12.3 Homework

① $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = 1\cdot 4 + 2\cdot 3 + 1\cdot 5 = 4+6+5 = \boxed{15}$

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⑬ $\langle 1,1,1\rangle,\ \langle 1,-2,-2\rangle$

$\rightarrow \cos\theta = |A\cdot B| / |A||B|$

$\rightarrow \langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle = 1\cdot 1 + 1\cdot(-2) + 1\cdot(-2) = 1-2-2 = \underline{-3}$

$\rightarrow |A| = \sqrt{1^2+1^2+1^2} = \underline{\sqrt{3}}$

$\rightarrow |B| = \sqrt{1^2+(-2)^2+(-2)^2} = \sqrt{9} = \underline{3}$

$\rightarrow \cos\theta = \frac{-\cancel{3}}{\cancel{3}\sqrt{3}} \Rightarrow \cos\theta = \frac{-1}{\sqrt{3}} \Rightarrow \theta = \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right) \Rightarrow \underline{\theta = 125.26^\circ}$

$\rightarrow$ The angle between these two vectors $\langle 1,1,1\rangle$ and $\langle 1,-2,-2\rangle$ is $\boxed{\text{obtuse}}$.

㉑ $i+j,\ j+2k \rightarrow \langle 1,1,0\rangle, \langle 0,1,2\rangle$

$\rightarrow \cos\theta = |A\cdot B| / |A||B|$

$\rightarrow \langle 1,1,0\rangle \cdot \langle 0,1,2\rangle = 1\cdot 0 + 1\cdot 1 + 0\cdot 2 = 0+1+0 = \underline{1}$

$\rightarrow |A| = \sqrt{1^2+1^2+0^2} = \sqrt{2}$

$\rightarrow |B| = \sqrt{0^2+1^2+2^2} = \sqrt{5}$

$\rightarrow \cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{5}} \Rightarrow \boxed{\cos\theta = \frac{1}{\sqrt{10}}}$

㉙

(a) $\langle b,3,2\rangle,\ \langle 1,b,1\rangle$

$\rightarrow \langle b,3,2\rangle \cdot \langle 1,b,1\rangle = b+3b+2 = 0 \rightarrow 4b = -2 \rightarrow \boxed{b = -\frac{1}{2}}$

(b) $\langle 4,-2,7\rangle,\ \langle b^2,b,0\rangle$

$\rightarrow \langle 4,-2,7\rangle \cdot \langle b^2,b,0\rangle = 4b^2-2b = 0 \rightarrow 2b(2b-1) = 0$

$2b = 0 \rightarrow \underline{b=0}$

$2b-1=0 \rightarrow \underline{b=\frac{1}{2}}$

$\boxed{b=0 \ \text{ or } \ b=\frac{1}{2}}$

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(31) Find two vectors that are not multiples of each other and are both orthogonal to $\langle 2,0,-3\rangle$.

$\rightarrow \langle 2,0,-3\rangle \cdot \langle a,b,c\rangle = 2a-3c=0 \rightarrow 2a=3c \rightarrow a=\frac{3c}{2}$

$\rightarrow a=\frac{3c}{2}$ where $c=1$ thus $a=\frac{3}{2} \rightarrow \boxed{\langle \frac{3}{2},1,1\rangle}$

$\rightarrow a=\frac{3c}{2}$ where $c=0$ thus $a=0 \rightarrow \boxed{\langle 0,0,0\rangle}$

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(57) $u=5i+7j-4k$, $v=k \longrightarrow u=\langle 5,7,-4\rangle$, $v=\langle 0,0,1\rangle$

$\rightarrow$ Equation: $u_{\parallel v} = \frac{u\cdot v}{\|v\|^2}\vec{v}$

$\rightarrow \langle 5,7,-4\rangle \cdot \langle 0,0,1\rangle = \underline{-4}$

$\rightarrow \|v\| = \underline{1}$

$\rightarrow \boxed{u_{\parallel v} = -4k}$

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(63) $u=\langle 3,5\rangle$, $v=\langle 8,2\rangle$

$\rightarrow$ Equation: $u_{\perp v} = \frac{u\cdot v}{\|v\|}$

$\rightarrow \langle 3,5\rangle \cdot \langle 8,2\rangle = 24+10 = \underline{34}$

$\rightarrow \|v\| = \sqrt{8^2+2^2} = \underline{\sqrt{68}}$

$\rightarrow u_{\perp v} = \frac{34}{\sqrt{68}} = \frac{34}{\sqrt{34}\sqrt{2}} = \frac{\sqrt{34}}{\sqrt{2}} = \sqrt{17}$

