14.3-14.5 (Oct. 4th)

14.3: #3, 5, 17, 19, 21, 27, 31, 39, 47

14.4: # 3, 5, 7, 13, 15, 17, 23, 25, 27

14.5: # 7, 11, 13, 19, 27, 31, 33, 37, 39, 41, 43

14.3: #3, 5, 17, 19, 21, 27, 31, 39, 47

3) $\dfrac{\frac{\partial}{\partial y}(y)(x+y) - \frac{\partial}{\partial y}(x+y)y}{(x+y)^2}$

$\frac{\partial}{\partial y}(y)=1 \quad \frac{\partial}{\partial y}(x+y)=1$

$= \dfrac{1\cdot(x+y) - 1\cdot y}{(x+y)^2} = \boxed{\dfrac{x}{(x+y)^2}}$

5) $2\cdot 3\cdot 1 = \boxed{6}$

17) $\frac{\partial}{\partial x}\left(\frac{x}{y}\right)$

$= \frac{1}{y}\frac{\partial}{\partial x}(x) = \frac{1}{y}\cdot 1 = \boxed{\frac{1}{y}}$

$\frac{\partial}{\partial y}\left(\frac{x}{y}\right)$

$= x\frac{\partial}{\partial y}\left(\frac{1}{y}\right)$

$= x\frac{\partial}{\partial y}(y^{-1}) = x(-1\cdot y^{-1-1}) = \boxed{\frac{-x}{y^2}}$

19) $\frac{\partial}{\partial x}\sqrt{9-x^2-y^2}$

$= \frac{1}{2\sqrt{9-x^2-y^2}}\frac{\partial}{\partial x}(9-x^2-y^2)$

$= \frac{1}{2\sqrt{9-x^2-y^2}}(-2x)$

$= \boxed{\frac{-x}{\sqrt{9-x^2-y^2}}}$

$\frac{\partial}{\partial y}\sqrt{9-x^2-y^2}$

$= \frac{1}{2\sqrt{9-x^2-y^2}}\frac{\partial}{\partial y}(9-x^2-y^2)$

$\frac{\partial}{\partial y}(9-x^2-y^2) = -2y$

$= \boxed{\frac{-y}{\sqrt{9-x^2-y^2}}}$

21) $\frac{\partial}{\partial x}\sin x \sin y$

$= \sin y \frac{\partial}{\partial x}\sin(x)$

$= \boxed{\sin(y)\cos(x)}$

$\frac{\partial}{\partial y}\sin x \sin y$

$= \sin x \frac{\partial}{\partial y}\sin y$

$= \boxed{\sin x \cos y}$

27) $\frac{\partial}{\partial r}e^{r+s}$

$e^{r+s}\frac{\partial}{\partial r}(r+s)$

$e^{r+s}\cdot 1 = \boxed{e^{r+s}}$

$\frac{\partial}{\partial s}e^{r+s}$

$e^{r+s}\frac{\partial}{\partial s}(r+s)$

$e^{r+s}\cdot 1 = \boxed{e^{r+s}}$

31) $\frac{\partial}{\partial x}e^{-x^2-y^2}$

$= e^{-x^2-y^2}\frac{\partial}{\partial x}(-x^2-y^2)$

$= e^{-x^2-y^2}(-2x)$

$= \boxed{-2e^{-x^2-y^2}x}$

$\frac{\partial}{\partial y}\left(e^{-x^2-y^2}\right)$

$= e^{-x^2-y^2}\frac{\partial}{\partial y}(-x^2-y^2)$

$= e^{-x^2-y^2}(-2y)$

$= \boxed{-2e^{-x^2-y^2}y}$

39) $\frac{\partial}{\partial L}\left(\frac{L}{M}e^{-\frac{Lt}{M}}\right)$

$= \frac{1}{M}\frac{\partial}{\partial L}\left(Le^{-\frac{Lt}{M}}\right)$

$= \frac{1}{M}\left(\frac{\partial}{\partial L}(L)e^{-\frac{Lt}{M}} + \frac{\partial}{\partial L}\left(e^{-\frac{Lt}{M}}\right)L\right)$

$= \frac{1}{M}\left(1\cdot e^{-\frac{Lt}{M}} + \left(-\frac{te^{-\frac{Lt}{M}}}{M}\right)L\right)$

