14.3 homework

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section22
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3. $\cancel{\frac{d}{dy}\frac{y}{x+y} = \frac{y'\cdot(x+y)-y\cdot y'}{(x+y)^2} = \frac{x\cdot y'}{(x+y)^2}}$

$$\frac{d}{dy}\frac{y}{x+y} = \frac{x}{(x+y)^2}.$$

5. calculate $f_z(2,3,1)$, where $f(x,y,z)=xyz$.

$$f(2,3,1)=6.$$

17. $z=\frac{x}{y}$

$$\frac{dz}{dx}=\frac{1}{y}$$

$$\frac{dz}{dy}=-\frac{x}{y^2}$$

19. $z=\sqrt{9-x^2-y^2}$

$$\frac{dz}{dx}=\frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}\cdot -2x = -x(9-x^2-y^2)^{-\frac{1}{2}}$$

$$\frac{dz}{dy}=\frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}\cdot -2y = -y(9-x^2-y^2)^{-\frac{1}{2}}.$$

21. $z=(\sin x)(\sin y)$

$$\frac{dz}{dx}=\cos x\sin y$$

$$\frac{dz}{dy}=\sin x\cos y$$

27. $w=e^{r+s}$

$$\frac{dw}{dr}=e^{r+s}$$

$$\frac{dw}{ds}=e^{r+s}.$$

31. $z=e^{-x^2-y^2}$

$$\frac{dz}{dx}=e^{-x^2-y^2}\cdot -2x$$

$$\frac{dz}{dy}=e^{-x^2-y^2}\cdot -2y.$$

39. $Q=\frac{L}{M}e^{-Lt/M}$

$$\frac{dQ}{dL}=\frac{L}{M}e^{-Lt/M}\cdot -\frac{t}{M} = -\frac{Lt}{M^2}e^{-Lt/M}$$

$$\frac{dQ}{dM}=\frac{L}{M}e^{-Lt/M}\cdot\frac{Lt}{M^2} = \frac{L^2t}{M^3}e^{-Lt/M}$$

$$\frac{dQ}{dL}=\frac{M-Lt}{M^2}e^{-Lt/M}$$

$$\frac{dQ}{dt}=-\frac{L^2}{M^2}e^{-Lt/M} \qquad \frac{dQ}{dM}=\frac{L(Lt-M)}{M^3}e^{-Lt/M}$$

47. ②

(a) $I(95,60) = 73.19125$.

(b) $\cancel{0.6845T + 0.0011HT^2}$

$\cancel{\frac{dI}{dT} = 0.6845 + 0.0022HT}$.

$\frac{dI}{dT}$; 1.66.

$\cancel{= 0.6845 + 0.0022 \times 95 \times 60}$.

14.4homework

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① 3、 $f(x,y)=x^2y+xy^3$, $(2,1)$

$\frac{dz}{dx}=2yx+y^3=4+1=5$

$\frac{dz}{dy}=x^2+3xy^2=4+6=10$

$z=6$

$z-6=5(x-2)+10(y-1)$

$z=5x-10+10y-10+6$

$z=5x+10y-14$

5、 $f(x,y)=x^2+y^{-2}$, $(4,1)$

$z=16+1=17$

$\frac{dz}{dx}=2x=8$

$\frac{dz}{dy}=-2y^{-3}=-2$

$z-17=8(x-4)+(-2)(y-1)$

$z=8x-32-2y+2+17$

$z=8x-2y-13$.

7、 $F(r,s)=r^2s^{-\frac{1}{2}}+s^{-3}$, $(2,1)$

$z=4\cdot1+1=5$.

$\frac{dz}{dr}=2s^{-\frac{1}{2}}r=2\cdot1\cdot2=4$

$\frac{dz}{ds}=-\frac{1}{2}r^2s^{-\frac{3}{2}}+(-3)s^{-4}=-\frac{1}{2}\cdot4\cdot1-3\cdot1$

$=-2-3$

$=-5$.

$z-5=4(r-2)+(-5)(s-1)$

$z-5=4r-8-5s+5$

$z=4r-5s+2$.

13、 ~~$f(x,y)$~~

$f(2,1)=4\cdot1=4$

$\frac{dz}{dx}=2xy^3=4$

$\frac{dz}{dy}=3y^2x^2=12$.

$L(x,y)=4+4(x-2)+12(y-1)$

$=-16+4x+12y$

$L(2.01,1.02)=4+4\times0.01+12\times0.02$

$=4.28$

$L(1.97,1.01)=4+4\times(-0.03)+12\times(0.01)$

$=4-0.12+0.12$

$=4$.

15. $f(x,y) = x^3y^{-4}$.

$\Delta f = f(2.03, 0.9) - f(2,1)$

$f(2,1) = 8 \cdot 1 = 8$

$\frac{dz}{dx} = 3x^2y^{-4} = 12$

$\frac{dz}{dy} = -4x^3y^{-5} = -32$.