$\rightarrow \boxed{u_{\perp v} = \sqrt{17}}$

12.4 Homework

① $\det\left(\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}\right) = 1\cdot 3 - 2\cdot 4 = 3-8 = \boxed{-5}$

---

⑤ $\det\left(\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix}\right) = (1)\left[(-3\cdot 1) - \cancel{(0\cdot 0)}\right] - (2)\left[(4\cdot 1) - \cancel{(0\cdot 1)}\right] + (1)\left[\cancel{4\cdot 0} - (-3\cdot 1)\right]$

$\cancel{-3} - 8 \cancel{+3} = \boxed{-8}$

---

⑬ $(i+j)\times K = i\times K + j\times K = -j + i = \boxed{i-j}$

㉑ $(u-2v)\times(u+2v) = u\times(u+2v) + (-2v)\times(u+2v)$

$u\times u + u\times 2v - 2v\times u - 2v\times 2v$

$0 + 2(u\times v) - 2(v\times u) - 2(v\times v)$

$2\langle 1,1,0\rangle + 2\langle 1,1,0\rangle = 4\langle 1,1,0\rangle = \boxed{\langle 4,4,0\rangle}$

㉕ Which of u and $-u$ is equal to $v\times w$?

$\boxed{v\times w = -u}$

㉗ $v = \langle 3,0,0\rangle$ and $w = \langle 0,1,-1\rangle$

$\rightarrow u = v\times w$

$\rightarrow u = v\times w$ is orthogonal to v and w.

$\rightarrow$ Since v lies along the x-axis, u must lie in the yz-plane.

$\rightarrow u = \langle 0, b, c\rangle$

$\rightarrow u\cdot w = \langle 0,b,c\rangle \cdot \langle 0,1,-1\rangle = \cancel{0\cdot 0} + b - c \rightarrow b - c = 0 \rightarrow b = c$

$\rightarrow v = \langle 0, b, b\rangle$

$\rightarrow \|v\| = \sqrt{3^2+0^2+0^2} = 3$

$\rightarrow \|w\| = \sqrt{0^2+1^2+(-1)^2} = \sqrt{2}$

$\rightarrow \|u\| = \sqrt{0^2+b^2+b^2} = \sqrt{2b^2} = |b|\sqrt{2}$

$\rightarrow \|v\|\|w\| \sin\frac{\pi}{2} = \|u\|$

$\rightarrow 3\sqrt{2}\cdot 1 = |b|\sqrt{2} \rightarrow b = \pm 3$

$\rightarrow$ Right-hand rule shows that u is in z-direction

$\rightarrow \boxed{u = \langle 0,3,3\rangle}$

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(39) Sketch and compute the volume of the parallelepiped spanned by:

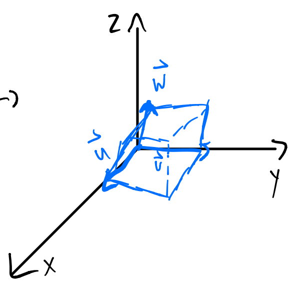
$u = \langle 1,0,0\rangle,\quad v = \langle 0,2,0\rangle,\quad w = \langle 1,1,2\rangle$

$\rightarrow V = |u\cdot(v \times w)|$

$\rightarrow \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} i - \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} j + \begin{vmatrix} 0 & 2 \\ 1 & 1 \end{vmatrix} k = 4i - 0j - 2k = \langle 4,0,-2\rangle$

$\rightarrow V = |\langle 1,0,0\rangle\cdot\langle 4,0,-2\rangle| \Rightarrow V = |(1\cdot 4)+(0\cdot 0)+(0\cdot -2)| \Rightarrow \boxed{V = 4 \text{ cubic units}}$

$\rightarrow$



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(41) Calculate the area of the parallelogram spanned by $u = \langle 1,0,3\rangle$ and $v = \langle 2,1,1\rangle$.

$\rightarrow \text{Area}(P) = \|v \times w\|$

$\rightarrow \begin{vmatrix} i & j & k \\ 2 & 1 & 1 \\ 1 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} i - \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} k = 3i - 5j - k = \langle 3,-5,-1\rangle$

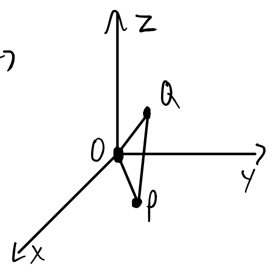
$\rightarrow \text{Area}(P) = \|\langle 3,-5,-1\rangle\| = \sqrt{9+25+1} = \boxed{\sqrt{35} \text{ squared units}}$

---

43. Sketch the triangle with vertices at the origin O, $P=(3,3,0)$ and $Q=(0,3,3)$, and compute its area using cross products.

