

Chs. 12.3, 12.4, 12.5 HW

9/20/20

12.3

1. $\langle 1, 2, 1 \rangle \cdot \langle 4, 3, 5 \rangle = 4 + 6 + 5 = 15$

13. $\langle 1, 1, 1 \rangle, \langle 1, -2, -2 \rangle \Rightarrow \langle 1, 1, 1 \rangle \cdot \langle 1, -2, -2 \rangle = 1 - 2 - 2 = -3$

Dot product is negative $\Rightarrow \angle$ between vectors is obtuse

★ $i+j, j+2k \Rightarrow$ Let: $v = i+j$ & $w = j+2k$

$\|v\| = \|i+j\| = \sqrt{1^2+1^2} = \sqrt{2}$

$\|w\| = \|j+2k\| = \sqrt{1^2+2^2} = \sqrt{5}$

$v \cdot w = (i+j) \cdot (j+2k) = i \cdot j + 2i \cdot k + j \cdot j + 2j \cdot k = \|j\|^2 = 1$

$\cos \theta = v \cdot w / \|v\| \|w\| = 1 / (\sqrt{2} \cdot \sqrt{5}) = 1 / \sqrt{10}$

29. (a) $\langle b, 3, 2 \rangle, \langle 1, b, 1 \rangle$; Vectors are orthogonal if scalar product = 0

$\langle b, 3, 2 \rangle \cdot \langle 1, b, 1 \rangle = 0$

$b \cdot 1 + 3 \cdot b + 2 \cdot 1 = 0$

$4b + 2 = 0 \Rightarrow b = -1/2$

(b) $\langle 4, -2, 7 \rangle, \langle b^2, b, 0 \rangle$

$\langle 4, -2, 7 \rangle \cdot \langle b^2, b, 0 \rangle = 0$

$4b^2 - 2b = 0$

$2b(2b-1) = 0 \Rightarrow b = 0$ or $b = 1/2$

31. Find 2 vectors that are not multiples of each other & are both orthogonal to $\langle 2, 0, -3 \rangle$

$\langle a, b, c \rangle \cdot \langle 2, 0, -3 \rangle = 0$

$2a - 3c = 0 \Rightarrow a = 3/2 c \Rightarrow \langle 3/2 c, b, c \rangle$

$c=0, b=1 \Rightarrow v_1 = \langle 0, 1, 0 \rangle$

$c=2, b=2 \Rightarrow v_2 = \langle 3, 2, 2 \rangle$

★ $u = 5i + 7j - 4k, v = k$

The projection of u along v is: $u_{\parallel v} = \left(\frac{u \cdot v}{v \cdot v} \right) v = \frac{-4k}{1} = -4k$

$u \cdot v = (5i + 7j - 4k) \cdot k = -4k \cdot k = -4$

$v \cdot v = \|v\|^2 = \|k\|^2 = 1$

★ Find the length of \overline{OP} in Figure 15. \Rightarrow component of $u = \langle 3, 5 \rangle$ along $v = \langle 8, 2 \rangle$

$u \cdot v = \langle 3, 5 \rangle \cdot \langle 8, 2 \rangle = 34$

$v \cdot v = \|v\|^2 = 8^2 + 2^2 = 68 \Rightarrow \left\| \left(\frac{u \cdot v}{v \cdot v} \right) v \right\| = \frac{34 \|v\|}{68} = \frac{34 \sqrt{68}}{68}$

12.4

1. $\begin{vmatrix} 1 & 2 \\ 4 & 3 \end{vmatrix} = 1 \cdot 3 - 2 \cdot 4 = 3 - 8 = -5$

$\begin{vmatrix} 1 & 2 & 1 \\ 4 & -3 & 0 \\ 1 & 0 & 1 \end{vmatrix} = 1 \begin{vmatrix} -3 & 0 \\ 0 & 1 \end{vmatrix} - 2 \begin{vmatrix} 4 & 0 \\ 1 & 1 \end{vmatrix} + 1 \begin{vmatrix} 4 & -3 \\ 1 & 0 \end{vmatrix}$
 $= 1 \cdot (-3 \cdot 1 - 0 \cdot 0) - 2 \cdot (4 \cdot 1 - 0 \cdot 1) + 1 \cdot (4 \cdot 0 - (-3) \cdot 1)$
 $= -3 - 8 + 3 = -8$

$(i+j) \times k = i \times k + j \times k = -j + i$
 $i \times k = -j$ & $j \times k = i$

$u \times v = \langle 1, 1, 0 \rangle$, $u \times w = \langle 0, 3, 1 \rangle$, $v \times w = \langle 2, -1, 1 \rangle$
 $(u - 2v) \times (u + 2v) = (u - 2v) \times u + (u - 2v) \times 2v$
 $= u \times u - 2v \times u + u \times 2v - 4v \times v = 0 + 2u \times v + 2u \times v - 0$
 $= 0 + 4u \times v = \langle 4, 4, 0 \rangle$

??? \star Let $v = \langle 3, 0, 0 \rangle$ & $w = \langle 0, 1, -1 \rangle$. Determine $u = v \times w$ using the geometric properties of the cross product rather than form.

