Fayed Raza
9/20/2020

12.3: 1, 13, 21, 29, 31, 37, 63

1. $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle$

$4\cdot 1 + 3\cdot 2 + 5\cdot 1 = 4+6+5 = 15$

13 $\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle \quad 1-2-2=-3$

$-3 \neq 0$ so its not orthogonal
and angle is obtuse

$1^2+1^2+1^2 = 1+1+1$

$\cos\Phi = \frac{u\cdot v}{\|u\|\|v\|} = \frac{-3}{3\sqrt{6}}$

$\sqrt{6}$

$1^2+4+4 = \sqrt{9}$

$= -\frac{1}{\sqrt{6}}$

$\cos^{-1}\left(-\frac{1}{\sqrt{6}}\right) = 114^\circ > 90^\circ$

21. $i+j$
$\langle 1,1,0\rangle$

$\langle 1,1,0\rangle \langle 0,1,2\rangle$

$j+2k \; \langle 0,1,2\rangle$

$0+1+0$

$\cos \frac{1}{(\sqrt{2})(\sqrt{5})} = \frac{1}{\sqrt{10}}$

29.

a. $\langle b, 3, 2\rangle \cdot \langle 1, b, 1\rangle$

$$b + 3b + 2 = 0$$

$$4b + 2 = 0$$

$$\frac{4b}{4} = \frac{-2}{4}$$

$$b = -\frac{1}{2}$$

b. $\langle -4, 2, 1\rangle \cdot \langle b^2, b, 0\rangle$

$$-4b^2 + 2b + 0 = 0$$

$$b(-4b + 2) = 0$$

$$-4b + 2 = 0$$

$$\frac{-4b}{-4} = \frac{-2}{-4}$$

$$b = 0$$

or

$$b = \frac{1}{2}$$

3) $\langle 2, 0, -3 \rangle \quad \langle i, j, k \rangle$

$2i + 0 + -3k = 0$

$\frac{2i}{2} = \frac{3k}{2}$

$k = 1 \rightarrow i = \frac{3}{2}$

$\langle \frac{3}{2}, 0, 1 \rangle$

$2(0) + 0 + 3(0) = 0$

$0 + 0 + 0 = 0$

$i = \frac{3}{2}(0) = 0$

$2(0) + 0j + 0 = 0$

$0j = 0$

$j = 1$

$u_1 = \langle \frac{3}{2}, 0, 1 \rangle$

$u_2 = \langle 0, 1, 0 \rangle$

57.

$u = 5i + 7j - 4k \rightarrow \langle 5, 7, -4 \rangle$

$\frac{u \cdot v}{\|v\|^2} \cdot v \qquad v = \langle 0, 0, 1 \rangle$

$\langle 5, 7, -4 \rangle \langle 0, 0, 1 \rangle$

$0 + 0 + -4 = -4$

$\frac{-4}{1} \langle 0, 0, 1 \rangle \qquad \|v\|^2 = \sqrt{0^2 + 0^2 + 1^2}^{\,2}$

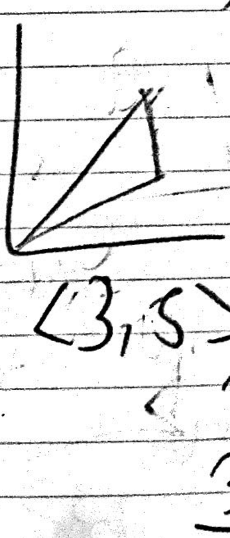
$\|v\|^2 = 1$

$\langle 0, 0, -4 \rangle$

$i \quad j \quad k$

$-4k$

63



$\sqrt{17}$

projection of u onto v

$\langle 3, 5 \rangle \cdot \langle 8, 2 \rangle \qquad \frac{u \cdot v}{\|v\|^2} \|v\|$

$24 + 10$

$\frac{34}{\sqrt{68}} \langle 8, 2 \rangle \qquad \sqrt{64 + 4} = \sqrt{68}$

$\frac{\sqrt{68}}{2} = \frac{\sqrt{4}\sqrt{17}}{2} = \frac{2\sqrt{17}}{2} = \sqrt{17} \qquad \frac{34}{\sqrt{68}} \cdot \sqrt{68} = \frac{34}{\sqrt{68}} \cdot \frac{\sqrt{68}}{\sqrt{68}} = \frac{34\sqrt{68}}{68} = \frac{\sqrt{68}}{2}$

