12.3 Homework

9/20/20

1. $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle$

$(1)(4)+(2)(3)+(1)(5)$

$4+6+5=\boxed{15}$

13. $\langle 1,1,1\rangle \cdot \langle 1,-2,-2\rangle$

$(1)(1)+(1)(-2)+(1)(-2)$

$1-2-2=-3 \rightarrow$ It is not orthogonal

Since $-3<0$, the angle is $\boxed{\text{OBTUSE}}$

21. $A\cdot B = |A||B|\cos\theta \rightarrow \cos\theta = \frac{A\cdot B}{|A||B|}$

$A\cdot B = \langle 1,1,0\rangle \cdot \langle 0,1,2\rangle = (1)(0)+(1)(1)+(0)(2)=1$

$\cos\theta = \frac{1}{\sqrt{5}\sqrt{2}} = \boxed{\frac{1}{\sqrt{10}}}$ $\quad |A| = \sqrt{(1)^2+(1)^2} = \sqrt{2} \quad |B| = \sqrt{5}$

29. (a) $\langle b,3,2\rangle \cdot \langle 1,b,1\rangle = 0$

$b+3b+2=0 \rightarrow 4b+2=0 \quad \boxed{b=-1/2}$

(b) $\langle 4,-2,7\rangle \cdot \langle b^2,b,0\rangle$

$4b^2-2b=0 \rightarrow 2b(2b-1) \quad \boxed{b=0,\ 1/2}$

31. $\langle 3,0,2\rangle$

$\langle 3,0,2\rangle \cdot \langle 2,0,-3\rangle = 6+0-6=0$ ✓

$\langle 0,0,0\rangle$

$\langle 0,0,0\rangle \cdot \langle 2,0,-3\rangle = 0+0+0=0$ ✓

57. Projection $= \frac{U\cdot V}{\|v\|^2}\vec{v}$ $\quad U=\langle 5,7,-4\rangle \quad V=\langle 0,0,1\rangle$

$U\cdot V = 0+0-4=-4$

$\|v\| = \sqrt{0^2+0^2+1^2} = 1$

$\frac{-4}{1}\langle 0,0,1\rangle = \boxed{\langle 0,0,-4\rangle}$

63. $V_{\|v} = \frac{U\cdot V}{\|v\|^2}$ $\quad U\cdot V = 34 \quad \|v\| = \sqrt{64+4} = \sqrt{61}$

$\frac{34}{61}$. I'm not sure how to do this :(

9/20/20

12.4 Homework

1. $(1)(3)-(2)(4)$

$3-8=\boxed{-5}$

13. $(\hat{i}+\hat{j})\times\hat{k}$

$\hat{i}\times\hat{k}+\hat{j}\times\hat{k}$

$-\hat{j}+\hat{i} \rightarrow \boxed{\hat{i}-\hat{j}}$

21. $(u-2v)\times(u+2v)$

$(u\times u)+(u\times 2v)+(-2v\times u)+(-2v\times 2v)$

$0+2(u\times v)-2(v\times u)-4(0)$

$0+2\langle 1,1,0\rangle-2\langle -1,-1,0\rangle-0$

$\langle 2,2,0\rangle+\langle 2,2,0\rangle$

$\boxed{\langle 4,4,0\rangle}$

5. $1[(-3)(1)-(0)(0)]-2[(4)(1)-(0)(1)]+1[(4)(0)-(-3)(1)]$

$1[-3]-2[4]+1[3]$

$-3-8+3=\boxed{-8}$

25. Using RHR, I find $-u$

27. We know that u has to be orthogonal to both v and w, which means that $u\cdot v=u\cdot w=0$

v is on x-axis, so $u_x=0$

$\langle 0,b,c\rangle\cdot\langle 0,1,-1\rangle$

$0+b-c=0$

$b=c$

$\langle 0,b,b\rangle=0$

$\langle 0,b,b\rangle\cdot\langle 3,0,0\rangle$

$0+0+0=0$

So, b and c have to be the same

39. First, find $V \times W$

$$\begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

$\hat{i}[(2)(2)-(0)(1)] - \hat{j}[(0)(2)-(0)(1)] + \hat{k}[(0)(1)-(2)(1)]$

$\hat{i}(4-0) - \hat{j}(0-0) + \hat{k}(0-2)$

$4\hat{i} - 2\hat{k} = \langle 4, 0, -2 \rangle$

Now, find $(V \times W) \cdot (U)$

$\langle 4, 0, -2 \rangle \cdot \langle 1, 0, 0 \rangle$

$= 4 + 0 + 0 = 4$

$\boxed{\text{Volume} = 4}$

41. $U = \langle 1, 0, 3 \rangle \quad V = \langle 2, 1, 1 \rangle$

$$U \times V = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{bmatrix}$$

$\hat{i}[(0)(1)-(3)(1)] - \hat{j}[(1)(1)-(3)(2)] + \hat{k}[(1)(1)-(0)(2)]$

$\hat{i}[0-3] - \hat{j}[1-6] + \hat{k}[1-0]$

$-3\hat{i} + 5\hat{j} + k = \langle -3, 5, 1 \rangle$

$\text{Area} = \|U \times V\| = \sqrt{(-3)^2 + (5)^2 + (1)^2} = \sqrt{9+25+1} = \boxed{\sqrt{35}}$

