

14.3

$$3. \frac{\partial}{\partial y} \frac{y}{x+y}$$

$$= \frac{1(x+y) - y(1)}{(x+y)^2}$$

$$= \frac{x+y-y}{(x+y)^2}$$

$$= \frac{x}{(x+y)^2}$$

17. $z = \frac{x}{y}$

$$\frac{\partial}{\partial x} = \frac{1 \cdot y - 0}{y^2} = \frac{1}{y}$$

$$\frac{\partial}{\partial y} = \frac{-x}{y^2}$$

21. $z = \sin x \sin y$

$$\frac{\partial}{\partial x} = \cos x \sin y$$

$$\frac{\partial}{\partial y} = \sin x \cos y$$

31. $z = e^{-x^2-y^2}$

$$\frac{\partial}{\partial x} = e^{-x^2-y^2} \cdot (-2x)$$

$$= -2xe^{-x^2-y^2}$$

$$\frac{\partial}{\partial y} = -2ye^{-x^2-y^2}$$

5. $f(x, y, z) = xyz$

$$f_z(2, 3, 1)$$

$$f_z = xy$$

$$= 2 \times 3 = 6$$

19. $z = \sqrt{9-x^2-y^2} = (9-x^2-y^2)^{\frac{1}{2}}$

$$\frac{\partial}{\partial x} = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}} \cdot (-2x)$$

$$= -x \cdot \frac{1}{\sqrt{9-x^2-y^2}}$$

$$\frac{\partial}{\partial y} = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}} \cdot (-2y)$$

$$= -y \cdot \frac{1}{\sqrt{9-x^2-y^2}}$$

27. $w = e^{rts}$

$$\frac{\partial}{\partial r} = e^{rts} \cdot 1 = e^{rts}$$

$$\frac{\partial}{\partial s} = e^{rts} \cdot 1 = e^{rts}$$

39. $Q = \frac{L}{M} e^{-\frac{Lt}{M}}$

$$\frac{\partial}{\partial L} = \frac{M-Lt}{M^2} e^{-\frac{Lt}{M}}$$

$$\frac{\partial}{\partial M} = \frac{L(1-t)}{M^3} e^{-\frac{Lt}{M}}$$

$$\frac{\partial}{\partial t} = -\frac{L^2}{M^2} e^{-\frac{Lt}{M}}$$



49.

(a) =

$$I(95, 50) = 45.33 + 0.6845 \times 95 + 5.758 \times 50 - 0.00365 \times 95^2 - 0.1565 \times 50 \times 95 + 0.00145 \times 50 \times 95^2$$
$$= 73.1913.$$

$$(b) \frac{\partial I}{\partial T} = 0.6845 - 0.0073T - 0.1565H + 0.0029 \times 2T$$
$$= 1.666$$



14. 4.

