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Exercise 14.3

Q3.

$$\frac{\partial}{\partial y}\frac{y}{x+y} = \frac{(x+y)-y(x+y)'}{(x+y)^2}$$

$$= \frac{x}{(x+y)^2}$$

Q21. $z=(\sin x)(\sin y)$

$$\frac{dz}{dx} = \cos x(\sin y)$$

$$\frac{dz}{dy} = \cos y(\sin x)$$

Q5. $f_z(2,3,1)$

$f(x,y,z) = xyz$

$f_z = 2\times3\times1 = 6$

Q27. $w = e^{r+s}$

$$\frac{dw}{dr} = e^{r+s}$$

$$\frac{dw}{ds} = e^{r+s}$$

Q17. $\frac{d}{dx} = 1\cdot\frac{1}{y} = \frac{1}{y}$

$$\frac{d}{dy} = x\cdot(-y^{-2}) = -xy^{-2}$$

Q31. $z = e^{-x^2-y^2}$

$$\frac{dz}{dx} = -2x\cdot e^{-x^2-y^2}$$

$$\frac{dz}{dy} = -2y\cdot e^{-x^2-y^2}$$

Q19. $z=(9-x^2-y^2)^{\frac{1}{2}}$

$$\frac{d}{dx} = (\tfrac{1}{2})\cdot -2x(9-x^2-y^2)^{-\frac{1}{2}}$$

$$= \frac{-x}{\sqrt{9-x^2-y^2}}$$

$$\frac{d}{dy} = (\tfrac{1}{2})\cdot -2y(9-x^2-y^2)^{-\frac{1}{2}}$$

$$= \frac{-y}{\sqrt{9-x^2-y^2}}$$

Q39. $Q = \frac{L}{M}\cdot e^{\frac{-Lt}{M}}$

$$\frac{dQ}{dL} = \frac{M-Lt}{M^2}\cdot e^{-\frac{Lt}{M}}$$

$$\frac{\partial Q}{dM} = \frac{L^2t - ML}{M^3}\cdot e^{-\frac{Lt}{M}}.$$

$$\frac{\partial Q}{dt} = -\frac{L^2}{M^2}e^{-\frac{Lt}{M}}$$

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Q47.

(a) $I(95,50)$

$= 45.33 + 0.6845 \cdot 95 + 5.758 \cdot 50 - 0.00365 \cdot 95^2 - 0.1565 \cdot 50 \cdot 95$
$+ 0.001 \cdot 50 \cdot 95^2$

$= 73.19125$

(b) $\frac{dI}{dT} = 0.6845 - 2 \times 0.00365 \times T - 0.1565H + 0.001 \times 2 \times T \times H$

$(95,50) = 1.666 \quad > 0$ increasing

$\frac{dI}{dH} = 5.758 - 0.1565T + 0.001T^2$

$= -0.0845 < 0$ decreasing

$\therefore$ therefore, $\frac{dI}{dT}$ tells us increase in I per degree increase in T. the partial derivatives is 1.666.