12.4 : 1, 5 > 13, 21, 25, 27, 37

41, 43, 45

1. $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$ = 5. $1(-3-0)$

$3-8 = -5$ $\quad -2[4-0]$

$+ 1[0-(-3)]$

$-3-8+3$

-8

13 $(i+j)\times k = ik + jk$

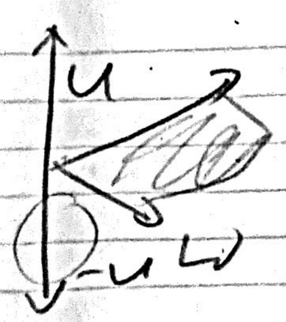
$i + j$

21. $u^2 + 2uv - 2uv - 2v^2$

$u^2 + 2\langle 1,1,\cancel{0}\rangle - 2\langle 1,1,0\rangle - \cancel{2v^2}$

$u^2 - 4v^2$ $\quad \langle 1, -4, 0\rangle$

25. $-u$



$-u$

27. $\begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 0 & 1 & -1 \end{vmatrix}$

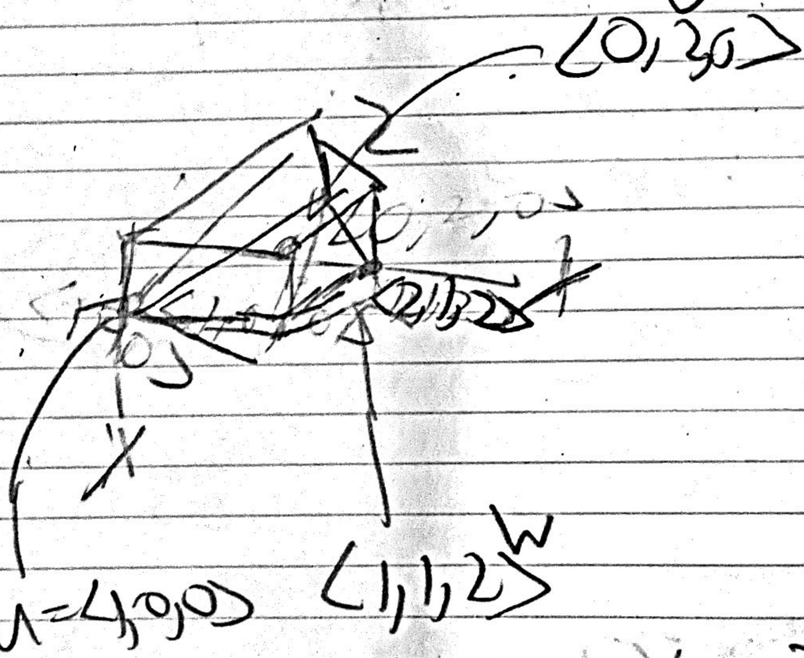
$3\begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix} - j[-3-0]$ $[3-0]k$

$3[0-0] \quad +3i \quad +3k$

$0+3i+3k$

$\langle 0,3,3\rangle$

39.



$\begin{vmatrix} i & j & k \\ 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{vmatrix}$

$1\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} - 0 + 0$

$1[4-0]$

$4-0+0$

4

41.

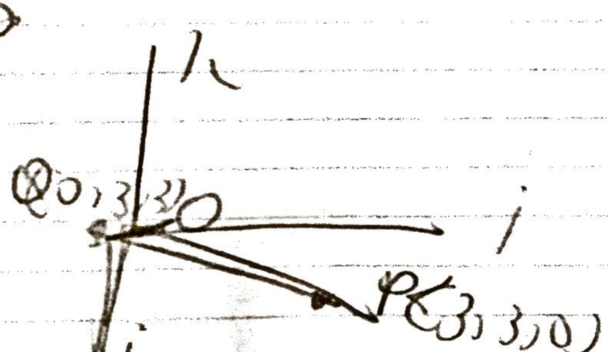
$$\begin{array}{ccc} i & j & k \end{array}$$
$$\begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix}$$

$$i[0-3] - j[1-6] + k[1-0]$$

$$= -3i + 5j + 1k = 7$$

$$\sqrt{3^2+5^2+1} = \sqrt{35} = 5.92$$

43.



$$\begin{array}{ccc} i & j & k \end{array}$$
$$\begin{matrix} 3 & 3 & 0 \\ 0 & 3 & 3 \end{matrix}$$

$$i[9-0] - j[9-0] + k[9-0]$$

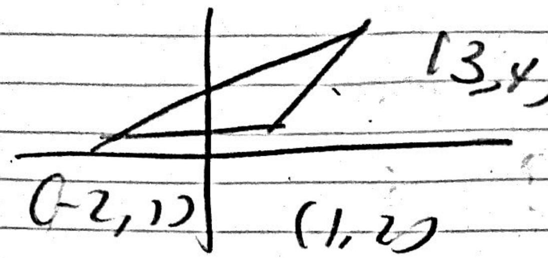
$$9i - 9j + 9k$$

$$\sqrt{9^2+9^2+9^2}$$

$$\sqrt{81+81+81}$$

$$\frac{\sqrt{243}}{2} = 7.8$$

45



$$\begin{array}{ccc} i & j & k \\ 2 & 8 & 2 \\ 1 & 3 & -2 \end{array}$$

$i(-8-6) - j(-4-2) + 6-4k$

$-14i + 6j + 2k$

$\sqrt{14^2+6^2+2^2} = 15.4$

12.5: 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

1.