43. $OP = \langle 3, 3, 0 \rangle \quad OQ = \langle 0, 3, 3 \rangle$

$$OP \times OQ = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{bmatrix}$$

$\hat{i}[(3)(3)-(0)(3)] - \hat{j}[(3)(3)-(0)(0)] + \hat{k}[(3)(3)-(3)(0)]$

$\hat{i}[9-0] - \hat{j}[9-0] + \hat{k}[9-0]$

$9\hat{i} - 9\hat{j} + 9\hat{k} = \langle 9, -9, 9 \rangle$

$\|OP \times OQ\| = \sqrt{81+81+81} = \sqrt{243}$

$\text{Area of Triangle} = \frac{\sqrt{243}}{2}$

45. $PQ = \langle 2, 2 \rangle \quad PR = \langle -3, 0 \rangle$

$\begin{matrix} 2 & 2 \\ -3 & 0 \end{matrix}$

$0 - (-6)$

$= 6$

$6/2 = \boxed{3}$

9/20/20

12.5 Homework

1. $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

$1(x-4)+3(y+1)+2(z-1)=0$

$x-4+3y+3+2z-2=0$

$x+3y+2z-3=0$

$\boxed{x+3y+2z=3}$

5. $n=\hat{\imath} \rightarrow \langle 1,0,0\rangle$

$1(x-3)+0(y-1)+0(z+9)$

$x-3=0$

$\boxed{x=3}$

9. $1(x-0)+1(y-0)+1(z-0)=0$

$\boxed{x+y+z=0}$

11. Since it is parallel to yz plane, normal vector is only in x-direction.

This means (ii) and (iv) are true.

13. $\langle 9,-4,-11\rangle$

15. $\langle 3,-8,11\rangle$

17. $PQ=\langle -1,2,-3\rangle \quad PR=\langle 1,2,-6\rangle$

$PQ\times PR=\begin{bmatrix}\hat{\imath} & \hat{\jmath} & \hat{k}\\ -1 & 2 & -3\\ 1 & 2 & -6\end{bmatrix}$ $\hat{\imath}[(2)(-6)-(-3)(2)]-\hat{\jmath}[(-1)(-6)-(-3)(1)]+\hat{k}[(-1)(2)-(2)(1)]$

$\hat{\imath}[(-12)+(+6)]-\hat{\jmath}[(6)+3]+\hat{k}[(-2)-2]$

$\hat{\imath}(-6)-\hat{\jmath}[9]+\hat{k}(-4)$

$-6\hat{\imath}-9\hat{\jmath}-4\hat{k} \rightarrow \langle -6,-9,-4\rangle$

$-6(x-1)-9(y-1)-4(z-1)=0$

$-6x+6-9y+9-4z+4=0$

$-6x-9y-4z+19=0$ $\boxed{-6x-9y-4z=-19}$

19. $PQ = \langle -1, 1, 1 \rangle \qquad PR = \langle 1, 0, 1 \rangle$

$$PQ \times PR = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} = \hat{i}[(1)(1)-(1)(0)] - \hat{j}[(-1)(1)-(1)(1)] + \hat{k}[(-1)(0)-(1)(1)]$$

$$\hat{i}[1] - \hat{j}[-2] + \hat{k}[-1]$$

$$\hat{i} + 2\hat{j} - k \rightarrow \langle 1, 2, -1 \rangle$$

$$1(x-1) + 2(y-0) - 1(z-0) = 0$$

$$x - 1 + 2y - z = 0$$

$$\boxed{x + 2y - z = 1}$$

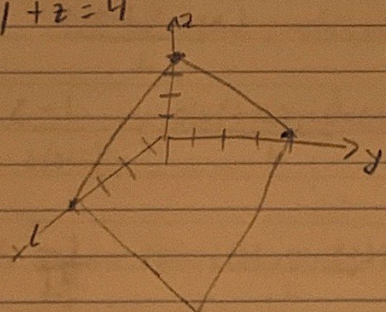
25. $\hat{i} + \hat{k} \rightarrow \langle 1, 0, 1 \rangle$

$$1(x+2) + 0(y+3) + 1(z-5) = 0$$

$$x + 2 + 0 + z - 5 = 0$$

$$x + z - 3 = 0 \rightarrow \boxed{x + z = 3}$$

31. $x + y + z = 4$



53. $3x + cy + 2z = 5 \qquad C = \text{ANYTHING!}$

1/20/20

13.1 Homework

5. $P + tv$

$P = (3, -5, 7) \qquad v = \langle 3, 0, 1 \rangle$

$\langle 3, -5, 7 \rangle + t\langle 3, 0, 1 \rangle$

$\langle 3, -5, 7 \rangle + \langle 3t, 0, t \rangle$

$\langle 3+3t, -5, 7+t \rangle$

17. $r(t) = (9\cos t)\hat{i} + (9\sin t)\hat{j} = \langle 9, 9, 0 \rangle + \langle \cos t, \sin t, 0 \rangle$

