12.3 | 12.4 | 12.5 Homework

12.3: 1, 13, 21, 29, 31, 57, 63
12.4: 1, 5, 13, 21, 25, 27, 39, 41, 43, 45
12.5: 1, 5, 9, 11, 13, 15, 17, 19, 25, 31, 53

12.3

1. $\langle 1,2,1\rangle \cdot \langle 4,3,5\rangle = 4+6+5 = \boxed{15}$

13. $A=\langle 1,1,1\rangle \quad B=\langle 1,-2,-2\rangle$

$A\cdot B = -3 \quad \boxed{\text{Obtuse}}$

21. $A = i+j \quad B = j+2k$

$$\cos\theta = \frac{A\cdot B}{|A||B|} = \frac{1}{\sqrt{2}\sqrt{5}} = \boxed{\frac{1}{\sqrt{10}}}$$

29. a. $A=\langle b,3,2\rangle \quad B=\langle 1,b,1\rangle$

$A\cdot B=0$

$b+3b+2=0 \quad \boxed{b=-\tfrac{1}{2}}$

$4b=-2$

b. $A=\langle 4,-2,7\rangle \quad B=\langle b^2,b,0\rangle$

$A\cdot B=0$

$4b^2-2b=0 \quad \boxed{b=0,\ \tfrac{1}{2}}$

$2b(2b-1)=0$

31. $\langle 3,0,2\rangle$
$\langle 3,7,2\rangle$

57. $v = \dfrac{A \cdot B}{|B|^2} B$ $\quad A = 5i + 7j - 4k$
$B = 0i + 0j + k$

$v = \dfrac{-4}{1} k$ $\quad \boxed{v = -4k}$

63. $A = \langle 3,5\rangle$ $\quad B = \langle 8,2\rangle$

$|v| = \dfrac{A \cdot B}{|B|} = \dfrac{34}{2\sqrt{17}} = \dfrac{17}{\sqrt{17}}$

$\boxed{|v| = \sqrt{17}}$

12.4

1. $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix}$ $\quad 3 - 8 = \boxed{-5}$

5.
$$1\begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2\begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 1\begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$$

$= -3 - 6 + 1 = \boxed{-8}$

i j k

13. $(i + j) \times k = i \times k + j \times k$

$= \langle 1,0,0 \rangle \times \langle 0,0,1 \rangle + \langle 0,1,0 \rangle \times \langle 0,0,1 \rangle$

$= \langle 0, 0-1, 0 \rangle + \langle 1-0, 0, 0 \rangle$

$= \langle 0, -1, 0 \rangle + \langle 1, 0, 0 \rangle$

$= \langle 1, -1, 0 \rangle$

$\boxed{= i - j}$

21. $(u-2v) \times (u+2v)$

$= u \times u + u \times 2v - 2v \times u - 2v \times 2v$

$0 + 2(u \times v) - 2(v \times u) - 0$

$= 2(u \times v) + 2(u \times v)$

$= 2\langle 1,1,0 \rangle + 2\langle 1,1,0 \rangle$

$\boxed{= \langle 4,4,0 \rangle}$

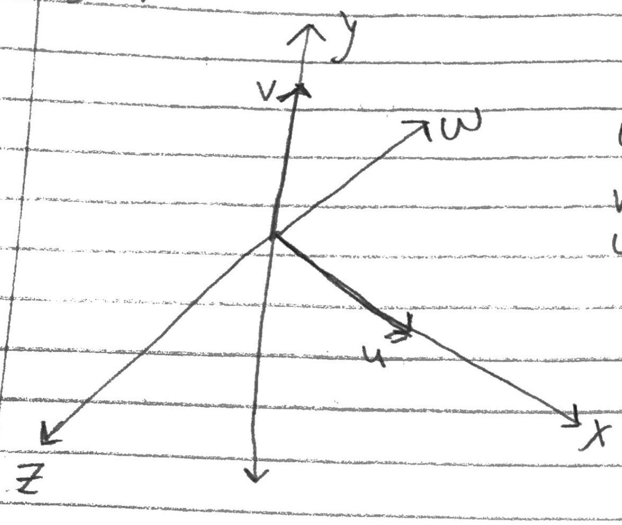
25. $-u$ (Right hand rule)

27. $v = \langle 3,0,0 \rangle$

$v \times w = 3\sqrt{2} \, (0, \sqrt{2}/2, \sqrt{2}/2)$

$\boxed{\langle 0,3,3 \rangle}$

39.