$$3. f(x,y) = x^2y + xy^3 \quad (2.11)$$

$$f'(x) = 2xy + y^3$$

$$f'(y) = x^2 + 3y^2x$$

$$f'(x) = 4 + 1 = 5$$

$$f'(y) = 4 + 6 = 10$$

$$z - 6 = 5(x-2) + 10(y-1)$$

$$z = 5x - 10 + 10y - 10 + 6$$

$$= 5x + 10y - 14$$

$$7. F(x,y) = x^2y^{-\frac{1}{2}} + y^{-3} \quad (2.11)$$

$$F'(x) = 2xy^{-\frac{1}{2}}$$

$$F'(y) = x^2 \cdot -\frac{1}{2}y^{-\frac{3}{2}} + -3y^{-4}$$

$$F'(x) = 4 \times 1 = 4$$

$$F'(y) = 4 \cdot (-\frac{1}{2}) + (-3)$$

$$= -5$$

$$z - 5 = 4(x-2) + -5(y-1)$$

$$= 4x - 8 - 5y + 5$$

$$z = 4x - 8 - 5y + 5 + 5$$

$$= 4x - 5y + 2$$

$$15. f(x,y) = x^3y^{-4} \quad (2.11)$$

$$f'(x) = 3x^2y^{-4}$$

$$f'(y) = -4x^3y^{-5}$$

$$f(2,1) = 8 \times 1^{-4} = 8$$

$$f'(x) = 3 \times 4 \times 1^{-4} = 12$$

$$f'(y) = -4 \times 8 \times 1^{-5} = -32$$

$$f(x,y) \approx 8 + 12(x-2) - 32(y-1)$$

$$f(2.3, 0.9) = 11.56$$

$$5. f(x,y) = x^2 + y^{-2} \quad (4.11)$$

$$f'(x) = 2x + 0$$

$$f'(y) = -2y^{-3}$$

$$f'(x) = 4 + 0$$

$$f'(y) = -2$$

$$z - 1 = 4(x-2) + -2(y-1)$$

$$z = 4x - 8 - 2y + 2 + 1$$

$$= 4x - 2y - 5$$

$$13. f(x,y) = x^2y^3 \quad (3.2.1)$$

$$f'(x) = 2x$$

$$f'(y) = 3x^2y^2$$

$$f(2,1) = 4 \times 1 = 4$$

$$f'(x) = 4 \quad f'(y) = 3 \times 4 \times 1 = 12$$

$$L(x,y) = 4 + 4(x-2) + 12(y-1)$$

$$f(x,y) \approx 4 + 4(x-2) + 12(y-1)$$

$$f(2.01, 1.02) = 4.28$$

$$f(1.97, 1.01) = 4$$

$$\Delta f = f(2.3, 0.9) - f(2, 1)$$

$$= 11.56 - 8$$

$$\approx 3.56$$



$$17. f(x,y) = e^{x^2+y} \quad (0,0)$$

$$f_x(x,y) = e^{x^2+y} \cdot 2x$$

$$f_y(x,y) = e^{x^2+y}$$

$$f(0,0) = e^0 = 1$$

$$f_x(0,0) = e^0 \cdot 0 = 0$$

$$f_y(0,0) = e^0 = 1$$

$$L(x,y) = 1 + 0(x-0) + 1(y-0)$$

$$f(x,y) \approx 1 + y$$

$$f(0.01, -0.02) = 1 - 0.02 \\ = 0.98$$

$$23. (2.01)^3 (1.02)^2$$

$$= 8.44$$

$$25. \sqrt{3.01^2 + 3.98^2}$$

$$= 4.998$$

$$27. \sqrt{1.9 \times 2.02 \times 4.05}$$

$$= 3.94$$



14.5

$$7. h(x, y, z) = xyz^{-3}$$

$$h_x = yz^{-3} \neq h_y = xz^{-3}$$

$$h_z = -3z^{-4}xy$$

$$\nabla h = \langle yz^{-3}, xz^{-3}, -3z^{-4}xy \rangle$$

$$13. f(x, y) = \sin(xy) \quad v(t) = (e^{2t}, e^{5t})$$

to

$$x = e^{2t} \quad y = e^{5t}$$

$$f(v(t)) = \sin(e^{2t} \cdot e^{5t})$$

$$= \sin e^{7t}$$

$$\frac{d}{dt} f(v(t)) = 7 \cos e^{7t} \cdot e^{7t}$$

to

$$\frac{d}{dt} f(v(t)) = 7 \cos 1 \times 1$$

$$= 7 \cos 1$$

$$= 4.999$$

$$21. f(x, y) = \ln(x^2 + y^2)$$

$$v = 3i - 2j \quad p = (1, 0)$$

$$f_x = \frac{2x}{x^2 + y^2} \quad f_y = \frac{2y}{x^2 + y^2}$$

$$\nabla f = \left\langle \frac{2x}{x^2 + y^2}, \frac{2y}{x^2 + y^2} \right\rangle$$

$$| \langle 3, -2 \rangle | = \sqrt{9 + 4} = \sqrt{13}$$

$$u = \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$$

$$\nabla f(1, 0) = \langle \frac{2}{1}, 0 \rangle = \langle 2, 0 \rangle$$

$$\nabla f \cdot u = \frac{2}{\sqrt{13}} \cdot \frac{3}{\sqrt{13}} + 0 = \frac{6}{13}$$