$L(x,y) = 8 + 12(x-2) + (-32)(y-1)$

$L(2.03, 0.9) = 8 + 12 \times 0.03 - 32 \times -0.1$

$= 8 + 0.36 + 3.2$

$= 11.56$

$\Delta f = 11.56 - 8 = 3.56$.

17. $f(x,y) = e^{x^2+y}$ at $(0,0)$ to estimate $f(0.01, -0.02)$.

$z = 1$.

$\frac{dz}{dx} = e^{x^2+y} \cdot 2x = 0$

$\frac{dz}{dy} = e^{x^2+y} = 1$.

$L(x,y) = 1 + 0 + (y-0)$

$= 1 + y$

$\boxed{L(0.01, -0.02) = 1 - 0.02 = 0.98}$.

? 23. $(2.01)^3(1.02)^2$

8.44

? 26. $\sqrt{3.01^2 + 3.99^2}$.

4.998

? 27 $\sqrt{(1.4)(2.02)(4.05)}$

3.945

14.5 homework

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7、 $h(x,y,z) = xyz^{-3}$

$f_x = yz^{-3}$

$f_y = xz^{-3}$

$f_z = -3xyz^{-4}$

$\nabla h(x,y,z) = \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle$

11、 $f(x,y) = x^2 - 3xy$, $r(t) = \langle \cos t, \sin t \rangle$, $t=0$

$\frac{d}{dt} f(r(t))$:

$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$

$\frac{dx}{dt} = -\sin t$

$\frac{dz}{dx} = 2x - 3y$

$\frac{dy}{dt} = \cos t$

$\frac{dz}{dy} = -3x$

$x = \cos t = \cos 0 = 1$

$y = \sin t = \sin 0 = 0$

$\frac{dz}{dt} = 2 \cdot -0 + (-3) \cdot \cos 0$

$= -3\cos 0$

$= -3 \cdot \cancel{\cos}(1)$

$= -3$

13、 $f(x,y) = \sin(xy)$,

$r(t) = \langle e^{2t}, e^{3t} \rangle$, $t=0$

$\frac{d}{dt} f(r(t))$:

$\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$

$\frac{dx}{dt} = 2e^{2t}$

$\frac{dz}{dx} = \cancel{\sin x y}\, y\cos(xy)$

$\frac{dy}{dt} = 3e^{3t}$

$\frac{dz}{dy} = x\cos(xy)$

$x = e^{2t} = 1$

$y = e^{3t} = 1$

$\frac{dz}{dt} = 2 \cdot \cos 1 + 3\cos 1$

$= 5\cos 1$

≈ 2.702

19. $g(x,y,z) = xyz^{-1}$, $r(t) = \langle e^t, t, t^2 \rangle$, $t=1$.

$\frac{d}{dt} f(r(t))$

$$\frac{dF}{dt} = \frac{dF}{dx} \cdot \frac{dx}{dt} + \frac{dF}{dy} \cdot \frac{dy}{dt} + \frac{dF}{dz} \cdot \frac{dz}{dt}$$

$\frac{dx}{dt} = e^t$ $\quad \frac{dy}{dt} = 1$ $\quad \frac{dz}{dt} = 2t$

$\frac{dF}{dx} = yz^{-1}$ $\quad \frac{dF}{dy} = xz^{-1}$ $\quad \frac{dF}{dz} = -xyz^{-2}$.

$x = e^t = e$

$y = t = 1$

$z = t^2 = 1$

$$\frac{dF}{dt} = e + e + 2 \cdot -e$$

$= 0$.

27. $f(x,y) = \ln(x^2+y^2)$, $v = 3i - 2j$, $P = (2,2)$

$f_x = \frac{2x}{x^2+y^2}$ $\quad f_y = \frac{2y}{x^2+y^2}$

$= \frac{4}{8}$ $\quad = \frac{4}{8}$

$= \frac{1}{2}$ $\quad = \frac{1}{2}$

$\nabla f = \langle \frac{1}{2}, \frac{1}{2} \rangle$

$u = \frac{\langle 3,2 \rangle}{\sqrt{13}} = \langle \frac{3\sqrt{13}}{13}, \frac{2\sqrt{13}}{13} \rangle$

$\frac{6}{\sqrt{16}}$

$u \cdot \nabla f = \frac{3\sqrt{13}}{26} + \frac{\sqrt{13}}{13} = \cancel{\frac{5\sqrt{13}}{26}}$

ANS: The directional ~~derivatz~~ derivative is $\frac{5\sqrt{13}}{26}$.

31. Find the directional derivative of $f(x,y)=x^2+4y^2$ at the point $P=(3,2)$ in the direction pointing to the origin.

$f_x = 2x = 6$

$f_y = 8y = 16$.