The cross product $u = v \times w$ is orthogonal to v .

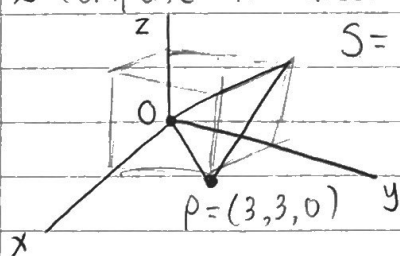
\star Sketch & compute the volume of the parallelepiped spanned by $u = \langle 1, 0, 0 \rangle$, $v = \langle 0, 2, 0 \rangle$, $w = \langle 1, 1, 2 \rangle$
 $u \cdot (v \times w) = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 1 & 1 & 2 \end{vmatrix} = 1(4 - 0) - 0 + 0 = 4$

\star Calc. area of the \square spanned by $u = \langle 1, 0, 3 \rangle$ & $v = \langle 2, 1, 1 \rangle$

$u \times v = \begin{vmatrix} i & j & k \\ 1 & 0 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} i - \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} j + \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} k = -3i - (1-6)j + k$
 $= -3i + 5j + k$

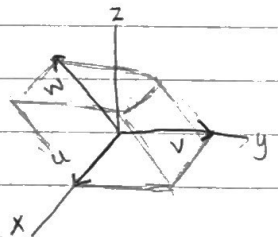
$A = \|u \times v\| = \sqrt{(-3)^2 + 5^2 + 1^2} = \sqrt{35}$

\star Sketch \triangle w/ vertices at the origin O , $P = (3, 3, 0)$, & $Q = (0, 3, 3)$, & compute its area using cross products.



$S = \frac{1}{2} \| \vec{OP} \times \vec{OQ} \|$

$\vec{OP} \times \vec{OQ} = \begin{vmatrix} i & j & k \\ 3 & 3 & 0 \\ 0 & 3 & 3 \end{vmatrix} = \begin{vmatrix} 3 & 0 \\ 3 & 3 \end{vmatrix} i - \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} j + \begin{vmatrix} 3 & 3 \\ 0 & 3 \end{vmatrix} k$
 $= 9i - 9j + 9k = 9 \langle 1, -1, 1 \rangle$



Chs. 12.3, 12.4, 12.5 HW Cont.

9/20/20

$$S = \frac{1}{2} \| 9 \langle 1, -1, 1 \rangle \| = \frac{9}{2} \| \langle 1, -1, 1 \rangle \| = \frac{9}{2} \sqrt{1^2 + (-1)^2 + 1^2} = \frac{9\sqrt{3}}{2}$$

45. Use cross products to find the area of the Δ in the xy -plane defined by $(1, 2)$, $(3, 4)$, & $(-2, 2)$

Let: $P = (1, 2, 0)$, $Q = (3, 4, 0)$, $R = (-2, 2, 0)$

$$T = \frac{1}{2} \| \vec{PQ} \times \vec{PR} \|$$

$$\vec{PQ} = \langle 3-1, 4-2, 0-0 \rangle = \langle 2, 2, 0 \rangle$$

$$\vec{PR} = \langle -2-1, 2-2, 0-0 \rangle = \langle -3, 0, 0 \rangle$$

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 2 & 0 \\ -3 & 0 & 0 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 0 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 2 & 0 \\ -3 & 0 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 2 & 2 \\ -3 & 0 \end{vmatrix} \mathbf{k} = 6\mathbf{k}$$

$$T = \frac{1}{2} \| 6\mathbf{k} \| = \frac{1}{2} \cdot 6 = 3$$

12.5

1. $n = \langle 1, 3, 2 \rangle$, $\langle 4, -1, 1 \rangle$

$$\langle 1, 3, 2 \rangle \cdot \langle x, y, z \rangle = \langle 1, 3, 2 \rangle \cdot \langle 4, -1, 1 \rangle = 4 - 3 + 2 = 3$$

$$x + 3y + 2z = 3$$

5. $n = \mathbf{i}$, $(3, 1, -9) \Rightarrow n = \langle 1, 0, 0 \rangle$

$$\langle 1, 0, 0 \rangle \cdot \langle x, y, z \rangle = \langle 1, 0, 0 \rangle \cdot \langle 3, 1, -9 \rangle = 3 + 0 + 0 = 3$$

$$x + 0 + 0 = 3 \Rightarrow x = 3$$

9. Write the eq. of any plane through the origin.

$$ax + by + cz = d \text{ that contains the point } (x, y, z) = (0, 0, 0)$$

$$\Rightarrow x + y + z = 0$$

Which of the following statements are true of a plane that is parallel to the yz -plane?