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Exercise 14.4

Q3. $f(x,y) = x^2y + xy^3$

$\frac{df}{dx} = 2xy + \cancel{x} y^3$

$\frac{df}{dy} = x^2\cancel{y} + 3y^2 \cdot x$

$f_x(2,1) = 4 + 1 = 5$

$f_y(2,1) = 4 + 6 = 10$

$f(2,1) = 6$

$z - 6 = 5(x-2) + 10(y-1)$

$z = 5x + 10y - 20 + 6$

$= 5x + 10 - 14$

Q5. $f(x,y) = x^2 + y^{-2}$ $(4,1)$

$\frac{d}{dx} = 2x$ $\quad \frac{d}{dx}(4,1) = 8$

$\frac{d}{dy} = -2y^{-3}$ $\quad \frac{d}{dy}(4,1) = -2$

$f(4,1) = 16 + 1 = 17$

$z - 17 = 8(x-4) + (-2)(y-1)$

$z = 8x - 2y - 13$

Q7. $F(r,s) = r^2 s^{-\frac{1}{2}} + s^{-3}$ $(2,1)$

$\frac{d}{dr} = 2r \cdot s^{-\frac{1}{2}}$ $\quad f_r(2,1) = 4$

$\frac{d}{ds} = -\frac{1}{2} \cdot s^{-\frac{3}{2}} \cdot r^2 - 3s^{-4}$

$f_s(2,1) = 4 \cdot (-\frac{1}{2}) - 3$

$= -5$

$f(2,1) = 4 + 1 = 5$

$z - 5 = 4(\cancel{x}\, r - 2) + (-5)(\cancel{y}\, s - 1)$

$z = \cancel{4x - 5y}$

$4r - 5s + 2$

Q13.

$L(x,y) \rightarrow f(x,y) = x^2y^3$

$\frac{d}{dx} = 2xy^3$

$\frac{d}{dy} = x^2 \cdot 3y^2$

$f_x(2,1) = 4$

$f_y(2,1) = 12$

$f(2,1) = 4$

$L(x,y) = 4 + 4(x-2) + 12(y-1)$

$= 4x + 12y - 16$

$f(2.01, 1.02) \approx 4.28$

$f(1.19, 1.01) \approx 4$

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Q15. $f(x,y) = x^3 y^{-4}$

$\frac{d}{dx} = 3x^2 y^{-4}$ $f_x(2,1) = 12$

$\frac{d}{dy} = x^3 y^{-5} \cdot (-4)$ $f_y(2,1) = -32$

$f(2,1) = 8$

$\therefore z - 8 = 12(x-2) + (-32)(y-1)$

$z = 12x - 32y + 16$

$\Delta f \approx 3.56$

Q17.

$f(x,y) = e^{x^2+y}$

$\frac{d}{dx} = 2x \cdot e^{x^2+y}$ $f_x(0,0) = 0$

$\frac{d}{dy} = e^{x^2+y}$ $f_y(0,0) = 1$

$f(0,0) = 1$

~~$z - 1 =$~~

$L(0,0) = 1 + 0(x-0) + 1(y-0)$

$L(x,y) = y + 1$

$L(0.01, -0.02) = 0.98$

Q23.

$(2.01)^3 (1.02)^2$

$x^3 y^2$

$\frac{d}{dx} = 3x^2 y^2$ $f_x(2.01, 1.02) = 12.609$

$\frac{d}{dy} = x^3 \cdot 2y$ $f_y(2.01, 1.02) = 16.566$

$L(x,y) = 8.448$ $f(2.01, 1.02) = 8.448$

the value is 8.448.

~~$L(2.01, 1.02)$~~

Q.25

$\sqrt{3.01^2 + 3.99^2} = 4.998$

$= \sqrt{x^2 + y^2}$

$\frac{d}{dx} = 2x \cdot (x^2+y^2)^{-\frac{1}{2}}$

$f_x = 1.204$

$\frac{d}{dy} = 2y \cdot (x^2+y^2)^{-\frac{1}{2}}$

$f_y = 1.596$

$f(x,y) = 4.998$

$L(x,y) = \cancel{1.596}$ $4.998 + 1.204(x-3.01) + 1.596(y-3.99)$

$\therefore$ the value is 4.998

Q27. $\sqrt{(1.9)(2.02)(4.05)}$

$= 3.945$

$\sqrt{xyz} \Rightarrow \frac{d}{dx} = (xyz)^{-\frac{1}{2}} \cdot yz$

$f_x = 2.075$ $\frac{d}{dy} = (xyz)^{-\frac{1}{2}} \cdot xz$

$f_y = 0.9245$

$f_z = 0.9734$ $\frac{d}{dz} = (xyz)^{-\frac{1}{2}} \cdot xy$

$f(x,y,z) = 3.945$

$L(x,y,z) = 3.945 + 2.075(x-1.9) + 0.9245(y-2.02) + 0.9734(z-4.05)$

$\therefore$ the value is 3.945.