$$11. f(x, y) = x^2 - 3xy$$

$$v(t) = \langle \cos t, \sin t \rangle \quad t=0$$

$$x = \cos t \quad y = \sin t$$

$$f(v(t)) = \cos^2 t - 3 \cos t \sin t$$

$$\frac{d}{dt} f(v(t)) = -2 \sin t \cos t - 3 \cos^2 t$$

$$t=0 \quad \frac{d}{dt} f(v(t)) = -3$$

$$19. g(x, y, z) = xyz^{-1}$$

$$v(t) = \langle e^t, t, t^2 \rangle \quad t=1$$

$$x = e^t \quad y = t \quad z = t^2$$

$$g(v(t)) = e^t \cdot t \cdot (t^2)^{-1}$$

$$= e^t \cdot t \cdot \frac{1}{t^2}$$

$$\frac{d}{dt} g(v(t)) = \frac{d}{dt} \left(\frac{e^t}{t} \right) = e^t \cdot (-2t^{-3})$$

$$\frac{d}{dt} g(v(t)) = e^1 \cdot (-2 \cdot 1^{-3})$$

$$= -5.44$$

$$31. f(x, y) = x^2 + 4y^2 \quad p = (3, 2)$$

$$\nabla f = \langle 2x, 8y \rangle$$

$$\langle 0, 0 \rangle - \langle 0, 3, 2 \rangle = \langle -3, -2 \rangle$$

$$| \langle -3, -2 \rangle | = \sqrt{13}$$

$$u = \left\langle \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$$

$$\nabla f(3, 2) = \langle 6, 16 \rangle$$

$$\nabla f \cdot u = 6 \cdot \frac{-3}{\sqrt{13}} + 16 \cdot \frac{-2}{\sqrt{13}} = -\frac{50}{\sqrt{13}}$$



38.

$$5-3=2$$

$$7-9=-2$$

$$3-4=-1$$

$$v = \langle 2, -2, -1 \rangle$$

$$\nabla T = \langle e^{x-z}, xe^{x-z}, -xe^{x-z} \rangle$$

$$|v| = \sqrt{4+4+1} = 3$$

$$u = \left\langle \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3} \right\rangle$$

$$\nabla T = \langle e^5, 3e^5, -3e^5 \rangle$$

$$\frac{2}{3}e^5 - \frac{6}{3}e^5 + e^5$$

$$= -e^5$$

39.

I know the way

to do it, but don't know

to calculate ~~the~~ ∇ with

the π .

$$43. \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1 \quad (1, 1, -2)$$

$$\frac{x}{2} + \frac{2y}{9} + 2z = 1$$

$$\text{grad } f = \left\langle \frac{x}{2}, \frac{2y}{9}, 2z \right\rangle = \langle 1, 1, -2 \rangle$$

$$\frac{x}{2} = c \quad \frac{2y}{9} = c \quad 2z = -2c \quad \text{seems correct.}$$

37.

$$D_{xy} = \langle 2, -4, 4 \rangle \cdot \langle 2, 1, 3 \rangle / \sqrt{4+16+16}$$

$$= 2 \times 2 - 4 + 12 / \sqrt{4+16+16}$$

$$= 4 - 4 + 12 / \sqrt{4+16+16}$$

$$= 12 / \sqrt{4+16+16}$$

increasing because

the direction is

positive.

$$41. x^2 + y^2 - z^2 = 6$$

$$f_x = 2x \quad (3, 1, 2)$$

$$f_y = 2y$$

$$f_z = -2z$$

$$\nabla f = \langle 2x, 2y, -2z \rangle$$

$$\langle 2 \times 3, 2 \times 1, -2 \times 2 \rangle$$

$$\langle 6, 2, -4 \rangle$$

$$x = 2c \quad y = \frac{1}{2}c \quad z = -\frac{1}{2}c$$

$$\frac{2c}{2} + \frac{2}{9} \cdot \frac{1}{2}c - \frac{1}{2}c = 1$$

$$c + c - \frac{1}{2}c = 1$$

$$\frac{3}{2}c = 1$$

