HW due 10/04/20

14.3 : 3, 5, 17, 19, 21, 27, 31, 39, 47

14.4 : 3, 5, 7, 13, 15, 17, 23, 25, 27

14.5 : 7, 11, 13, 19, 27, 31, 33, 37, 39, 41, 43

14.3

3. $\frac{d}{dy}\left(\frac{y}{x+y}\right) = \frac{(x+y)\frac{d}{dy}(y) - y\frac{d}{dy}(x+y)}{(x+y)^2}$

$= \frac{(x+y)(1) - y(1)}{(x+y)^2} = \boxed{\frac{x}{(x+y)^2}}$

5. $\frac{d}{dz}(xyz) = xy = (2)(3) = \boxed{6}$

17. $z = \frac{x}{y}$ $\quad \frac{dz}{dx} = \frac{1}{y}$ $\quad \frac{dz}{dy} = \frac{-x}{y^2}$

19. $z = \sqrt{9-x^2-y^2}$

$\frac{dz}{dx} = \frac{1}{2(9-x^2-y^2)^{\frac{1}{2}}} \cdot (-2x) = \frac{-x}{\sqrt{9-x^2-y^2}}$

$\frac{dz}{dy} = \frac{-y}{\sqrt{9-x^2-y^2}}$

21. $z = (\sin x)(\sin y)$

$\frac{dz}{dx} = \cos x \sin y$ $\quad \frac{dz}{dy} = \sin x \cos y$

27. $W = e^{r+s} = e^r \cdot e^s$

$\frac{dW}{dr} = e^{r+s}$ $\quad \frac{dW}{ds} = e^{r+s}$

31. $z = e^{-x^2-y^2}$

$\frac{dz}{dx} = -2xe^{-x^2-y^2}$ $\quad \frac{dz}{dy} = -2ye^{-x^2-y^2}$

39. $Q = \frac{L}{M}e^{-Lt/M}$

$\frac{dQ}{dL} = \frac{1}{M}\left[Le^{-Lt/M}\left(-\frac{t}{M}\right) + e^{-Lt/M}\right]$

$= \frac{M-Lt}{M^2}e^{-Lt/M}$

$\frac{dQ}{dM} = \frac{Le^{-Lt/M}}{M^2}\left[-ML t\left(-\frac{1}{M^2}\right) - 1\right]$

$= \frac{Le^{-Lt/M}}{M^2}\left[\frac{Lt-M}{M}\right] = \frac{Le^{-Lt/M}(Lt-M)}{M^3}$

$\frac{dQ}{dt} = \frac{L}{M}e^{-Lt/M}\left(-\frac{L}{M}\right) = \frac{-L^2}{M^2}e^{-Lt/M}$

