Math 251 Shaun Goda Section 23 HW #13

14.3:

3) $\frac{\partial}{\partial y}\frac{y}{x+y} = \frac{(1)(x+y)-(y)(1)}{(x+y)^2} = \boxed{\frac{x}{(x+y)^2}}$

5) $f_z(x,y,z) = xy \quad f_z(2,3,1) = \boxed{6}$

17) $\frac{\partial}{\partial x}\left(\frac{x}{y}\right) = \frac{1}{y} \quad \frac{\partial}{\partial y}\left(\frac{x}{y}\right) = \boxed{-\frac{x}{y^2}}$

19) $\frac{\partial}{\partial x}\left((9-x^2-y^2)^{\frac{1}{2}}\right) = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}(-2x) = \boxed{\frac{-x}{\sqrt{9-x^2-y^2}}}$

$\frac{\partial}{\partial y}\left((9-x^2-y^2)^{\frac{1}{2}}\right) = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}(-2y) = \boxed{\frac{-y}{\sqrt{9-x^2-y^2}}}$

21) $\frac{\partial}{\partial x}(\sin x \sin y) = \cos x \sin y + \sin x(0) = \boxed{\cos x \sin y}$

$\frac{\partial}{\partial y}(\sin x \sin y) = (0)\sin y + \sin x \cos y = \boxed{\sin x \cos y}$

27) $\frac{\partial}{\partial r} e^{r+s} = e^{r+s}(1) = \boxed{e^{r+s}} \quad \frac{\partial}{\partial s} e^{r+s} = e^{r+s}(1) = \boxed{e^{r+s}}$

31) $\frac{\partial}{\partial x} e^{-x^2-y^2} = e^{-x^2-y^2}(-2x) = \boxed{-2x e^{-x^2-y^2}}$

$\frac{\partial}{\partial y} e^{-x^2-y^2} = e^{-x^2-y^2}(-2y) = \boxed{-2y e^{-x^2-y^2}}$

39) $\frac{\partial}{\partial L}\left(\frac{L}{m}e^{-Lt/m}\right) = \left(\frac{1}{m}\right)\left(e^{-Lt/m}\right) + \left(\frac{L}{m}\right)\left(e^{-Lt/m}\right)\left(-\frac{t}{m}\right)$

$= \frac{e^{-Lt/m}\,m}{m^2} - \frac{Lt\,e^{-Lt/m}}{m^2} = \boxed{\frac{m-Lt}{m^2}\left(e^{-Lt/m}\right)}$

$\frac{\partial}{\partial m}\left(\frac{L}{m}e^{-Lt/m}\right) = \left(-\frac{L}{m^2}\right)\left(e^{-Lt/m}\right) + \left(\frac{L}{m}\right)\left(e^{-Lt/m}\right)\left(\frac{Lt}{m^2}\right)$

$= \frac{-L e^{-Lt/m}\,m}{m^3} + \frac{L^2t\,e^{-Lt/m}}{m^3} = \boxed{\frac{L^2t-Lm}{m^3}\left(e^{-Lt/m}\right)}$

$\frac{\partial}{\partial t}\left(\frac{L}{m}e^{-Lt/m}\right) = (0)\left(e^{-Lt/m}\right) + \left(\frac{L}{m}\right)\left(e^{-Lt/m}\right)\left(-\frac{L}{m}\right)$

$= \boxed{-\frac{L^2}{m^2}\left(e^{-Lt/m}\right)}$

47) (a) (evaluated using calculator)

$\boxed{173.19}$

(b) partial derivative of $I(T, H)$ in respects to T can find the answer.

14.4:

$f(2,1) = 14$

3) $f_x(x,y) = 2xy + y^3$ $f_y(x,y) = x^2 + 3xy^2$

$f_x(2,1) = 2(2)(1) + (1)^3 = 5$ $f_y(2,1) = (2)^2 + 3(2)(1)^2 = 10$

$\boxed{z = 5x + 10y - 14}$

5) $f_x(x,y) = 2x$ $f_y(x,y) = -2y^{-3}$ $f(4,1) = 4^2 + 2(1)^{-2} = 14$

$f_x(4,1) = 8$ $f_y(4,1) = \frac{-2}{1^3} = -2$

$\boxed{z = -2x + 8y - 14}$

7) $F_r(r,s) = 2rs^{-\frac{1}{2}}$ $F_s(r,s) = -\frac{1}{2}s^{-\frac{3}{2}}r^2 - 3s^{-4}$

$F(2,1) = 2$ $F_r(2,1) = 2(2)(1) = 4$ $F_s(2,1) = -\frac{1}{2}(1)2^2 - 3(1)$

$= -2 - 3 = -5$

$\boxed{z = 4r - 5s + 2}$

13) $f_x(x,y) = 2xy^3$ $f_y(x,y) = 3x^2y^2$ $f(2,1) = 2^2(1)^3 = 4$

$f_x(2,1) = 2(2)(1)^3 = 4$ $f_y(2,1) = 3(2)^2(1)^2 = 12$

$\boxed{z = 12y + 4x + 4}$

$f(2.01, 1.02) \approx 24.28$ $f(1.97, 1.01) \approx 24$

15) $\Delta f = 4.75$

used calculator

17) $f(0,0) = 1$ $f_x(x,y) = e^{x^2+y}(2x)$ $f_y(x,y) = e^{x^2+y}(1)$

$f_x(0,0) = 0$ $f_y(0,0) = 1$

$L(x,y) = 1 + 0(x+0) + 1(y+0)$

$= 1 + y$

$\boxed{f(0.01, -0.02) = 0.98}$

? 23) $\boxed{8.44867}$

? 25) $\boxed{4.99801}$

? 27) $\boxed{3.94257}$

14.5:

7) $h_x(x,y,z) = yz^{-3}$ $h_y(x,y,z) = xz^{-3}$

$h_z(x,y,z) = -3xyz^{-4}$

$\boxed{\nabla h = \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle}$

11) $r(0) = \langle \cos(0), \sin(0) \rangle = \langle 1, 0 \rangle$

$r'(t) = \langle -\sin(t), \cos(t) \rangle$ $r'(0) = \langle 0, 1 \rangle$

$f_x(x,y) = 2x - 3y$ $f_y(x,y) = -3x$

$f_x(1,0) = 2 - 0$ $f_y(1,0) = -3$

$\frac{d}{dt} f(r(t)) = 2(0) - 3(1) = \boxed{-3}$

13) $r(0) = \langle 1, 1 \rangle$ $r'(t) = \langle 2e^{2t}, 3e^{3t} \rangle$ $r'(0) = \langle 2(1), 3(1) \rangle = \langle 2, 3 \rangle$

$f_x(x,y) = \cos(xy)y$ $f_y(x,y) = \cos(xy)x$

$f_x(1,1) = \cos(1)(1) = \cos(1)$ $f_y(1,1) = \cos(1)$

$2\cos(1) + 3\cos(1) = \boxed{5\cos(1)}$

19) $r(1) = \langle e, 1, 1\rangle$ $r'(t) = \langle e^t, 1, 2t\rangle$ $r'(1) = \langle e, 1, 2\rangle$

$g_x(x,y,z) = yz^{-1}$ $g_y(x,y,z) = xz^{-1}$ $g_z(x,y,z) = -xyz^{-2}$

$g_x(e,1,1) = 1$ $g_y(e,1,1) = e$ $g_z(e,1,1) = -e$

$1(e) + e(1) - e(2) = \boxed{0}$

27) $\nabla f(x,y) = \langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}\rangle$

$|v| = \sqrt{3^2+2^2} = \sqrt{13}$ $u = \langle \frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\rangle$

$\nabla f(1,0) = \langle \frac{2}{1}, 0\rangle = \langle 2, 0\rangle$

$\langle \frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\rangle \cdot \langle 2, 0\rangle = \boxed{\frac{6}{\sqrt{13}}}$

31) $\nabla f(x,y) = 2xi + 8yj = \langle 2x, 8y\rangle$

$|v| = \sqrt{3^2+2^2} = \sqrt{13}$ $u = \langle \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\rangle$

$\nabla f(3,2) = \langle 6, 16\rangle$

$\langle \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\rangle \cdot \langle 6, 16\rangle = \frac{-18}{\sqrt{13}} - \frac{32}{\sqrt{13}} = \boxed{\frac{-50}{\sqrt{13}}}$

33) $\nabla T(x,y,z) = \langle e^{y-z}, xe^{y-z}, -xe^{y-z}\rangle$

$v = \langle 2, -2, -1\rangle$ $|v| = \sqrt{2^2+2^2+1^2} = \sqrt{9}$

$u = \langle \frac{2}{\sqrt{9}}, -\frac{2}{\sqrt{9}}, -\frac{1}{\sqrt{9}}\rangle$

$\nabla T(3,9,4) = \langle e^5, 3e^5, -3e^5\rangle$

$\langle e^5, 3e^5, -3e^5\rangle \cdot \langle \frac{2}{\sqrt{9}}, -\frac{2}{\sqrt{9}}, -\frac{1}{\sqrt{9}}\rangle$

$= \frac{2e^5}{\sqrt{9}} - \frac{6e^5}{\sqrt{9}} + \frac{3e^5}{\sqrt{9}} = \frac{-e^5}{\sqrt{9}} = \boxed{-\frac{e^5}{3}}$

$^{\triangle}$37) $\boxed{\text{Increasing}}$

39) $\nabla f(x,y,z) = \langle \cos(xy+z)\,y,\ \cos(xy+z)\,x,\ \cos(xy+z) \rangle$

$\cancel{u} = \langle \cos 30°, \sin 30° \rangle = \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle \quad u = \langle \frac{\sqrt{3}}{2}, \frac{1}{2}, 0 \rangle$

$|\cancel{u}| = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = \sqrt{1} = 1$

$\nabla f(0,-1,\pi) = \langle -\cos(\pi), 0, \cos(\pi) \rangle$

$\langle -\cos(\pi), 0, \cancel{x}\cos(\pi) \rangle \cdot \langle \frac{\sqrt{3}}{2}, \frac{1}{2}, 0 \rangle$

$= \boxed{-\frac{\sqrt{3}}{2}\cos(\pi)}$

41) $\nabla f(x,y,z) = \langle 2x, 2y, -2z \rangle$

$\nabla f(3,1,2) = \boxed{\langle 6, 2, -4 \rangle}$

43) $\cancel{\nabla f(x,y,z) = \langle \frac{x}{2}, \frac{2y}{9}, 2z \rangle}$

$\cancel{|v| = \sqrt{1^2+1^2+2^2} = \sqrt{6} \quad u = \langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \rangle}$

$\cancel{\frac{x^2}{4}}$ $\nabla f(x,y,z) = \langle \frac{x}{2}, \frac{2y}{9}, 2z \rangle$

$\langle \frac{x}{2}, \frac{2y}{9}, 2z \rangle = k\langle 1, 1, -2 \rangle$

$x = 2k \quad y = \frac{9k}{2} \quad z = -k$

$\cancel{\ldots}$ $k^2 + \frac{9}{4}k^2 + k^2 = 1 \ / \pm\frac{2}{\sqrt{17}}$

$(x,y,z) = \boxed{\pm\langle \frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, \frac{-2}{\sqrt{17}} \rangle}$