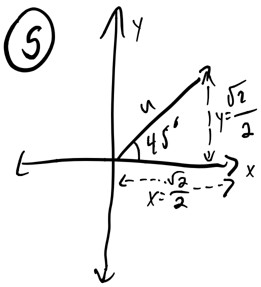
12.1 - Vectors in the Plane (Homework)

(5)



$$\boxed{\left\langle \frac{\sqrt{2}}{2}\|u\|, \frac{\sqrt{2}}{2}\|u\| \right\rangle}$$

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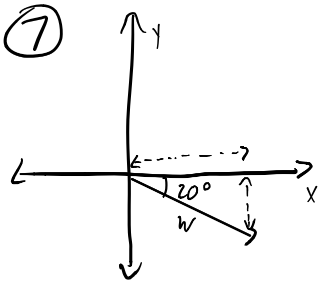
(7)



$$\boxed{\left\langle \cos(340°)\|w\|, \sin(340°)\|w\| \right\rangle}$$

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(9) $P = (3,2)$, $Q = (2,7)$

$$\langle (2-3), (7-2) \rangle = \boxed{\langle -1, 5 \rangle}$$

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(11) $P = (3,5)$, $Q = (1,-4)$

$$\langle (1-3), (-4-5) \rangle = \boxed{\langle -2, -9 \rangle}$$

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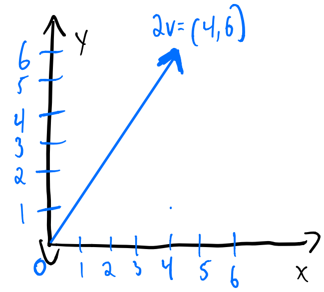
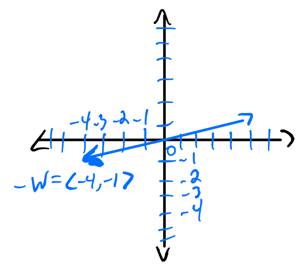
(15) $5\langle 6,2 \rangle = \boxed{\langle 30, 10 \rangle}$

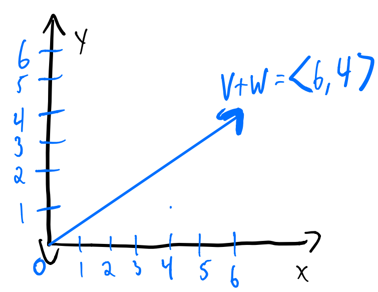
(21) $v = \langle 2,3 \rangle$

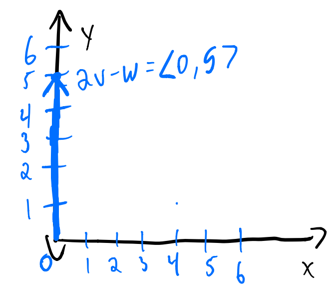
$w = \langle 4,1 \rangle$

$$\boxed{\begin{aligned} 2v &= \langle 4,6 \rangle \\ -w &= \langle -4,-1 \rangle \\ v+w &= \langle 6,4 \rangle \\ 2v-w &= \langle 0,5 \rangle \end{aligned}}$$









(41) Unit vector e_v where $v = \langle 3,4 \rangle$

$\rightarrow$ Magnitude $= \sqrt{9+16} = 5$

$$\boxed{e_v = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle}$$

(47) Unit vector e making an angle of $\frac{4\pi}{7}$ with the x-axis

$$\boxed{e = \left\langle \cos\left(\frac{4\pi}{7}\right), \sin\left(\frac{4\pi}{7}\right) \right\rangle}$$

12.2 - Vectors in Three Dimensions (Homework)

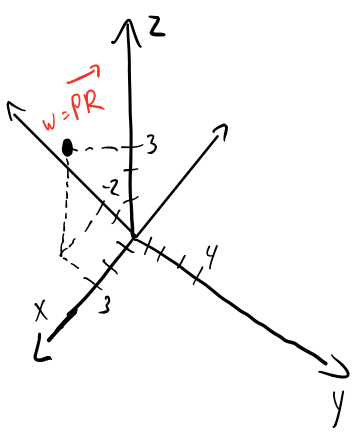
(11) Find the point P such that $w = \overrightarrow{PR}$ has components $\langle 3, -2, 3 \rangle$ and sketch w.