Radius $= 9$, xy plane $(z=0)$ at origin

13.2 Homework

9/20/20

3. $\lim_{t\to 0} e^{2t}\hat{i} + \ln(t+1)\hat{j} + 4\hat{k}$ Plug in $t=0$

$\langle 1, 0, 4\rangle$ or $\boxed{\hat{i} + 4\hat{k}}$

5. $\lim_{h\to 0} \frac{r(t+h) - r(t)}{h}$ $r(t) = \langle t^{-1}, \sin t, 4\rangle$

$\lim_{h\to 0} \frac{\langle (t+h)^{-1}, \sin(t+h), 4\rangle - \langle t^{-1}, \sin t, 4\rangle}{h}$

$\langle \frac{(t+h)^{-1} - t^{-1}}{h}, \frac{\sin(t+h) - \sin(t)}{h}, \frac{4-4}{h} \rangle$

$\langle \frac{1/(t+h) - 1/t}{h}, \frac{\sin(t+h) - \sin(t)}{h}, 0 \rangle$

$\langle \frac{t-t-h/t(t+h)}{h}, \frac{\sin(t+h) - \sin(t)}{h}, 0 \rangle$

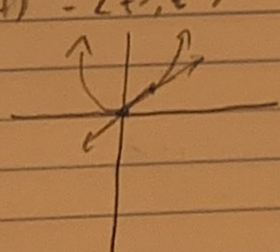
$\langle \frac{-1}{t(t+h)}, \frac{\sin(t+h) - \sin(t)}{h}, 0 \rangle$

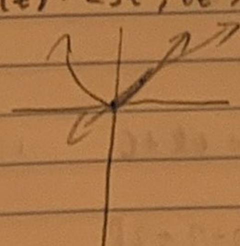
$\langle \frac{-1}{t^2}, \frac{\sin(t+h-\sin(t))}{h}, 0 \rangle$

7. $r(t) = \langle t, t^2, t^3\rangle$ $r'(t) = \langle 1, 2t, 3t^2\rangle$

15. $r_1(t) = \langle t, t^2\rangle$ $r_1'(t) = \langle 1, 2t\rangle$ $r_1'(1) = \langle 1, 2\rangle$

$r_2(t) = \langle t^3, t^6\rangle$ $r_2'(t) = \langle 3t^2, 6t^5\rangle$ $r_2'(1) = \langle 3, 6\rangle$





31. $r(t) = \langle 1-t^2, 5t, 2t^3 \rangle$ at $t=2$

$r(2) = \langle -3, 10, 16 \rangle$

$r'(t) = \langle -2t, 5, 6t^2 \rangle$

Direction of tangent line: $r'(2) = \langle -4, 5, 24 \rangle$

Tangent Line: $\langle -3, 10, 16 \rangle + t\langle -4, 5, 24 \rangle$

$\langle -3, 10, 16 \rangle + \langle -4t, 5t, 24t \rangle$

$= \langle -3-4t, 10+5t, 16+24t \rangle$

$\boxed{x(t) = -3-4t \quad y(t) = 10+5t \quad z(t) = 16+24t}$

33. $r(s) = 4s^{-1}\hat{i} - 8/_3 s^{-3}\hat{k}$ at $s=2$

$r(2) = 2\hat{i} - 1/_3\hat{k}$

$r'(s) = \frac{-4}{s^2}\hat{i} + \frac{24}{3s^4}\hat{k}$

$r'(2) = -\hat{i} + 1/_2\hat{k}$. = Direction of Tangent Line

$\langle 2, 0, -1/_3 \rangle + t\langle -1, 0, 1/_2 \rangle$

$\langle 2, 0, -1/_3 \rangle + \langle -t, 0, t/_2 \rangle$

$\langle 2-t, 0, -1/_3 + t/_2 \rangle$

$\boxed{x(t) = 2-t \quad y(t) = 0 \quad z(t) = -1/3 + t/2}$

41. $\int_{-2}^{2} (u^3\hat{i} + u^5\hat{j}) = \left. \frac{u^4}{4}\hat{i} + \frac{u^6}{5}\hat{j} \right]_{-2}^{2}$

$\cancel{(4\hat{i} + \frac{64}{5}\hat{j}) - (4\hat{i} - \frac{64}{5}\hat{j})}$

$\cancel{(\frac{64}{5}\hat{j})}$

$\boxed{\langle 0, 0 \rangle}$

49. $\int t^2\hat{i} + 5t\hat{j} + k = \frac{t^3}{3}\hat{i} + \frac{5t^2}{2}\hat{j} + tk + C$ $\quad r(1) \langle 1, \ldots \rangle$

$1/_3\hat{i} + 5/_2\hat{j} + \hat{k} + C = 0 + \hat{j} + 2\hat{k}$

$1/_3\hat{i} + 3/_2\hat{j} - \hat{k} = C$

~~$1/_3\hat{i} + 5/_2\hat{j} + k$~~

$\boxed{\frac{t^3}{3}\hat{i} + \frac{5t^2}{2}\hat{j} + tk + \hat{j} + 2\hat{k}}$