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14.5

Q7. $h(x,y,z)=xyz^{-3}$

$h_x = yz^{-3}$

$h_y = xz^{-3}$

$h_z = -3z^{-4}\cdot xy$

$\nabla F \Rightarrow \nabla h = (yz^{-3}, xz^{-3}, -3z^{-4}\cdot xy)$

Q11. $f(x,y)=x^2-3xy$

$r(t)=(\cos t, \sin t)$ $t=0$

$\frac{d}{dt}f(r(t))$?

Answer: $r_x(t)=\cos t$, $r_y(t)=\sin t$

$f(r_x(t), r_y(t)) = (\cos t)^2 - 3\cos t\cdot \sin t$

$f'(r(t)) = (-\sin t)\cdot 2\cos t + 3\sin t^2 - 3\cos t^2$

when $t=0$

$f'(r(t)) = -3$

Q13. $f(x,y)=\sin(xy)$

$r(t)=(e^{2t}, e^{3t})$ $t=0$

$\frac{d}{dt}f(r(t))$?

Answer:

$f(r(t)) = \sin(e^{2t}\cdot e^{3t})$

$= \sin e^{5t}$

$f'(r(t)) = \cos e^{5t}\cdot 5$

$= 5\cos 1$

≈ 2.7015

Q19. $g(x,y,z)=xyz^{-1}$

$r(t)=(e^t, t, t^2)$ $t=1$

$\frac{d}{dt}g(r(t))$?

Answer:

$g(r(t)) = (e^t\cdot t\cdot t^{-2})$

$= (e^t\cdot t^{-1})$

$g'(r(t)) = -t^{-2}\cdot e^t + e^t\cdot t^{-1}$

$= e^t(t^{-1} + (-t^{-2}))$

~~$(t=1) = e^{10}(0^-$~~

$t=1 \Rightarrow 0$

$\therefore g'(r(t))=0$

Q27. $f(x,y)=\ln(x^2+y^2)$

$v=3i-2j$ $P=(1,0)$

$\therefore$ direction $(3,-2)$

$\nabla F = \left(\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}\right)$

~~$\nabla F(1,0) = \frac{3-2}{}$~~

$|(3,-2)| = \sqrt{9+4} = \sqrt{13}$

$u = \frac{3,-2}{\sqrt{13}} = \left(\frac{3\sqrt{13}}{13}, \frac{-2\sqrt{13}}{13}\right)$

$\nabla F(1,0) = (2,0)$

$u\cdot\nabla F = \left(\frac{3\sqrt{13}}{13}, \frac{-2\sqrt{13}}{13}\right)\cdot(2,0)$

$= \left(\frac{6\sqrt{13}}{13} + 0\right)$

$= \frac{6\sqrt{13}}{13}$

Q31. $f(x,y)=x^2+4y^2$

$P=(3,2)$ ~~direction (0,0)~~

$\therefore \nabla F=(2x,8y)$

$\nabla F(+3,+2)=(+6,+16)$

$\cancel{u\ ||(0,0)||\ 0}$ $(0,0)-(3,2)$

$=(-3,-2)$

$\cancel{u=(0,0,0)}$

$|(-3,-2)|=\sqrt{13}$

$u=\dfrac{(3,2)}{\sqrt{13}}=\left(\dfrac{-3}{\sqrt{13}},\dfrac{-2}{\sqrt{13}}\right)$

$u\cdot\nabla F=\left(\dfrac{-3}{\sqrt{13}},\dfrac{-2}{\sqrt{13}}\right)\cdot(6,16)$

$=\dfrac{-18}{\sqrt{13}}+\dfrac{-32}{\sqrt{13}}$

$=\dfrac{-50}{\sqrt{13}}$

Q33.