$x + 3y + 2z = 3$

$\langle 1, -1, 1\rangle \cdot \langle 1, 3, 2\rangle$

$4 - 3 + 2$

$1+2$

5. $n = i$

$\langle 1,0,0\rangle$

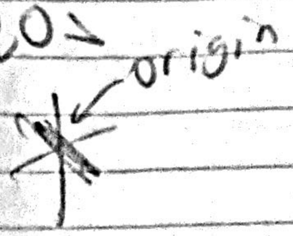
$x = 3$

$\langle 1,0,0\rangle \langle 3,1,-9\rangle$

$3+0+0 = 3$

9. Origin $\langle 0,0,0 \rangle$

$X=0$



11.

a False

b True

c False

d True

13 $\langle 9, -8, -11 \rangle$

15. $3(x-4) - 8(y-1) + 11z = 0$

$3x - 12 - 8y + 8 + 11z$

$3x - 8y + 11z = 4$ $\quad \langle 3, -8, 11 \rangle$

17. PQ $(1-2, 1-(-1), 1-4) = (3, 2, 5)$

QR $(3-1, 1-1, -2-1) = (2, 0, -3)$

$$\begin{array}{ccc} i & j & k \\ 3 & 2 & 5 \\ 2 & 0 & -3 \end{array}$$

$-6i - (-9-10)j + k(0-4)$

$-6i + 19j - 4k$

$(-6, 19, -4)$

19. $PQ = (-1, 1, 1)$
$PR = (2, -1, 0)$

$$\begin{array}{ccc} i & j & k \\ -1 & 1 & 1 \\ 2 & -1 & 0 \end{array}$$

$i(0 - -1)$ $0-2$ $1-2$

i $-2j$ k

$(1, -2, -1)$

25. $\langle -2, -3, 5 \rangle$ $\langle 1, 0, 1 \rangle$

$\langle -2, -3, 5 \rangle$ $\langle 1, 0, 1 \rangle$

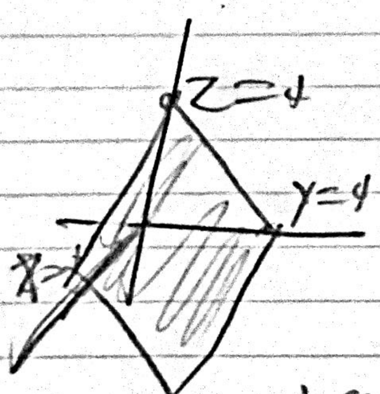
$(-2, 3, 5) * (i + 0 + j)$

$4 + 9 + 25$

$(-2x) + (3) + (5z) = 0$

$-2x + 5z = -3$

31.



$x = 4$
$y = 4$
$z = 4$

53. becomes zero

$ax + by + cz = d$

$az + cz = d$

$a = 3x$ $c = 2x$ $d = 5x$

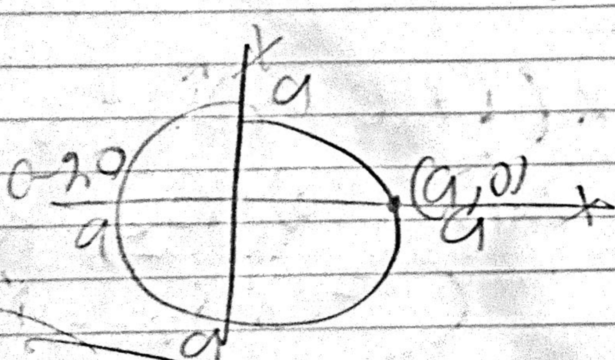
$(3x)x + bx + (2x)z = 5x$

13, 1 5, 17

5. $\langle 3, -5, 1 \rangle + t\langle 3, 0, 1 \rangle$

$r(t) = (3+3t)i - 5j + (1+t)k$

17. $r(t) = (9\cos t)i + (9\sin t)j$



Radius: 9

Center (0, 0)

