Chs. 12.1, 12.2 HW

9/13/20

12.1

5. $\|u\|\langle \cos 45°, \sin 45° \rangle = \|u\| \langle \sqrt{2}/2, \sqrt{2}/2 \rangle$

7. $\|w\|\langle \cos(-20°), \sin(-20°) \rangle = \|w\| \langle \cos 20°, -\sin 20° \rangle$

9. $P = (3,2)$, $Q = (2,7)$

$\vec{PQ} = \langle 2-3, 7-2 \rangle = \langle -1, 5 \rangle$

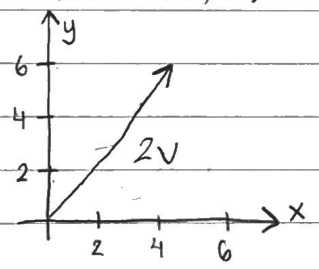
11. $P = (3,5)$, $Q = (1,-4)$

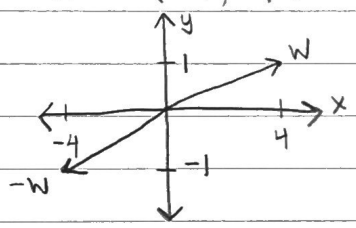
$\vec{PQ} = \langle 1-3, -4-5 \rangle = \langle -2, -9 \rangle$

15. $5\langle 6, 2 \rangle = \langle 5 \cdot 6, 5 \cdot 2 \rangle = \langle 30, 10 \rangle$

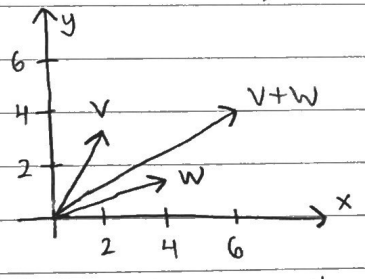
21. $v = \langle 2, 3 \rangle$, $w = \langle 4, 1 \rangle$

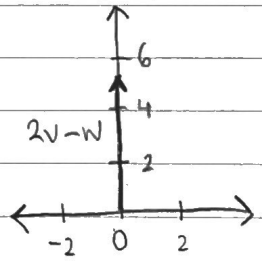
$2v = \langle 4, 6 \rangle$ $\qquad -w = \langle -4, -1 \rangle$





$v + w = \langle 6, 4 \rangle$ $\qquad 2v - w = \langle 4, 6 \rangle - \langle 4, 1 \rangle = \langle 0, 5 \rangle$





Unit vector e_v where $v = \langle 3, 4 \rangle$

$e_v = \dfrac{1}{\|v\|} v$; $\|v\| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$; $e_v = \dfrac{1}{5}\langle 3, 4 \rangle = \left\langle \dfrac{3}{5}, \dfrac{4}{5} \right\rangle$

47. Unit vector e making an angle of $4\pi/7$ with the x-axis

$e = \left\langle \cos \dfrac{4\pi}{7}, \sin \dfrac{4\pi}{7} \right\rangle = \langle -0.22, 0.97 \rangle$

12.2

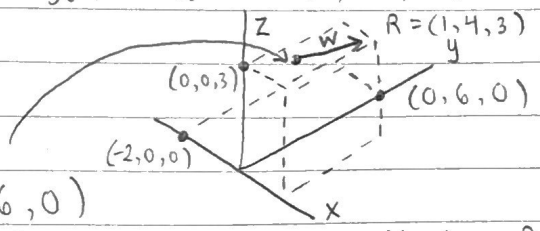
Find the point P such that $w = \vec{PR}$ has components $\langle 3, -2, 3 \rangle$, and sketch w.

$P = (x_0, y_0, z_0) \Rightarrow \vec{PR} = \langle 1 - x_0, 4 - y_0, 3 - z_0 \rangle = \langle 3, -2, 3 \rangle$

$1 - x_0 = 3$

$4 - y_0 = -2$

$3 - z_0 = 3$



$x_0 = -2,\ y_0 = 6,\ z_0 = 0 \Rightarrow P = (-2, 6, 0)$

13. Let $v = \langle 4, 8, 12 \rangle$. Which of the following vectors is $\parallel$ to v? Which point in the same direction?

(a) $\langle 2, 4, 6 \rangle \Rightarrow$ (+) scalar multiple of $v \Rightarrow \parallel$ & same direction

(b) $\langle -1, -2, 3 \rangle \Rightarrow$ not $\parallel$

(c) $\langle -7, -14, -21 \rangle \Rightarrow -7/4\, v \Rightarrow \parallel$ & opposite direction

(d) $\langle 6, 10, 14 \rangle \Rightarrow$ not a constant multiple of $v \Rightarrow$ not $\parallel$

19. $-2\langle 8, 11, 3 \rangle + 4\langle 2, 1, 1 \rangle = \langle -16, -22, -6 \rangle + \langle 8, 4, 4 \rangle$

$= \langle -8, -18, -2 \rangle$

25. $u = \langle 4, 2, -6 \rangle,\ v = \langle 2, -1, 3 \rangle \Rightarrow$ not $\parallel$

27. $u = \langle -3, 1, 4 \rangle,\ v = \langle 6, -2, 8 \rangle \Rightarrow$ not $\parallel$

★ Unit vector in the direction opposite to $v = \langle -4, 4, 2 \rangle$

$-e_v = -\dfrac{1}{\|v\|} v$; $\|v\| = \sqrt{(-4)^2 + 4^2 + 2^2} = 6$; $-e_v = -\dfrac{1}{6}\langle -4, 4, 2 \rangle =$

$\langle -4/-6, 4/-6, 2/-6 \rangle = \langle 2/3, -2/3, -1/3 \rangle$

★ Find 2 diff. vector parametrizations of the line through $P = (5, 5, 2)$ w/ dir. vector $v = \langle 0, -2, 1 \rangle$.

$r_1(t) = \langle 5, 5, 2 \rangle + t\langle 0, -2, 1 \rangle$

$r_2(t) = \langle 5, 5, 2 \rangle + t\langle 0, -20, 10 \rangle$

★ Show that the lines $r_1(t) = \langle -1, 2, 2 \rangle + t\langle 4, -2, 1 \rangle$ & $r_2(t) = \langle 0, 1, 1 \rangle + t\langle 2, 0, 1 \rangle$ do not intersect.

They intersect if there exist parameter values t_1 & t_2 s.t.

$\langle -1, 2, 2 \rangle + t_1\langle 4, -2, 1 \rangle = \langle 0, 1, 1 \rangle + t_2\langle 2, 0, 1 \rangle$

$\langle -1 + 4t_1, 2 - 2t_1, 2 + t_1 \rangle = \langle 2t_2, 1, 1 + t_2 \rangle$

$-1 + 4t_1 = 2t_2 \Rightarrow t_2 = 1/2$

$2 - 2t_1 = 1 \Rightarrow t_1 = 1/2$

$2 + t_1 = 1 + t_2 \Rightarrow t_2 = 3/2$

Equations do not have solutions $\Rightarrow$ 2 lines do not intersect