12.4 HW — #1, 5, 13, 21, 25, 27, 39, 41, 43, 45

pg 2

① $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = (3)(1) - (2)(4)$
$= 3 - 8 = -5$

⑤ $\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1(-3-0) - 2(4-0) + 1(0+3)$
$= -3 - 8 + 3 = -8$

⑬ $(\hat{i} + \hat{j}) \times \hat{k} = \langle 1,1,0\rangle \times \langle 0,0,1\rangle$

$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = (0)\hat{i} - \hat{j} + (0)\hat{k}$
$= -1\hat{j}$

㉑ $(u - 2v) \times (u + 2v)$

$= (u-2v)\times u + (u-2v)\times 2v$

$= (u\times u)(-2v\times u) + (u \times 2v) - (2v \times 2v)$

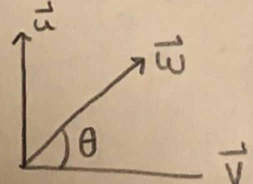
$\underset{=0}{(u\times u)}$ $\quad = -2(v\times u) = 2(u\times v)$ $\quad \underset{=0}{(2v\times 2v)}$

$\underbrace{(v\times u) \quad 2(u\times v)}$ equal

$= 2(u\times v) + 2(u\times v)$
$= 4(u\times v)$

㉕ $-u \quad (-\hat{k})$

㉗ $V = \langle 3,0,0\rangle$
$W = \langle 0,1,-1\rangle$



determine $u = v \times w$

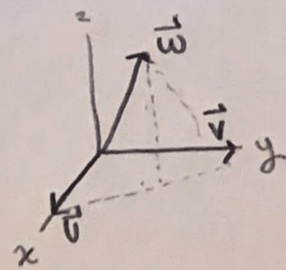
$v \cdot w = |\vec{v}||\vec{w}|\cos\theta \rightarrow (3)(0) + (0)(1) + (0)(-1) = 3 \times \sqrt{2}\cos\theta$
$0 = 3\sqrt{2}\cos\theta$
$\cos\theta = 0 \rightarrow \theta = 90°$

$\vec{U} = |\vec{v}||\vec{w}|\sin\theta\,\hat{r}$

$\vec{U} = |3||\sqrt{2}|\sin(90)\,\hat{r}$

$\vec{U} = 0$

㊴ $\vec{U} = \langle 1,0,0\rangle$
$\vec{V} = \langle 0,2,0\rangle$
$\vec{W} = \langle 1,1,2\rangle$



volume of parallelpiped $= \vec{U} \cdot |\vec{V} \times \vec{W}|$

$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 4\hat{i} - 0\hat{j} + (-2)\hat{k}$

$\langle 4,0,-2\rangle \cdot \langle 1,0,0\rangle = 4$

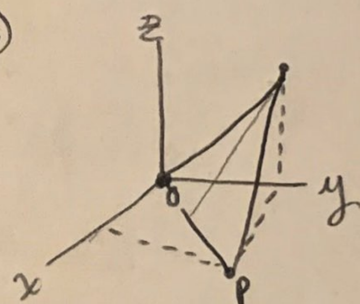
㊶ $U = \langle 1,0,3\rangle$ area of parallelogram:
$V = \langle 2,1,1\rangle$ $\quad |A\times B| = |A||B|\sin\theta$

$\cos\theta = \dfrac{2+0+3}{\sqrt{10}\times 2} = \dfrac{5}{2\sqrt{10}} \rightarrow \theta = 37.76°$

area $= |\sqrt{10}||2|\sin(37.76)$

area $= 3.87$ units^2

㊸



Area $\Delta = \frac{1}{2}|v\times u|$

where...
$V = \overrightarrow{OP}$
$u = \overrightarrow{OQ}$

$OP = \langle 3,0,-3\rangle$
$OQ = \langle 0,3,3\rangle$

$\cos\theta = \dfrac{A\cdot B}{|A||B|}$

$\cos\theta = \dfrac{-9}{\sqrt{18}\sqrt{18}} = -\frac{1}{2}$

$\theta = 120°$

$|v\times u| = |v||u|\sin\theta$
$= \sqrt{18}\sqrt{18}\sin(120) = 15.59$

Area $\Delta = \frac{1}{2}(15.59) = 7.8$ units^2

㊺ $P = (1,2)$, $Q = (3,4)$, $R = (-2,2)$

$\overrightarrow{PQ} = \langle 2,2\rangle$
$\overrightarrow{PR} = \langle -3,0\rangle$

$\cos\theta = \dfrac{A\cdot B}{|A||B|} = \dfrac{-6}{2\times 3} = -1$

$\theta = 180°$

Area $\Delta = \dfrac{|A||B|\sin\theta}{2} = 0$

Calculus III HW (sections 12.3, 12.4, 12.5, 13.1, 13.2):