Plane: XY Plane

13.2 : 3, 5, 7, 15, 31, 33, 41, 49

3 $\lim_{t \to 0} e^{2t} i + \ln(t+1) j + 4k$

$e^0 + \ln(1) + 4k$

$1 + \ln(1) + 4k$ $i + 4k$

5 $\lim_{h \to 0} \frac{r(t+h) - r(t)}{h}$

$\lim_{h \to 0} \frac{\frac{1}{t+h} - \frac{1}{t}}{h}$

$\lim_{h \to 0} \frac{1}{h(t+h)} - \frac{t+h}{h(t+h)}$

$\lim_{t \to 0} \frac{\sin(t+h)}{h}$

$\lim_{h \to 0} \frac{\cancel{t}+h}{h(t+h)} = \frac{1}{h} = 1$

$\langle 1, 1, 0 \rangle$

$\lim_{t \to 0} \frac{\sin(t+h)}{h}$

$\lim_{h \to 0} \frac{4-4}{h} = \frac{0}{h} = 0$

7. $\frac{dr}{dt} = \langle 1, 2t, 3t^2 \rangle$

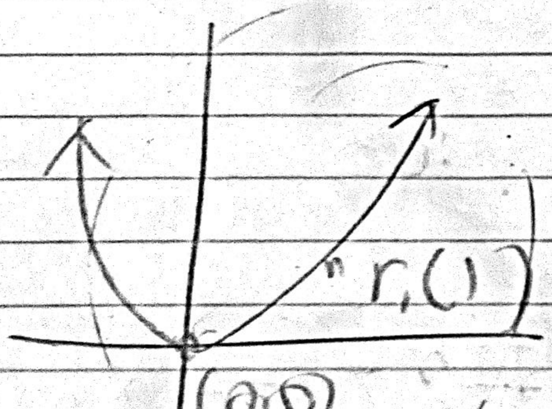
15.

$x(t) = t$

$y(t) = t^2$

$X(t) = t^3$

$x = t^6$

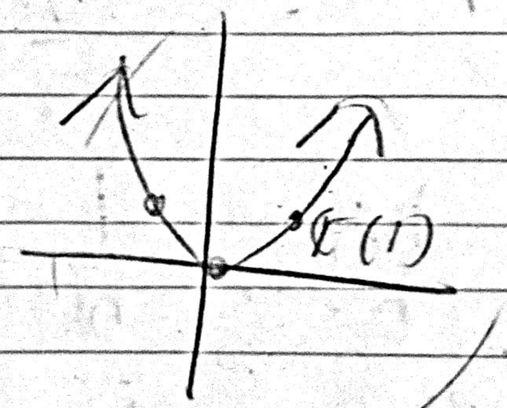


$\phi(0) = 0$

$y(0) = 0$

$x = 1$

$y =$



31.

$X(t) = 1 - t^2$

$Y(t) = 5t$

$z(t) = 2t^3$

$r'(t) = \langle -2t, 5, 6t^2 \rangle$

$r(2) = \langle -3, 10, 16 \rangle$

$r'(2) = \langle -4, 5, 24 \rangle$

$l(t) = \langle -3 - 4t, 10 + 5t, 16 + 24t \rangle$

33

$r(t) = \langle 4s^{-1}, 0, -\frac{8}{3}s^{-3} \rangle$ $l(t) = \langle 2 - t, 0, -\frac{1}{3} - \frac{t}{2} \rangle$

$r(\frac{1}{2}) = \langle 2, 0, -\frac{1}{3} \rangle$

$r'(t) = \langle -4s^{-2}, 0, -8s^{-4} \rangle$

$r'(2) = \langle -1, 0, -\frac{1}{2} \rangle$

41. $\frac{u^4}{4} + \frac{u^6}{6} \Big|_{-2}^{2}$

$\langle u^3, u^5 \rangle$

$\cancel{\left(\frac{2^4}{4} + \frac{2^6}{6}\right)} - \cancel{\left(\frac{2^4}{4} + \frac{-2^6}{6}\right)}$

$\langle \frac{u^4}{4} \Big|, \frac{u^6}{6} \Big|_{-2}^{2} \rangle$

$\langle 0, 0 \rangle$

49. $r(1) = j + 2k$

$r(t) = \frac{t^3 i}{3} + \frac{5t^2 j}{2} + \frac{k^2}{2} + C$

$j + 2k = \frac{1}{3} i + \frac{5}{2} j + \frac{1}{2} k + C$

$-\frac{1}{3} i - \frac{3}{2} j - \frac{3}{2} k = C$

$r(t) = \cancel{\frac{1}{3} i} - \cancel{\frac{1}{3} i} + \frac{5}{2} j - \frac{3}{2} j + \frac{1}{2} k - \frac{3}{2} k$

$r(t) = 2j - k$