$u = \langle 1, 0, 0 \rangle$
$v = \langle 0, 1, 0 \rangle$
$w = \langle 1, 1, 2 \rangle$

Volume: $|u \times v|$ $\quad \dfrac{|w \cdot (u \times v)|}{|u \times v|}$

Volume $= |w \cdot (u \times v)|$

Volume $= 4$

41. $A = |u \times v|$ $\boxed{= \sqrt{35}}$

43. $(0,0,0)$, $(3,3,0)$, $(0,3,3)$

$u = \langle 3, 3, 0 \rangle$ $\quad v = \langle 0, 3, 3 \rangle$

$\frac{1}{2} |u \times v| = \text{Area} = \frac{1}{2} \, 9\sqrt{3}$

$\boxed{\text{Area} = \frac{9}{2}\sqrt{3}}$

P Q R

45. (1,2) (3,4) (-2,2)

PQ PR

$A = \frac{1}{2}|u \times v|$ $u = \langle 2, 2 \rangle$ $v = \langle -3, 0 \rangle$

$\boxed{A = 3}$

12.5

1. $x + 3y + 2z = d$ $\boxed{x + 3y + 2z = 3}$
@ 4, -1, 1; $d = 3$

5. $x = d$ @ (3, 1, -4); $d = 3$

$\boxed{x = 3}$

9. $\langle 1, 0, 0 \rangle$ (0, 0, 0)

$\boxed{x = 0}$

11. B, D

13. $\langle 9, -4, -11 \rangle$

15. $3x - 12 - 8y + 8 + 11z = 0$ $\boxed{\langle 3, -8, 11 \rangle}$
$3x - 8y + 11z = 4$

17. $P(2,-1,4)$ $\quad ax+by+cz=d$

$Q(1,1,1)$

$R(3,1,-2)$

$\overrightarrow{PQ} = u = \langle -1, 2, -3\rangle$

$\overrightarrow{PR} = v = \langle 1, 2, -6\rangle$

$|u \times v| = \langle -6, -9, -4\rangle$

$-6x - 9y - 4z = d$ $\quad \boxed{6x + 9y + 4z = 19}$

$-6 - 9 - 4 = d$

$-19 = d$

19. $P(1,0,0)$

$Q(0,1,1)$

$R(2,0,1)$

$\overrightarrow{PQ} = u = \langle -1, 1, 1\rangle$

$\overrightarrow{PR} = v = \langle 1, 0, 1\rangle$

$u \times v = \langle 1, 2, -1\rangle$

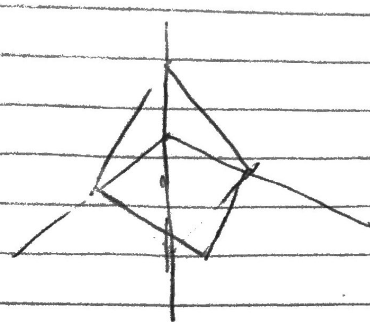
$x + 2y - z = d$ $\quad d = 1$

$\boxed{x + 2y - z = 1}$

25. $x + z = d$ $\quad d = 3$

$\boxed{x + z = 3}$

31.



53. $30x + by + 20z = 50$
where σ is a real number $\neq 0$

13.1 Textbook Notes Space Curves

$\langle f(t), g(t), h(t) \rangle$ expressed as a vector called vector function
can also be thought of as:
$x = f(t)$, $y = g(t)$, $z = h(t)$

13.2 Textbook Notes Calculus w/ Vector Functions

$r'(t) = \langle f'(t), g'(t), h'(t) \rangle$

$v(t) = |r'(t)|$ - called velocity vector

Angle between 2 curves where ~~ic~~ they meet is the angle between their tangent vectors at that point