$\rightarrow$ Area $(T) = \frac{1}{2}\|v \times u\|$

$\rightarrow$



$\rightarrow$ Vector $v = \overrightarrow{OP} = v = \langle 3, 3, 0 \rangle$

$\rightarrow$ Vector $u = \overrightarrow{OQ} = u = \langle 0, 3, 3 \rangle$

$\rightarrow \begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 3 & 3 \end{vmatrix} i - \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} j + \begin{vmatrix} 3 & 3 \\ 0 & 3 \end{vmatrix} k = 9i - 9j + 9k = \langle 9, -9, 9 \rangle$

$\rightarrow \frac{1}{2}\|\langle 9,-9,9\rangle\| = \frac{1}{2}\|\sqrt{81+81+81}\| = \frac{1}{2} \cdot 9\sqrt{3} = \boxed{\frac{9\sqrt{3}}{2} \text{ squared units}}$

---

45. Use cross products to find the area of the triangle in the xy-plane defined by $P(1,2)$, $Q(3,4)$, and $R(-2,2)$.

$\rightarrow$ Area $(T) = \frac{1}{2}\|v \times u\|$

$\rightarrow \overrightarrow{QP} = v = \langle -2, -2, 0 \rangle$

$\rightarrow \overrightarrow{QR} = u = \langle -5, -2, 0 \rangle$

$\rightarrow \begin{vmatrix} i & j & k \\ -2 & -2 & 0 \\ -5 & -2 & 0 \end{vmatrix} = \begin{vmatrix} -2 & 0 \\ -2 & 0 \end{vmatrix} i - \begin{vmatrix} -2 & 0 \\ -5 & 0 \end{vmatrix} j + \begin{vmatrix} -2 & -2 \\ -5 & -2 \end{vmatrix} k = 0i - 0j - 6k = \langle 0, 0, -6 \rangle$

$\rightarrow$ Area $(T) = \frac{1}{2}\|\langle 0, 0, -6 \rangle\| = \frac{1}{2}\sqrt{36} = \boxed{3 \text{ squared units}}$

12.5 Homework

(1) $n = \langle 1, 3, 2 \rangle$, $(4, -1, 1)$

$\to 1(x-4) + 3(y+1) + 2(z-1) = 0$

$\to x - 4 + 3y + 3 + 2z - 2 = 0$

$\to \boxed{x + 3y + 2z = 3}$

---

(5) $n = i$, $(3, 1, -9)$

$\to 1(x-3) + \cancel{0(y-1)} + \cancel{0(z+9)} = 0$

$\to x - 3 = 0$

$\to \boxed{x = 3}$

---

(9) Write down the equation of any plane through the origin.

$\to \langle 1, 0, 0 \rangle$, $(0, 0, 0)$

$\to 1(x-0) + \cancel{0(y-0)} + \cancel{0(z-0)} = 0$

$\to \boxed{x = 0}$

---

(11) Which of the following statements are true of a plane that is parallel to the yz-plane?

(a) $n = \langle 0, 0, 1 \rangle$ is a normal vector.

(b) $n = \langle 1, 0, 0 \rangle$ is a normal vector.

(c) The equation has the form $ay + bz = d$.

(d) The equation has the form $x = d$.

$\to$ $\boxed{\text{(b) and (d)}}$

---

(13) $9x - 4y - 11z = 2$

$\to$ Normal vector : $\boxed{\langle 9, -4, -11 \rangle}$

---

(15) $3(x-4) - 8(y-1) + 11z = 0$

$\rightarrow 3x - 12 - 8y + 8 + 11z = 0$

$\rightarrow 3x - 8y + 11z = 4$

$\rightarrow$ Normal vector : $\boxed{\langle 3, -8, 11 \rangle}$

---

(17) $P(2,-1,4)$, $Q(1,1,1)$, $R(3,1,-2)$

$\rightarrow u = \overrightarrow{PQ} = Q - P = (1,1,1) - (2,-1,4) = \underline{\langle -1, 2, -3 \rangle}$

$\rightarrow v = \overrightarrow{PR} = R - P = (3,1,-2) - (2,-1,4) = \underline{\langle 1, 2, -6 \rangle}$