$R = (1, 4, 3)$

$R - P = \langle 3, -2, 3 \rangle$

$P = R - \langle 3, -2, 3 \rangle$

$\boxed{P = (-2, 6, 0)}$



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(13) $v = \langle 4, 8, 12 \rangle$

$v = 4\langle 1, 2, 3 \rangle$

(a) $\langle 2, 4, 6 \rangle$

$2\langle 1, 2, 3 \rangle$

$\Rightarrow$ The scalar 2 is positive.

$\boxed{\text{(a) is parallel and points in the same direction.}}$

(b) $\langle -1, -2, 3 \rangle$

$-1 \langle 1, 2, -3 \rangle$

Not parallel

(c) $\langle -7, -14, -21 \rangle$

$-7 \langle 1, 2, 3 \rangle$

→ The scalar -7 is negative.

(c) is parallel but it does not point in the same direction.

(d) $\langle 6, 10, 14 \rangle$

$2 \langle 3, 5, 7 \rangle$

Not parallel

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19 $-2\langle 8, 11, 3 \rangle + 4\langle 2, 1, 1 \rangle$

$\langle -16, -22, -6 \rangle + \langle 8, 4, 4 \rangle = \boxed{\langle -8, -18, -2 \rangle}$

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25 $u = \langle 4, 2, -6 \rangle$ $v = \langle 2, -1, 3 \rangle$

$u = 2\langle 2, 1, -3 \rangle$

Not parallel

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27 $u = \langle -3, 1, 4 \rangle$ $v = \langle 6, -2, 8 \rangle$

$v = 2\langle 3, -1, 4 \rangle$

Not parallel

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31 Unit vector in the direction opposite to $v = \langle -4, 4, 2 \rangle$

Unit vector of v:

$\text{length} = \sqrt{16 + 16 + 4} = \sqrt{36} = 6$

$e_v = \frac{1}{6}\langle -4, 4, 2\rangle$

$e_v = \langle \frac{-4}{6}, \frac{4}{6}, \frac{2}{6}\rangle$

$e_v = \langle \frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\rangle$

Opposite direction; you can mult. by -1:

ans: $-1\langle \frac{-2}{3}, \frac{2}{3}, \frac{1}{3}\rangle$

$\boxed{\text{ans: } \langle \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\rangle}$

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(49) $P = (5, 5, 2)$

direction vector $v = \langle 0, -2, 1\rangle$

$r(t) = r_0 + tv$

$r_1(t) = \langle 5, 5, 2\rangle + t\langle 0, -2, 1\rangle$

$\boxed{r_1(t) = \langle 5, 5-2t, 2+t\rangle}$

$r_2(t) = \langle 5, 5, 2\rangle + 2t\langle 0, -2, 1\rangle$

$\boxed{r_2(t) = \langle 5, 5-4t, 2+2t\rangle}$

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(51) $r_1(t) = \langle -1, 2, 2\rangle + t\langle 4, -2, 1\rangle = \langle 4t-1, -2t+2, t+2\rangle$

$r_2(s) = \langle 0, 1, 1\rangle + s\langle 2, 0, 1\rangle = \langle 2s, 1, s+1\rangle$

$r_1(t) = r_2(s)$

$\langle 4t-1 = 2s, \ -2t+2 = 1, \ t+2 = s+1\rangle$

$\downarrow$ $t = \frac{1}{2}$ $\qquad \downarrow \qquad \downarrow$ $t = \frac{1}{2}, s = \frac{1}{2}$

$2 - 1 = 2s \qquad t = \frac{1}{2} \qquad \frac{1}{2} + \frac{4}{2} = \frac{1}{2} + \frac{2}{2}$

$s = \frac{1}{2} \qquad \frac{5}{2} \neq \frac{3}{2}$

$r_1(t)$ and $r_2(s)$ don't intersect because the z-coordinate does not equate to a solution. Since r_1 and r_2 don't have the same z-coordinate, they do not intersect.