47a. $(T, H) = (95, 50)$

$I(95,50) = 45.33 + 0.6845(95) + 5.758(50)$

$-0.00365(95)^2 - 0.1565(50)(95) + 0.001(50)(95)^2$

$\approx \boxed{73.1913}$

47b. $I_T(T,H) = 0.6845 - 0.0073T - 0.1565H + 0.002HT$

$I_T(95,50) = 0.6845 - 0.0073(95) - 0.1565(50)$

$+ 0.002(95)(50) \approx \boxed{1.666}$

14.4

3. $f(x,y) = x^2y + xy^3$ $\quad (2,1)$

$f(2,1) = 4 + 2 = 6$

$f_x(x,y) = 2xy + y^3 = 4 + 1 = 5$

$f_y(x,y) = x^2 + 3xy^2 = 4 + 6 = 10$

$z = 6 + 5(x-2) + 10(y-1)$

$z = 5x + 10y - 14$

5. $f(x,y) = x^2 + y^{-2}$ $\quad (4,1)$

$f(4,1) = 4^2 + 1 = 17$

$f_x(x,y) = 2x = 8$

$f_y(x,y) = -2y^{-3} = -2$

$z = 17 + 8(x-4) - 2(y-1)$

$z = 8x - 2y - 13$

7. $f(r,s) = \frac{r^2}{\sqrt{s}} + \frac{1}{s^3}$ $\quad (2,1)$

$f(2,1) = 4 + 1 = 5$

$f_r(r,s) = \frac{2r}{\sqrt{s}} = 4$

$f_s(r,s) = -\frac{1}{2}r^2s^{-3/2} - 3s^{-4} = -2 - 3 = -5$

$z = 5 + 4(r-2) - 5(s-1)$

$z = 4r - 5s + 2$

15. $f(x,y) = x^3y^{-4}$ $\quad \Delta f = f(2.03, 0.9) - f(2,1)$

$f(2,1) = 8$

$f_x(x,y) = 3x^2y^{-4} = 12$ $\quad f_y(x,y) = -4x^3y^{-5} = -32$

$\Delta x = 2.03 - 2 = 0.03$ $\quad \Delta y = 0.9 - 1 = -0.1$

$\Delta f = (12)(0.03) + (-32)(-0.1) = 0.36 + 3.2 = \boxed{3.56}$

17. $f(x,y)=e^{x^2+y}$ $(0,0)$ $f(0.01,-0.02)$

$f(0,0)=1$

$f_x(x,y)=2xe^{x^2+y}=0$

$f_y(x,y)=e^{x^2+y}=1$

$f(0.01,-0.02)=e^{(0.01)^2-0.02}\approx\boxed{0.9803}$

23. $f(x,y)=x^3y^2$ $(2.01,1.02)$

$f(2,1)=8$

$f_x(x,y)=3x^2y^2=12$ $f_y(x,y)=2x^3y=16$

$f(2.01,1.02)=8+12(.01)+16(.02)=\boxed{8.44}$

25. $f(x,y)=\sqrt{x^2+y^2}$ $(3.01,3.99)$

$f(3,4)=5$

$f_x(x,y)=\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\cdot 2x=0.6$

$f_y(x,y)=\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\cdot 2y=0.8$

$f(3.01,3.99)=5+0.6(0.1)+0.8(-0.01)=\boxed{4.998}$

27. $f(x,y,z)=\sqrt{xyz}$ $(2,2,4)$

$f(2,2,4)=4$

$f_x(x,y,z)=\frac{1}{2}\sqrt{\frac{yz}{x}}=1$

$f_y(x,y,z)=\frac{1}{2}\sqrt{\frac{xz}{y}}=1$

$f_z(x,y,z)=\frac{1}{2}\sqrt{\frac{xy}{z}}=\frac{1}{2}$

$f(1.9,2.02,4.05)=4+(1)(-0.1)+(1)(0.02)+(\frac{1}{2})(0.05)$

$=\boxed{3.945}$

14.5

7. $h(x,y,z)=xyz^{-3}$

$h_x(x,y,z)=yz^{-3}$ $h_y(x,y,z)=xz^{-3}$

$h_z(x,y,z)=-3xyz^{-4}$

$\nabla h=\langle yz^{-3}, xz^{-3}, -3xyz^{-4}\rangle$

11. $f(x,y)=x^2-3xy$ $r(t)=\langle \cos t,\sin t\rangle$

$r(0)=\langle 1,0\rangle$ $\nabla f=\langle 2x-3y,-3x\rangle$

$\nabla f_{r(0)}=\langle 2,-3\rangle$

$r'(0)=\langle 0,1\rangle$

$\frac{d}{dt}f(r(t))\big|_{t=0}=0-3=\boxed{-3}$

13. $f(x,y)=\sin(xy)$ $r(t)=\langle e^{2t},e^{3t}\rangle$

$r(0)=\langle 1,1\rangle$ $\nabla f=\langle y\cos(xy),x\cos(xy)\rangle$

$\nabla f_{r(0)}=\langle \cos 1,\cos 1\rangle$

$r'(t)=\langle 2e^{2t},3e^{3t}\rangle$ $r'(0)=\langle 2,3\rangle$

$\frac{d}{dt}f(r(t))\big|_{t=0}=2\cos 1+3\cos 1=5\cos 1=\boxed{2.702}$

19. $g(x,y,z)=xyz^{-1}$ $r(t)=\langle e^t,t,t^2\rangle$

$\nabla f=\langle yz^{-1},xz^{-1},-xyz^{-2}\rangle$

$\nabla f_{r(1)}=\langle 1,e,-e\rangle$

$r'(t)=\langle e^t,1,2t\rangle$ $r'(1)=\langle e,1,2\rangle$

$\frac{d}{dt}f(r(t))\big|_{t=1}=\langle 1,e,-e\rangle\cdot\langle e,1,2\rangle$

$=(1)(e)+(e)(1)+(-e)(2)=\boxed{0}$

27. $f(x,y)=\ln(x^2+y^2)$ $v=3i-2j$ $P=(1,0)$

$f_x(x,y)=\frac{2x}{x^2+y^2}$ $f_y(x,y)=\frac{2y}{x^2+y^2}$

$\nabla f(1,0)=\langle 2,0\rangle$

$\|v\|=\sqrt{3^2+(-2)^2}=\sqrt{9+4}=\sqrt{13}$

$u=\frac{v}{\|v\|}=\frac{\langle 3,-2\rangle}{\sqrt{13}}=\langle\frac{3}{\sqrt{13}},\frac{-2}{\sqrt{13}}\rangle$

$\langle 2,0\rangle\cdot\langle\frac{3}{\sqrt{13}},\frac{-2}{\sqrt{13}}\rangle=\boxed{\frac{6}{\sqrt{13}}}$

31. $f(x,y)=x^2+4y^2$ $P=(3,2)$

$f_x(x,y)=2x$ $f_y(x,y)=8y$

$\nabla f(3,2)=\langle 6,16\rangle$

$v=\langle 0,0\rangle-\langle 3,2\rangle=\langle -3,-2\rangle$

$\langle 6,16\rangle\cdot\langle -3,-2\rangle=-18-32=-50$

$\frac{1}{\|v\|}=\frac{1}{\sqrt{13}}$ $\frac{1}{\sqrt{13}}(-50)=\boxed{\frac{-50}{\sqrt{13}}}$

37. $\nabla f_P=(2,-4,4)$ $v=(2,1,3)$

$u=\frac{v}{\|v\|}=\frac{(2,1,3)}{\sqrt{2^2+1^2+3^2}}=\frac{(2,1,3)}{\sqrt{14}}$

$\nabla f_P\cdot u=(2,-4,4)\frac{(2,1,3)}{\sqrt{14}}=\frac{4-4+12}{\sqrt{14}}=\boxed{\frac{12}{\sqrt{14}}}$

39. $f(x,y,z)=\sin(xy+z)$ $P=(0,-1,\pi)$

$f_x(x,y,z)=y\cos(xy+z)$ $f_y(x,y,z)=x\cos(xy+z)$

$f_z(x,y,z)=\cos(xy+z)$

$\nabla f_P(0,-1,\pi)=\langle 1,0,-1\rangle$ $\|\nabla f\|=\sqrt{1+0+1}=\sqrt{2}$

$\|\nabla f_P\|\cdot\cos\theta=\sqrt{2}\cos 30=\sqrt{2}\cdot\frac{\sqrt{3}}{2}=\boxed{\frac{\sqrt{6}}{2}}$

41 $f(x,y,z)=x^2+y^2-z^2-6$ $P=(3,1,2)$

$\nabla f=\langle 2x,2y,-2z\rangle$ $\nabla f_P=\boxed{\langle 6,2,-4\rangle}$