A: Start point $=(3,9,4)$

B: end point $=(5,7,3)$

$\therefore B-A=(2,-2,-1)$

unit direction vector $=\dfrac{(2,-2,-1)}{3}$

$\nabla T=(e^{y-z},\ x\cdot e^{y-z},\ x\cdot(-1)\cdot e^{y-z})$

$\nabla T(3,9,4)=(e^5,\ 3e^5,\ -3e^5)$

$u\cdot\nabla T=(e^5,3e^5,-3e^5)\cdot\left(\frac{2}{3},-\frac{2}{3},-\frac{1}{3}\right)$

$=\left(\dfrac{2e^5}{3},-2e^5,e^5\right)$

$=\frac{2}{3}e^5-e^5$

$=-\frac{1}{3}e^5\approx-49.5$

Q37. $\nabla f_p=(2,-4,4)$

$v=(2,1,3)$

$||v||=\sqrt{4+1+9}=\sqrt{14}$

$u=\dfrac{(2,1,3)}{\sqrt{14}}$

$u\cdot\nabla f_p=\left(\frac{2}{\sqrt{14}},\frac{1}{\sqrt{14}},\frac{3}{\sqrt{14}}\right)\cdot(2,-4,4)$

$=\dfrac{4}{\sqrt{14}}+\left(-\dfrac{4}{\sqrt{14}}\right)+\dfrac{12}{\sqrt{14}}$

$=\dfrac{12}{\sqrt{14}}$

$\Rightarrow \dfrac{12}{\sqrt{14}}>0$

$\therefore$ it is increasing

Q39. $f(x,y,z)=\sin(xy+z)$

$P=(0,-1,\pi)$

$u=(\cos 30°,\ \sin 30°)$

$=\left(\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

$\nabla f=(\cos(xy+z)\cdot y,$
$\cos(xy+z)\cdot x,$
$\cos(xy+z))$

$\nabla f(0,-1,\pi)$

$=(-\cos\pi,0,\cos\pi)$

$=(+1,0,-1)$

~~$\nabla f\cdot u=$~~

$|\langle 1,0,-1\rangle|\cdot\cos 30°$

$=\cancel{\frac{\sqrt{3}}{2}+0+}$ $=\dfrac{\sqrt{2}\cdot\sqrt{3}}{2}$

$\sqrt{2}\cdot\frac{\sqrt{3}}{2}$

$\therefore =\dfrac{\sqrt{6}}{2}$

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Q41. $x^2+y^2+(-z^2)=6$

$P=(3,1,2)$

$\nabla(f) = 2x+2y-2z = C^*(3,1,2)$

$2x=3C \quad 2y=C \quad -2z=2C$

$x=\frac{3C}{2} \quad y=\frac{C}{2} \,.\, z=\frac{2C}{-2}=-C$

$(\frac{3C}{2})^2+(\frac{C}{2})^2-(-C)^2=6$

$\frac{9C^2}{4}+\frac{C^2}{4}-C^2=6$

$\frac{6}{4}C^2=6$

$C^2=4$

$C=2$

$(2x,2y,-2z)=(3C, C,-2C)=(6,2,-4).$

Q43. $\frac{x^2}{4}+\frac{y^2}{9}+z^2=1$

$V=(1,1,-2)$

$\nabla f = \frac{1}{2}x+\frac{2}{9}y+2z = C^*(1,1,-2)$

$\frac{1}{2}x = C \quad \frac{2}{9}y=C \quad 2z=-2C$

$x = 2C \quad y=\frac{9}{2}C \quad z=-C$

$\frac{4C^2}{4}\cancel{\frac{2a}{4}}+\frac{81C^2}{4}\cdot\frac{1}{9}+C^2=1$

$C^2\cancel{\frac{1}{2}C}+\frac{9}{4}C^2+C^2=1$

$\frac{17}{4}C^2=1$

$C^2=\frac{4}{17}$

$C=\sqrt{\frac{4}{17}}=\frac{2}{\sqrt{17}}$ or $\frac{-2}{\sqrt{17}}$

$\therefore$ point $=(\frac{4}{\sqrt{17}},\frac{9}{\sqrt{17}},-\frac{2}{\sqrt{17}})$ or $(-\frac{4}{\sqrt{17}},-\frac{9}{\sqrt{17}},\frac{2}{\sqrt{17}})$