$= \boxed{\frac{M-Lt}{M^2}e^{-\frac{Lt}{M}}}$

$\frac{\partial}{\partial M}\left(\frac{L}{M}e^{-\frac{Lt}{M}}\right)$

$= L\frac{\partial}{\partial M}\left(\frac{1}{M}e^{-\frac{Lt}{M}}\right)$

$= L\left(\frac{\partial}{\partial M}\left(\frac{1}{M}\right)e^{-\frac{Lt}{M}} + \frac{\partial}{\partial M}\left(e^{-\frac{Lt}{M}}\frac{1}{M}\right)\right)$

$= \boxed{\frac{L(Lt-M)}{M^3}e^{-Lt/M}}$

47) a) $I(95,50) = 45.33 + 0.6845(95) + 5.758(50) - 0.00365(95)^2 - 0.1565(95)(50)$

$+ 0.001(50)(95)^2 = 73.19$

b) $I(T,H) = 45.33 + 0.6845T + 5.758H - 0.00365T^2 - 0.1565TH + 2(0.001)H$

$I_T(95,50) = 0.6845 - 2(0.00365)(95) - 0.1565(50) + 2(0.001)(50) = \boxed{1.66}$

14.4: # 3, 5, 7, 13, 15, 17, 23, 25, 27

3) $f(x,y) = x^2y + xy^3$, $(2,1)$

$f_x = 2xy + y^3 \Rightarrow f_x(2,1) = 4+1 = 5$

$f_y = x^2 + x3y^2 \Rightarrow f_y(2,1) = 4 + 2(3) = 10$

$5(x-2) + 10(y-1) = 0$

$5x - 10 + 10y - 10 = 0$

$5x + 10y - 20$

5) $f(x,y) = x^2 + y^{-2}$, $(4,1)$

$f_x = 2x \Rightarrow f_x(4,1) = 8$

$f_y = -2y^{-3} \Rightarrow f_y(4,1) = -2$

$8(x-4) - 2(y-1) = 0$

$8x - 32 - 2y + 2 = 0$

$8x - 2y - 30 - 17 = \boxed{8x - 2y - 13}$

7) $F(r,s) = r^2s^{-1/2} + s^{-3}$, $(2,1)$

$f_r = 2rs^{-1/2} + s^{-3} \Rightarrow 5$

$f_s = r^2 - \frac{1}{2}s^{-3/2} + -3s^{-4} \Rightarrow -5$

$5(x-2) - 5(y-1) + 5 = 0$

$5x - 10 - 5y + 5 + 5 = 0$

$5x - 5y$ $\boxed{4r - 5s + 2}$

13) $f_x(x,y) = 2xy^3$

$f_y(x,y) = 3x^2y^2$

$f(2,1) = 4$, $f_x(2,1) = 4$, $f_y(2,1) = 12$

$L(x,y) = 4 + 4(x-2) + 12(y-1)$

$f(x,y) = 4x + 12y - 16$

$f(2+0.01, 1+0.02) = f(2.01, 1.02)$

$\Delta x = 0.03$ $= 4 + 0.04 + 0.24 = 4.28$

$f(1.97, 1.01) \approx 4 + 4 \times (-0.03) + 12 \times 0.01 = 4$

$\frac{3.998 - 4}{3.998} \times 100 = 0.05\%$

15) $(a,b) = (2,1)$

$\Delta x = 2.03 - 2 = 0.03$

$\Delta y = 0.9 - 1 = -0.1$

$f_x = 3x^2y^{-4}$ $f_y = x^3 - 4y^{-5}$

$f_x(2,1) = 3(2)^2(1)^{-4} = 12$ $= 2^3 - 4(1)^{-5} = -32$

$12(0.03) - 32(-0.1) = 3.56$

17) $f(0,0) = e^{0+0} = e^0 = 1$

$f(x,y) = 2x\, e^{x^2+y}$ $\quad f_x(0,0) = 0$

$f_y(x,y) = e^{x^2+y}$ $\quad f_y(0,0) = 1$

$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0)$

$+ f_y(x_0,y_0)(y-y_0)$

$L(x,y) = 1 + 0(x-0) + 1(y-0)$

$L(x,y) = 1+y$

$f(0.01, -0.02)$

$L(x,y) = 0.98$

23) $f(x) = x^3$

$x = 2,\ dx = 0.01$

$f(x+dx) \simeq f(x) + f'(x)\,dx$

$= f(2) + (3x^2)(0.01)$

$= 2^3 + 3(2)^2(0.01)$

$= 8.12$