$\rightarrow u \times v = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ 2 & -6 \end{vmatrix} i - \begin{vmatrix} -1 & -3 \\ 1 & -6 \end{vmatrix} j + \begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix} k = -6i - 9j - 4k = \underline{\langle -6, -9, -4 \rangle}$

$\rightarrow$ Choosing point $Q(1,1,1)$: $-6(x-1) - 9(y-1) - 4(z-1) = 0$

$\rightarrow -6x + 6 - 9y + 9 - 4z + 4 = 0$

$\rightarrow -6x - 9y - 4z = -19$

$\rightarrow \boxed{6x + 9y + 4z = 19}$

---

(19) $P(1,0,0)$, $Q(0,1,1)$, $R(2,0,1)$

$\rightarrow u = \overrightarrow{PQ} = Q - P = (0,1,1) - (1,0,0) = \underline{\langle -1, 1, 1 \rangle}$

$\rightarrow v = \overrightarrow{PR} = R - P = (2,0,1) - (1,0,0) = \underline{\langle 1, 0, 1 \rangle}$

$\rightarrow u \times v = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} i - \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} j + \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} k = i + 2j - k = \underline{\langle 1, 2, -1 \rangle}$

$\rightarrow$ Choosing any point $Q(0,1,1)$: $1(x-0) + 2(y-1) - 1(z-1) = 0$

$\rightarrow x + 2y - 2 - z + 1 = 0$

$\rightarrow \boxed{x + 2y - z = 1}$

---

(25) Passes through $(-2,-3,5)$ and has normal vector $i+k$ $\langle 1,0,1 \rangle$

$\rightarrow$ Equation of plane : $x + z = d$

$\rightarrow -2 + 5 = d$

$\rightarrow d = 3$

→ $\boxed{x+z=3}$

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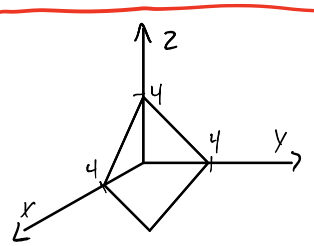
(31) $x+y+z=4$

→ $(4,0,0)$, $(0,4,0)$, $(0,0,4)$

→ 

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(53) Find all planes in R^3 whose intersection with the xz-plane is the line with equation $3x+2z=5$.

→ Equation of a plane: $ax+by+cz=d$

→ When $z=0$, $ax+by=d$

→ when $x=0$, $by+cz=d$

→ When $y=0$, $ax+cz=d$

→ In $3x+2z=5$, y is equal to 0.

→ $3x+2z=5$; $ax+cz=d$

→ $a=3\lambda$, $c=2\lambda$, $d=5\lambda$, $\lambda\neq 0$

→ $\boxed{(3\lambda)x+by+(2\lambda)z=5\lambda,\ \lambda\neq 0}$

13.1 Homework

(5) Find a vector parametrization of the line through $P=(3,-5,7)$ the direction $v=\langle 3,0,1\rangle$.

$\rightarrow (3,-5,7)+t\langle 3,0,1\rangle$

$\rightarrow (3,-5,7)+\langle 3t,0,t\rangle$

$\rightarrow \langle 3+3t,-5,7+t\rangle$

$\rightarrow \boxed{x(t)=3+3t,\quad y(t)=-5,\quad z(t)=7+t}$

---

(17) $r(t)=(9\cos t)i+(9\sin t)j$

$\rightarrow \langle 0,0,0\rangle + 9\langle \cos(t),\ \sin(t),\ 0\rangle$

$\rightarrow$ $\boxed{\text{Center is } (0,0,0)}$

$\rightarrow$ $\boxed{\text{Radius is } 9}$

$\rightarrow$ $\boxed{\text{Plane xy plane}}$

13.2 Homework

③ $\lim_{t\to 0}\left(e^{2t}i + \ln(t+1)j + 4K\right) = i + 0 + 4K = \boxed{i + 4K}$

---

⑤ $\lim_{h\to 0}\left(\frac{r(t+h)-r(t)}{h}\right)$ for $r(t) = \langle t^{-1}, \sin(t), 4\rangle$

$\rightarrow \boxed{r'(t) = \langle -\frac{1}{t^2}, \cos(t), 0\rangle}$

---

⑦ $r(t) = \langle t, t^2, t^3\rangle$

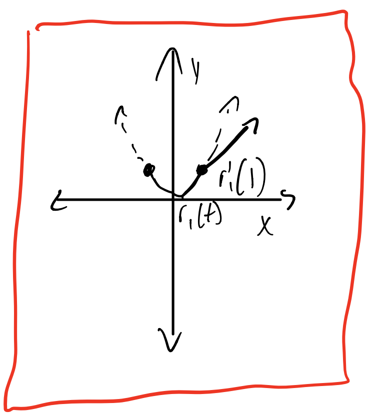
$\rightarrow \boxed{r'(t) = \langle 1, 2t, 3t^2\rangle}$

