12.1 Homework

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5. X-component: $||v|| \cos 45$
   y-component: $||v|| \sin 45$
   $\langle ||v|| \sqrt{2}/2, ||v|| \sqrt{2}/2 \rangle$

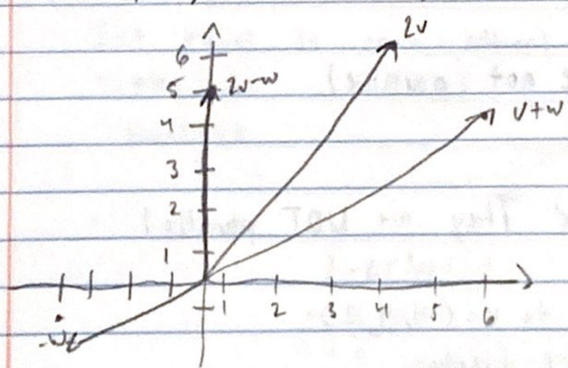
7. X-component: $||w|| \cos -20$
   y-component: $||w|| \sin -20$
   $\langle ||w|| \cos(-20), ||w|| \sin(-20) \rangle$

9. $P = (3,2)$ $Q = (2,7)$
   $\overrightarrow{PQ} = \langle -1, 5 \rangle$

11. $P = (3,5)$, $Q = (1,-4)$
    $\overrightarrow{PQ} = \langle -2, -9 \rangle$

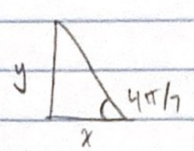
15. $5\langle 6,2 \rangle = \langle 5 \cdot 6, 5 \cdot 2 \rangle = \langle 30, 10 \rangle$

21. $2v = \langle 4,6 \rangle$, $-w = \langle -4,-1 \rangle$, $v+w = \langle 6,4 \rangle$, $2v-w = \langle 0,5 \rangle$



41. $v = \langle 3,4 \rangle$ $||v|| = \sqrt{3^2+4^2} = 5$ $e_v = \frac{1}{||v||} v$
    $e_v = \langle 3/5, 4/5 \rangle$

47.



$\langle \cos 4\pi/7, \sin 4\pi/7 \rangle$

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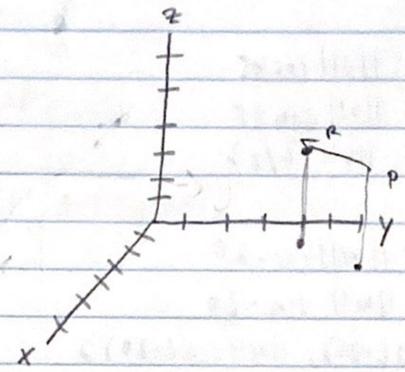
12.2 Homework

11. $R = (1, 4, 3)$
$w = \overrightarrow{PR} = \langle 3, -2, 3 \rangle$
$1 - x = 3 \rightarrow x = -2$
$4 - y = -2 \rightarrow y = 6$
$3 - z = 3 \rightarrow z = 0$
$P = (-2, 6, 0)$



13. $v = \langle 4, 8, 12 \rangle$
(a) $\langle 2, 4, 6 \rangle$ $4/2 = 2$, $8/4 = 2$, $12/6 = 2$ ✓ It is parallel
(b) $\langle -1, -2, 3 \rangle$ $4/-1 = -4$, $8/-2 = -4$, $12/3 = 4$ ✗ It is NOT parallel
(c) $\langle -7, -14, -21 \rangle$ $4/-7 = -4/7$, $8/-14 = -4/7$, $12/-21 = -4/7$ ✓ It is parallel
(d) $\langle 6, 10, 14 \rangle$ $4/6 = 4/6$, $8/10 = 4/5$ ✗ It is NOT parallel

19. $-2\langle 8, 11, 3 \rangle + 4\langle 2, 1, 1 \rangle$
$\langle -16, -22, -6 \rangle + \langle 8, 4, 4 \rangle$
$\langle -8, -18, -2 \rangle$

25. $u = \langle 4, 2, -6 \rangle$, $v = \langle 2, -1, 3 \rangle$
$4/2 = 2$; $2/-1 = -2$ ✗ They are not parallel

27. $u = \langle -3, 1, 4 \rangle$, $v = \langle 6, -2, 8 \rangle$
$-3/6 = -1/2$; $1/-2 = -1/2$; $4/8 = 1/2$ ✗ They are NOT parallel

31. Unit vector in direction opposite to $v = \langle -4, 4, 2 \rangle$
First, find unit vector in same direction:
$e_v = \frac{1}{\|v\|} v$ $\|v\| = \sqrt{(-4)^2 + (4)^2 + 2^2} = \sqrt{36} = 6$
$e_v = \frac{1}{6} \langle -4, 4, 2 \rangle = \langle -2/3, 2/3, 1/3 \rangle$

Opposite Direction: $\langle 2/3, -2/3, -1/3 \rangle$

49. Find two different vector parametrizations of line through $P=(5,5,2)$ with direction vector $v=\langle 0,-2,1\rangle$

Vector Parametrization: $r(t) = \langle x_0, y_0, z_0\rangle + t\langle a,b,c\rangle$

$r_1(t) = \langle 0,-2,1\rangle + t\langle 5,5,2\rangle$

$= \langle 0,-2,1\rangle + \langle 5t,5t,2t\rangle$

$\boxed{r_1(t) = \langle 5t, -2+5t, 1+2t\rangle}$

Parallel Vector: $2\cdot\langle 0,-2,1\rangle = \langle 0,-4,2\rangle$

$r_2(t) = \langle 0,-2,1\rangle + t\langle 0,-4,2\rangle$

$= \langle 0,-2,1\rangle + \langle 0,-4t,2t\rangle$

$\boxed{r_2(t) = \langle 0, -2-4t, 1+2t\rangle}$

51. Show $r_1(t) = \langle -1,2,2\rangle + t\langle 4,-2,1\rangle$ and $r_2(t) = \langle 0,1,1\rangle + t\langle 2,0,1\rangle$ do not intersect

$r_1(t) = \langle -1,2,2\rangle + \langle 4t,-2t,t\rangle$

$r_1(t) = \langle -1+4t, 2-2t, 2+t\rangle$

$r_2(s) = \langle 0,1,1\rangle + s\langle 2,0,1\rangle$

$= \langle 0,1,1\rangle + \langle 2s,0,s\rangle$

$r_2(s) = \langle 2s, 1, 1+s\rangle$

Set equal to each other!

$-1+4t = 2s$ $\qquad 2-2t=1$ $\qquad 2+t = 1+s$

$s = \frac{-1+4t}{2}$ $\qquad\qquad 2+t = 1 - \frac{1+4t}{2}$

$4+2t = \frac{1}{2} - 1 + 4t$

$2-2(3/8) \stackrel{?}{=} 1$

$2 - 6/8 = 1$ $\qquad 4 = \frac{-1}{2} + 4t$

???

Therefore, they do NOT intersect $\qquad 6/4 = 4t \qquad 6/16 = t \qquad t = 3/8$