left(\frac{t^3}{3} - \frac{1}{3}\right) i + \left(\frac{5t^2}{2} - \frac{3}{2}\right) j + (t+1) k$