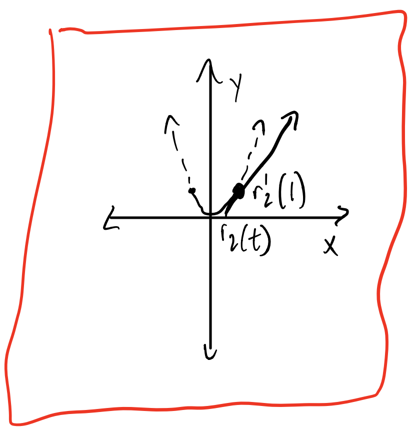
---

⑮ Sketch the curve parametrized by $r_1(t) = \langle t, t^2\rangle$ together with its tangent vector at $t=1$. Then do the same for $r_2(t) = \langle t^3, t^6\rangle$.

$\rightarrow r_1'(1) = \langle 1, 2\rangle$

$\rightarrow r_2'(1) = \langle 3, 6\rangle$





㉛ $r(t) = \langle 1-t^2, 5t, 2t^3\rangle$, $t=2$

$\rightarrow r'(t) = \langle -2t, 5, 6t^2\rangle$

$\rightarrow r'(2) = \langle -4, 5, 24\rangle$

$\rightarrow r(2) = \langle -3, 10, 16\rangle$

$\rightarrow \ell(t) = (-3,10,16) + t\langle -4,5,24\rangle$

$\rightarrow \ell(t) = (-3,10,16) + \langle -4t, 5t, 24t\rangle$

$\rightarrow \boxed{\ell(t) = \langle -3-4t,\ 10+5t,\ 16+24t\rangle}$

---

(33) $r(s) = \langle \frac{4}{s}, 0, \frac{-8}{3s^3}\rangle,\quad s=2$

$\rightarrow r(2) = \langle 2, 0, -\frac{1}{3}\rangle$

$\rightarrow r'(s) = \langle -\frac{4}{s^2}, 0, \frac{8}{s^4}\rangle$

$\rightarrow r'(2) = \langle -1, 0, \frac{1}{2}\rangle$

$\rightarrow (2,0,-\frac{1}{3}) + t\langle -1, 0, \frac{1}{2}\rangle$

$\rightarrow (2,0,-\frac{1}{3}) + \langle -t, 0, \frac{t}{2}\rangle$

$\rightarrow \boxed{\ell(t) = \langle 2-t, 0, -\frac{1}{3}+\frac{t}{2}\rangle}$

---

(41) $\int_{-2}^{2}(u^3 i + u^5 j)\,du$

$\rightarrow i\int_{-2}^{2} u^3\,du + j\int_{-2}^{2} u^5\,du = i\left[\frac{u^4}{4}\Big|_{-2}^{2}\right] + j\left[\frac{u^6}{6}\Big|_{-2}^{2}\right]$

$\rightarrow i\left[(4-4)\right] + j\left[\left(\frac{32}{3} - \frac{32}{3}\right)\right] = 0i + 0j$

$\rightarrow \boxed{\langle 0,0\rangle}$

---

(49) $r'(t) = t^2 i + 5tj + K,\quad r(1) = j + 2K$

$\rightarrow \int r'(t)\,dt = r(t) = i\int t^2\,dt + 5j\int t\,dt + k\int 1\,dt$

$\rightarrow \boxed{r(t) = i\left(\frac{t^3}{3}\right) + j\left(\frac{5t^2}{2}\right) + K(t) + C}$

$\rightarrow r(1) = i\left(\frac{1}{3}\right) + j\left(\frac{5}{2}\right) + K + C$

$\rightarrow \frac{1}{3}i + \frac{5}{2}j + k + C = j + 2k$

$\rightarrow C = -\frac{1}{3}i - \frac{3}{2}j + k$

$\rightarrow r(t) = i\left(\frac{t^3}{3}\right) + j\left(\frac{5t^2}{2}\right) + k(t) - \frac{1}{3}i - \frac{3}{2}j + k$

$\rightarrow \boxed{r(t) = i\left(\frac{t^3}{3} - \frac{1}{3}\right) + j\left(\frac{5t^2}{2} - \frac{3}{2}\right) + k(t+1)}$