$8.12(1.04) = 8.445$ approx

let $g(x) = x^2$

$x = 1,\ dx = 0.02$

$g(x+dx) \simeq g(x) + g'(x) \cdot dx$

$= g(1) + 2x \cdot dx$

$= 1^2 + 2(1)(0.02)$

$= 1.04$

25) $f(x,y) = \sqrt{x^2+y^2}$

$f_x(x,y) = \dfrac{x}{\sqrt{x^2+y^2}}$

$f_y(x,y) = \dfrac{y}{\sqrt{x^2+y^2}}$

$f(3,4) = 5,\ f_x(3,4) = \frac{3}{5}\quad f_y(3,4) = \frac{4}{5}$

$f(x,y) \simeq f(3,4) + f_x(3,4)(x-3) + f_y(3,4)(y-4)$

$f(3+0.01, 4-0.01) = f(3.01, 3.99)$

$= 5 + \frac{0.03}{5} - \frac{0.04}{5} = 4.998$

27) $f(x,y,z) = \sqrt{xyz}$

$f_x(x,y,z) = \frac{yz}{2\sqrt{xyz}}$

$f_y(x,y,z) = \frac{xz}{2\sqrt{xyz}}$

$f_z(x,y,z) = \frac{xy}{2\sqrt{xyz}}$

$f(2,2,4) = 4$, $f_x(2,2,4) = 1$, $f_y(2,2,4) = 1$, $f_z(2,2,4) = \frac{1}{2}$

$f(2+\Delta x, 2+\Delta y, 4+\Delta z) = \sqrt{(2+\Delta x)(2+\Delta y)(4+\Delta z)}$

$\Delta x = -0.1$, $\Delta y = 0.02$, $\Delta z = 0.05$

$\sqrt{1.9(2.02)(4.05)} \approx 4 - 0.1 + 0.02 + \frac{0.05}{2} = \boxed{3.945}$

14.5: # 7, 11, 13, 19, 27, 31, 33, 37, 39, 41, 43

7) $h_x = yz^{-3}$ $\quad h_y = xz^{-3}$

$h_z = -3xyz^{-4}$

gradient $= \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle$

11) $f(x,y) = x^2 - 3xy$

$r(t) = \langle \cos t, \sin t \rangle$ $\quad t=0$

$f(r(t)) = \cos^2 t - 3\cos t \sin t$

$= \cos^2(t) - \frac{3}{2}\sin(2t)$

$\frac{d}{dt} f(r(t)) = 2\cos(t) \cdot -\sin(t) - \frac{3}{2}\cos(2t) \cdot 2$

$= -2\sin(t)\cos(t) - 3\cos(2t)$

$= \boxed{-3}$

13) $f(x,y) = \sin(xy)$, $r(t) = \langle e^{2t}, e^{3t} \rangle$; $t=0$

$\frac{\partial}{\partial x} \sin(xy)$

$= \cos(xy) \frac{\partial}{\partial x}(xy)$

$= \cos(xy)\, y \frac{\partial}{\partial x}(x)$

$= y\cos(xy)$

$\frac{\partial}{\partial y} = \frac{\partial}{\partial y} \sin(xy)$

$= \cos(xy) \frac{\partial}{\partial y}(xy)$

$= \cos(xy)\, x \frac{\partial}{\partial y}(y)$

$= x\cos(xy)$

$\frac{d}{dt}(e^{2t}) = 2e^{2t}$

$\frac{dy}{dt} = \frac{d}{dt}(e^{3t})$

$= 3e^{3t}$

$\frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

$= y\cos(xy)(2e^{2t}) + (x\cos(xy))(3e^{3t})$

$= e^{3t}\cos(e^{2t}\cdot e^{3t})(2e^{2t}) + (e^{2t}\cos(e^{2t}\cdot e^{3t})(3e^{3t})$

$= 5e^{5t}\cos(e^{5t})$

$\frac{d}{dt}f(r(t)) = 5e^{5(0)}\cos(e^{5(0)})$

$= 5\cos(1)$

19) $g(x,y,z) = xyz^{-1}$, $r(t) = \langle e^t, t, t^2\rangle$, $t=1$

$\frac{\partial}{\partial x} xyz^{-1}$ $\quad \frac{\partial}{\partial y} = xyz^{-1}$ $\quad \frac{\partial}{\partial z} = xyz^{-1}$