$\overrightarrow{AB} = B-A = \langle 0,0\rangle - \langle 3,2\rangle = \langle -3,-2\rangle$

$u = \dfrac{\langle -3,-2\rangle}{\sqrt{13}} = \langle -\frac{3\sqrt{13}}{13}, -\frac{2\sqrt{13}}{13}\rangle$

~~gra~~ $\nabla f = \langle 6, 16\rangle$

$\nabla f \cdot u = \langle 6,16\rangle \cdot \langle -\frac{3\sqrt{13}}{13}, -\frac{2\sqrt{13}}{13}\rangle$

$= -\frac{18\sqrt{13}}{13} - \frac{32\sqrt{13}}{13}$

$= -\frac{50\sqrt{13}}{13}$.

ANS: The directional derivative is $-\frac{50}{13}\sqrt{13}$.

33. A bug located at $(3,9,4)$ begins walking in a straight line toward $(5,7,3)$. At what rate is the bug's temperature changing if the temperature is $T(x,y,z) = xe^{y-z}$? Unit are in meter and degrees Celsius.

$f_x = e^{y-z}$ $\quad f_y = xe^{y-z}$ $\quad f_z = -xe^{y-z}$ $\quad$ The Ans is $\frac{17}{83}\sqrt{83}e^5$

$\nabla T = \langle e^5, 3e^5, -3e^5\rangle$ $\quad -\dfrac{e^5}{3} = -49.47$

$u = \dfrac{\langle 5,7,3\rangle}{\sqrt{83}} = \langle \frac{5\sqrt{83}}{83}, \frac{7\sqrt{83}}{83}, \frac{3\sqrt{83}}{83}\rangle$

$\nabla T \cdot u = \frac{5\sqrt{83}}{83}e^5 + \frac{21\sqrt{83}}{83}e^5 - \frac{9\sqrt{83}}{83}e^5 = \frac{17\sqrt{83}}{83}e^5$

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37. Suppose that $\nabla f_P = \langle 2,-4,4\rangle$. Is f increasing or decreasing at P in the direction $v = \langle 2,1,3\rangle$?

~~$\nabla f_P \cdot u_v =$~~

$$u_v = \frac{\langle 2,1,3\rangle}{\sqrt{14}} = \left\langle \frac{\sqrt{14}}{7}, \frac{\sqrt{14}}{14}, \frac{3\sqrt{14}}{14} \right\rangle$$

$$\nabla f_P \cdot u_v = \langle 2,-4,4\rangle \cdot \left\langle \frac{\sqrt{14}}{7}, \frac{\sqrt{14}}{14}, \frac{3}{14}\sqrt{14} \right\rangle$$

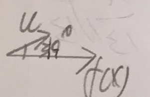
$$= \frac{2\sqrt{14}}{7} - \frac{2\sqrt{14}}{7} + \frac{6}{7}\sqrt{14}$$

$$= \frac{6}{7}\sqrt{14} > 0$$

Ans: increasing.

39. Let $f(x,y,z) = \sin(xy+z)$ and $P = (0,-1,\pi)$. Calculate $D_u f(P)$, where u is a unit vector making an angle $\theta = 30^\circ$ with ∇f_P.

$f_x = \cos(xy+z)\cdot y = 1 \qquad f_y = \cos(xy+z)\cdot x = 0 \qquad f_z = \cos(xy+z) = -1$



$$\nabla f = \langle 1, 0, -1\rangle$$

$$D_u f(P) = \nabla f \div \cos 30^\circ$$

$$= \langle 1,0,-1\rangle \div \frac{\sqrt{3}}{2}$$

$$= \langle 1,0,-1\rangle \times \frac{2}{\sqrt{3}}$$

$$= \left\langle \frac{2\sqrt{3}}{3}, 0, -\frac{2}{3}\sqrt{3} \right\rangle.$$

$\frac{\sqrt{6}}{2}$

ANS: $D_u f(P) = \left\langle \frac{2}{3}\sqrt{3}, 0, -\frac{2}{3}\sqrt{3} \right\rangle$.

41. Find a vector normal to the surface $x^2+y^2-z^2=6$ at $P=(3,1,2)$ 5

$\langle 6, 2, -4\rangle$

43. Find the two points on the ellipsoid

$$\frac{x^2}{4}+\frac{y^2}{9}+z^2=1.$$

where the tangent plane is normal to $v=\langle 1,1,-2\rangle$

$f_x=\frac{x}{2}$ $f_y=\frac{2}{9}y$ $f_z=2z$

$\left(\frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, -\frac{2}{\sqrt{17}}\right)$ and $\left(-\frac{4}{\sqrt{17}}, -\frac{9}{\sqrt{17}}, \frac{2}{\sqrt{17}}\right)$.