12.3 HW — #1, 13, 21, 29, 31, 57, 63

(1) $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = (1)(4)+(2)(3)+(1)(5)$
$= 4+6+5$
$= 15$

(13) orthogonal → ⊥ to each other

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|A||B|}$ $\quad \vec{A} = \langle 1,1,1\rangle$ $\quad \vec{B} = \langle 1,-2,-2\rangle$

$\cos\theta = \dfrac{\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle}{(\sqrt{1^2+1^2+1^2})(\sqrt{(1)^2+(-2)^2+(-2)^2})}$

$\cos\theta = \dfrac{(1)(1)+(-2)(1)+(-2)(1)}{\sqrt{3}\sqrt{9}}$

$\cos\theta = \dfrac{-3}{3\sqrt{3}} = \dfrac{-1}{\sqrt{3}}$

$\arccos\left(\dfrac{-1}{\sqrt{3}}\right) = \underline{125.26°}$

↳ obtuse ∠, not orthogonal

(21) Find $\cos\theta$

$\vec{A} = \hat{i}+\hat{j} = \langle 1,1,0\rangle$

$\vec{B} = \hat{j}+2\hat{k} = \langle 0,1,2\rangle$

$\cos\theta = \dfrac{\langle 1,1,0\rangle \cdot \langle 0,1,2\rangle}{(\sqrt{1^2+1^2})(\sqrt{1^2+2^2})}$

$\cos\theta = \dfrac{(1)(0)+(1)(1)+(2)(0)}{\sqrt{2}\sqrt{5}}$

$\cos\theta = \dfrac{1}{\sqrt{10}}$

(29) Find all values of b which the vectors are orthogonal.

a) $\langle b,3,2\rangle$ $\quad \perp \to \cos(1) = 0$
$\langle 1,b,1\rangle$

$0 = \dfrac{b+3b+2}{\sqrt{b^2+9+4}\sqrt{b^2+2}} = \dfrac{4b+2}{\sqrt{b^2+13}\sqrt{b^2+2}}$

$\to 0 = 4b+2$

$\underline{b = -1/2}$

b) $\langle 4,-2,7\rangle$
$\langle b^2,b,0\rangle$

$0 = \dfrac{4b^2-2b}{\sqrt{69}\sqrt{b^2+b}}$ $\to 0 = 4b^2-2b$
$0 = 2b(2b-1)$
$\underline{b = 0,\ 1/2}$

(31) $\langle 2,0,-3\rangle$

$0 = 2x+0-3z \to 0 = 2x-3z$
$\langle 3,0,2\rangle$
$\langle 6,0,4\rangle$

(57) proj. of u along v

$\text{proj} = \dfrac{A\cdot B}{|B|^2}B = \dfrac{u\cdot v}{|v|^2}v$, where...
$u = \langle 5,7,-4\rangle$
$v = \langle 0,0,1\rangle$

$= \dfrac{(-4)}{1}\langle 0,0,1\rangle$

$= \langle 0,0,-4\rangle$

(63) $|u - v_{proj}|^2 + |v_{proj}|^2 = |u|^2 \longrightarrow |\sqrt{899+11}|^2 + \cancel{\sqrt{\quad}}\ ^{|v|^2} = \sqrt{34}^2$

$u = \langle 3,5\rangle$ $\quad v_{proj} = \dfrac{34}{\sqrt{68}}\langle 8,2\rangle$ $\quad |v|:$

$v = \langle 8,2\rangle$

$= \dfrac{34}{2\sqrt{17}}\langle 8,2\rangle = \left\langle \dfrac{136}{\sqrt{17}}, \dfrac{34}{\sqrt{17}}\right\rangle$ $\quad u-v = \left\langle 3-\dfrac{136}{\sqrt{17}},\ 5-\dfrac{34}{\sqrt{17}}\right\rangle$

13.1 HW - 5,17

5 $P = (3, -5, 7)$

directio of $v = \langle 3, 0, 1 \rangle$

$\langle 3, -5, 7 \rangle + t \langle 3, 0, 1 \rangle$

$= \langle 3 + 3t, -5, 7 + t \rangle$

17 $r(t) = (9\cos t)\hat{\imath} + 9(\sin t)\hat{\jmath}$

center $= \langle 0, 0, 0 \rangle$

radius $= 9$ units

plane $= z$

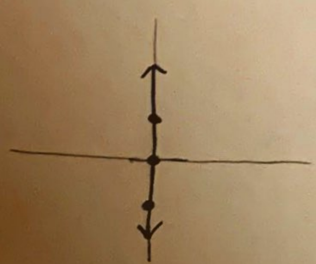
13.2 HW - # 3,5,7,15,31,33,41,49

3 $\langle 1, 0, 4 \rangle$

5 $\langle -t^{-2}, \cos t, 0 \rangle$

7 $r'(t) = \langle 1, 2t, 3t^2 \rangle$

15 $r_1(t) = \langle 1 - t^2, t \rangle \quad -1 \leq t \leq 1$



Tangent Line to curve:

$\langle x, y, z \rangle = P + tD$

$r'(t) = \langle -2t, 1 \rangle$

direction vector

$(1, 0) + t * \langle 2t, 1 \rangle$

$= \langle 1 + 2t^2, t \rangle$

31 $r(t) = \langle 1 - t^2, 5t, 2t^3 \rangle \quad t = 2$

$P = \langle -3, 10, 16 \rangle$

direction $\vec{v}$: $r'(t) = \langle -2t, 5, 6t^2 \rangle$

$P + t * D \Rightarrow \langle -3, 10, 16 \rangle + t * \langle -2t, 5, 6t^2 \rangle$

$= \langle -3 - 2t^2, 10 + 5t, 16 + 16t^3 \rangle$

33 $r(s) = 4s^{-1}\hat{\imath} - \frac{8}{3}s^{-3}\hat{k} \quad s = 1$

$P = (\frac{1}{4}, 0, -\frac{3}{8})$

direction $\vec{v}$: $\langle -4s^{-2}, 0, 8s^{-4} \rangle$

$(\frac{1}{4}, 0, -\frac{3}{8}) + s * \langle -4s^{-2}, 0, 8s^{-4} \rangle = \langle \frac{1}{4} - 4s^{-1}, 0, -\frac{3}{8} + 8s^{-3} \rangle$

47 $\int_{-2}^{2} (u^3 \hat{\imath} + u^5 \hat{\jmath})\, du$

$\langle \frac{u^4}{4}, \frac{u^6}{6}, u \rangle \Big|_{-2}^{2}$

$= \langle 0, 0, 4 \rangle$

49 $r'(t) = \langle t^2, 5t, 1 \rangle$

$r(t) = \langle \frac{1}{3}t^3, \frac{5}{2}t^2, t \rangle$

$r(1) = \hat{\jmath} + 2\hat{k} = (0, 1, 2 \rangle$

$r(t) = \langle \frac{1}{3}t^3 - \frac{1}{3}, \frac{5}{2}t^2 - \frac{3}{2}, t + 1 \rangle$

12.5 HW - # 1, 5, 9, 11, 13, 15, 17, 19, 25, 31

① $n = \langle 1, 3, 2 \rangle$ $P = (4, -1, 1)$

from origin:

$O = (0,0,0)$
$P = (4,-1,1)$] $\vec{v} = (4-0, -1-0, 1-0)$
$= (4, -1, 1)$

$ax + by + cz = d$

$d = \langle 1, 3, 2 \rangle \cdot \langle 4, -1, 1 \rangle$

$= (4) - (3) + (2) = 3$

$d = 3 \longrightarrow$ scalar form:

$4x - y + z = 3$

⑤ $n = \langle 1, 0, 0 \rangle$ $P = (3, 1, -9)$

fromorigin:

$O = (0,0,0)$
$P = (3,1,-9)$] $\vec{v} = \langle 3-0, 1-0, -9-0 \rangle$
$= \langle 3, 1, -9 \rangle$

$d = \langle 1, 0, 0 \rangle \cdot \langle 3, 1, -9 \rangle$

$d = 3 \longrightarrow$ scalar form:

$3x + y - 9z = 3$

⑨ $ax + by + cx = d$

⑪ (B)

⑬ $9x - 4y - 11z = 2$

$ax + by + cz = d$

$n = \langle 9, -4, -11 \rangle$

⑮ $3(x-4) - 8(y-1) + 11z = 0$

$3x - 12 - 8y + 1 + 11z = 0$

$3x - 8y + 11z = 11$

$n = \langle 3, -8, 11 \rangle$

⑰ $P = (2, -1, 4)$

$Q = (1, 1, 1)$

$R = (3, 1, -2)$

$\vec{PQ} = \langle -1, 0, -3 \rangle$

$\vec{PR} = \langle 1, 0, -6 \rangle$

cross product:

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -3 \\ 1 & 0 & -6 \end{vmatrix} = 0\hat{i} - 9\hat{j} - 0\hat{k}$$

Plane $= \langle 0, -9, 0 \rangle$

⑲ $P = (1, 0, 0)$ $\vec{PQ} = \langle -1, 1, 1 \rangle$

$Q = (0, 1, 1)$ $\vec{PR} = \langle 1, 0, 1 \rangle$

$R = (2, 0, 1)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = 1\hat{i} + 2\hat{j} - 1\hat{k}$$

$\langle 1, 2, -1 \rangle$

㉕ $P = (-2, -3, 5)$

$n = \langle 1, 0, 1 \rangle$

$\langle 1, 0, 1 \rangle \cdot \langle x+2, y+3, z-5 \rangle$

$= (x+2) + (z-5) \rightarrow x + z = 3$

㉛