$= yz^{-1}$ $\quad = xz^{-1}$ $\quad = -xyz^{-2}$

$\frac{d}{dt} = e^t$, $\frac{dy}{dt} = 1$, $\frac{dz}{dt} = 2t$

$yz^{-1}(e^t) + xz^{-1}(1) + (-xyz^{-2})(2t)$

$t(t^2)^{-1}(e^t) + e^t(t^2)^{-1}(1) + (-e^t t(t^2)^{-2})(2t)$

$t^{-1}e^t + e^t t^{-2} + -2te^t 2t^{-1}$

$= e + e + -2e(2) = -e$

27) $f(x,y) = \ln(x^2+y^2)$, $v = 3i - 2j$, $P = (1,0)$

$\frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y}$

$= \frac{2x}{x^2+y^2} + \frac{2y}{x^2+y^2}j$

$2i \cdot \left(\frac{3i-2j}{\sqrt{3^2+2^2}}\right) = \frac{6}{\sqrt{13}}$

31) $f(x,y) = x^2 + 4y^2$, $P = (3,2)$

$\vec{v} = \vec{PO} = -3i - 2j$

$= i(2x) + j(8y)$

$(6i + 16j) \cdot \frac{(-3i-2j)}{\sqrt{13}} = \frac{-18-32}{\sqrt{13}} = \boxed{\frac{-50}{\sqrt{13}}}$

33) $v = \overrightarrow{PQ} = \langle 5-3, 7-9, 3-4 \rangle =$

$v = \langle 2, -2, -1 \rangle$

$\nabla T = \langle e^{y-z}, xe^{y-z}, -xe^{y-z} \rangle$

$\nabla T(2,9,4) = \langle e^{9-4}, 3e^{9-4}, -3e^{9-4} \rangle$

$$D_u T(2,9,4) = \frac{2e^5 - 6e^5 + 3e^5}{\sqrt{2^2 + (-2)^2 + (-1)^2}}$$

$= -\frac{e^5}{3} \approx \boxed{-49.47^\circ \text{ per meter}}$

34) $v = \langle 2, 1, 3 \rangle$

$|v| = \sqrt{2^2 + 1^2 + 3^2} = \sqrt{14}$

$u = \frac{v}{|v|} = \frac{1}{\sqrt{14}} \langle 2, 1, 3 \rangle$

$D_u f_p = \langle 2, -4, 4 \rangle \cdot \frac{1}{\sqrt{14}} \langle 2, 1, 3 \rangle$

$= \frac{12}{\sqrt{14}}$

$\boxed{\nabla f_p \text{ is increasing at } P \text{ in the direction of } v}$

39) $f(x,y,z) = \sin(xy+z)$

$\nabla f(x,y,z) = f_x(x,y,z)i + f_y(x,y,z)j + f_z(x,y,z)k$

$\nabla f(x,y,z) = y\cos(xy+z)i + x\cos(xy+z)j + \cos(xy+z)k$

$\nabla f(0,-1,\pi) = i - k$

$\|\nabla f(0,-1,\pi)\| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$

$D_u f(0,-1,\pi) = \sqrt{2}\cos(\frac{\pi}{6}) = \sqrt{2}(\frac{\sqrt{3}}{2})$ $\frac{\sqrt{6}}{2}$

41) gradient $= 2xi + 2yj - 2zk$

$2 \times 3i + 2 \times 1j - 6 \times 2k$

$= 6i + 2j - 12k$

$\boxed{\langle 6, 2, -12 \rangle}$

43) $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$ $\quad v = \langle 1, 1, -2 \rangle$

$\langle \frac{x_0}{4}, \frac{y_0}{9}, 2z_0 \rangle = k\langle 1, 1, -2 \rangle$

$\frac{16k^2}{4} + \frac{81k^2}{9} + 4k^2 = 1$

$17k^2 = 1$

$k^2 = \frac{1}{17}, \quad k = \pm\frac{1}{\sqrt{17}}$

$\boxed{\left(\pm\frac{4}{\sqrt{17}}, \pm\frac{9}{\sqrt{17}}, \pm\frac{-2}{\sqrt{17}}\right)}$