(a) $n = \langle 0, 0, 1 \rangle$ is a normal vector \Rightarrow \parallel to xy -plane FALSE

(b) $n = \langle 1, 0, 0 \rangle$ is a normal vector \Rightarrow \parallel to yz -plane TRUE

(c) The eq. has the form $ay + bz = d$ FALSE

(d) The eq. has the form $x = d$. $\Rightarrow \langle 1, 0, 0 \rangle$ TRUE

13. $9x - 4y - 11z = 2 \Rightarrow n = \langle 9, -4, -11 \rangle$

15. $3(x-4) - 8(y-1) + 11z = 0 \Rightarrow n = \langle 3, -8, 11 \rangle$

$P = (2, -1, 4)$, $Q = (1, 1, 1)$, $R = (3, 1, -2)$

$$a = \vec{PQ} = \langle 1-2, 1-(-1), 1-4 \rangle = \langle -1, 2, -3 \rangle$$

$$b = \vec{PR} = \langle 3-2, 1-(-1), -2-4 \rangle = \langle 1, 2, -6 \rangle$$

$$n = a \times b = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ 2 & -6 \end{vmatrix} i - \begin{vmatrix} -1 & -3 \\ 1 & -6 \end{vmatrix} j + \begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix} k$$

$$= -6i - 9j - 4k = \langle -6, -9, -4 \rangle$$

Choose a point $\Rightarrow Q = (1, 1, 1)$

$$n \cdot \langle x, y, z \rangle = n \cdot \langle x_0, y_0, z_0 \rangle$$

$$\langle -6, -9, -4 \rangle \cdot \langle x, y, z \rangle = \langle -6, -9, -4 \rangle \cdot \langle 1, 1, 1 \rangle$$

$$-6x - 9y - 4z = -6 - 9 - 4 = -19$$

$$6x + 9y + 4z = 19$$

★ $P = (1, 0, 0), Q = (0, 1, 1), R = (2, 0, 1)$

$$n \cdot \langle x, y, z \rangle = d$$

$$\vec{PQ} = \langle 0, 1, 1 \rangle - \langle 1, 0, 0 \rangle = \langle -1, 1, 1 \rangle$$

$$\vec{PR} = \langle 2, 0, 1 \rangle - \langle 1, 0, 0 \rangle = \langle 1, 0, 1 \rangle$$

$$n = \vec{PQ} \times \vec{PR} = \begin{vmatrix} i & j & k \\ -1 & 1 & 1 \\ 1 & 0 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} i - \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} j + \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix} k$$

$$= i + 2j - k = \langle 1, 2, -1 \rangle$$

$$P = (1, 0, 0)$$

$$d = n \cdot \vec{op} = \langle 1, 2, -1 \rangle \cdot \langle 1, 0, 0 \rangle = 1 \cdot 1 + 2 \cdot 0 + (-1) \cdot 0 = 1$$

$$\langle 1, 2, -1 \rangle \cdot \langle x, y, z \rangle = 1$$

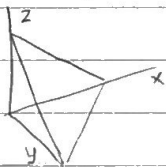
$$x + 2y - z = 1$$

??? ★ Passes through $(-2, -3, 5)$ & has normal vector $i+k$

$$\langle 1, 0, 1 \rangle \cdot \langle x, y, z \rangle = \langle 1, 0, 1 \rangle \cdot \langle -2, -3, 5 \rangle = -2 + 0 - 5 = -7$$

$$\Rightarrow x + 0 + z = -7 \Rightarrow x + z = -7$$

31. $x + y + z = 4$



??? ★ Find all planes in \mathbb{R}^3 whose intersection w/ xz -plane is the line w/ eq. $3x + 2z = 5$

$$ax + by + cz = d, \quad y = 0 \Rightarrow ax + cz = d$$

$$a = 3\lambda, \quad c = 2\lambda, \quad d = 5\lambda$$

$$(3\lambda)x + by + (2\lambda)z = 5\lambda, \quad \lambda \neq 0$$