14.3 HW
Due 10/4

Rahul Paleja

14.3 - # 3, 5, 17, 19, 21, 27, 31, 39, 47:

(3) $\frac{d}{dy} \frac{y}{x+y}$ $u = y$, $u' = 1$ $v = x+y$, $v' = 1$

$$\frac{vu' - uv'}{v^2} = \frac{x+y-y}{(x+y)^2} = \boxed{\frac{x}{(x+y)^2}}$$

(5) $f_z(2,3,1)$ $f(x,y,z) = (xy)z$ $f_z = xy$

$$f_z(2,3,1) = 2 \cdot 3 = \boxed{6}$$

(17) $\frac{d}{dx} = \frac{1}{y} \cdot (x) = \boxed{\frac{1}{y}}$ $\frac{d}{dy} = x\left(\frac{1}{y}\right) = x \cdot y^{-1}$

$$= \boxed{\frac{-x}{y^2}}$$

(19) $z = \sqrt{9-x^2-y^2}$

$$\frac{d}{dx} = (9-x^2-y^2)^{1/2} \cdot -2x = \frac{-\cancel{2}x}{\cancel{2}\sqrt{9-x^2-y^2}}$$

$$= \boxed{-\frac{x}{\sqrt{9-x^2-y^2}}}$$

$$\frac{d}{dy} = (9-x^2-y^2)^{1/2} \cdot -2y = \frac{-\cancel{2}y}{\cancel{2}\sqrt{9-x^2-y^2}} = \boxed{\frac{-y}{\sqrt{9-x^2-y^2}}}$$

(21) $\frac{d}{dx} = \sin y \cos(x)$ $\frac{d}{dy} = \sin(x)\cos(y)$

(27) $w = e^{r+s}$ $\frac{d}{dr} = e^{r+s} \cdot (1) = e^{r+s}$

$$\frac{d}{ds} = e^{r+s}$$

(31) $\frac{d}{dx} = e^{-x^2-y^2}(-x^2-y^2)' = -2x\,e^{-x^2-y^2}$

$\frac{d}{dy} = e^{-x^2-y^2}(-x^2-y^2)' = -2y\,e^{-x^2-y^2}$

(39) $Q = \frac{L}{M} e^{-Lt/M}$ $\quad \frac{d}{dL}$ $\quad u = \frac{L}{m}$ $\quad u' = \frac{1}{m}$ $\quad v = e^{-Lt/m}$ $\quad v' = -\frac{t}{m} e^{-Lt/m}$

$uv' + vu' = \frac{L}{m} \cdot \frac{-t}{m} e^{-Lt/m} + \frac{1}{m} e^{-Lt/m}$

$= \frac{-Lt}{m^2} e^{-Lt/m} + \frac{m}{m^2} e^{-Lt/m}$

$\frac{d}{dL} = \boxed{\frac{-Lt+m}{m^2} e^{-Lt/m}}$

$\frac{d}{dm} =$ $\quad u = \frac{L}{m}$ $\quad u' = \frac{-L}{m^2}$ $\quad v = e^{-Lt/m}$ $\quad v' = \frac{Lt}{m^2} e^{-Lt/m}$

$\frac{L^2t}{m^3} e^{-Lt/m} - \frac{L}{m^2} e^{-Lt/m} = \boxed{\frac{(L^2t - Lm)e^{-Lt/m}}{m^3}}$

$\frac{d}{dt} = \frac{L}{m} e^{-Lt/m} \cdot \frac{-L}{m} = \boxed{\frac{-L^2}{m^2} e^{-Lt/m}}$

(47) a) $I(95,50) = \boxed{73.19125}$

b) $\frac{d}{dT} = .6845 - .0073T - .1565H + .002HT$

$\frac{d}{dT}(95,50) = \boxed{1.666}$

14.4 HW
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14.4: #3, 5, 7, 13, 15, 17, 23, 25, 27:

(3) $f(x,y) = x^2y + xy^3 \quad (2,1) \qquad f(2,1) = 6$

$f_x = 2yx + y^3 \qquad f_y = x^2 + 3xy^2$

$f_x(2,1) = 2(1\cdot 2) + (1)^3 \qquad f_y(2,1) = 2^2 + 3(2\cdot 1^2)$

$= 5 \qquad = 10$

$z - 6 = 5(x-2) + 10(y-1) = z - 6 = 5x - 10 + 10y - 10$

$z - 6 = 5x + 10y - 20$

$+6 \qquad +6$

$\boxed{z = 5x + 10y - 14}$

(5) $f(x,y) = x^2 + y^{-2} \quad (4,1)$

$f(4,1) = 16 + 1 = 17$

$f_x = 2x \qquad f_y = \frac{-2}{y^3} \qquad f_y(4,1) = \frac{-2}{1^3} = -2$

$f_x(4,1) = 2(4) = 8$

$z - 17 = 8(x-4) - 2(y-1) = z - 17 = 8x - 32 - 2y + 2$

$z - 17 = 8x - 2y - 30$

$+17 \qquad +17$

$\boxed{z = 8x - 2y - 13}$

(7) $F(r,s) = r^2 s^{-1/2} + s^{-3} \quad (2,1)$

$F(2,1) = 5 \qquad F_r = \frac{2}{\sqrt{s}} r \qquad F_r(2,1) = \frac{2}{\sqrt{1}} \cdot 2 = \frac{4}{\sqrt{1}}$

$F_s = -\frac{1}{2} r^2 \frac{1}{\sqrt{s^3}} - \frac{3}{s^4} \qquad F_s(2,1) = -\frac{1}{2}(4) - \frac{3}{16} = \frac{-2}{1} - \frac{3}{16}$

$= -\frac{32}{16} - \frac{3}{16} = \frac{-35}{16}$

$z = \frac{4}{\sqrt{1}}(r-2) - \frac{35}{16}(s-1) + 5$

(13) $(x_0, y_0) = (2,1)$ $\quad L_z = z_0 + f_x(x_0, y_0)\cdot(x-x_0) + f_y(x_0,y_0)\cdot(y-y_0)$

$z_0 = f(2,1) = 2^2 \cdot 1^3 = 4$

$f_x = 2y^3 x \quad f_x(2,1) = 2(1)^3(2) = 4$
$f_y = 3x^2y^2 \quad f_y(2,1) = 3(2)^2(1)^2 = 12$

$z - 4 = 4(x-2) + 12(y-1) + 4$
$+4$

$z = 4x - 8 + 12y - 12 + 4 \rightarrow \boxed{z = 4x + 12y - 16}$

$L(2.01, 1.02) = 4.28$
$L(1.97, 1.01) = 4$

(15) $df = f_x(x,y)\,dx + f_y(x,y)\,dy$
$f(x,y) = x^3 y^{-4} \quad \Delta f = f(2.03, .99) - f(2,1)$
$f(2,1) \quad \Delta x = .03$
$\Delta y = -.01$

$f_x(x,y) = 3y^{-4}x^2 \quad f_x(2,1) = 3(2)^2 = 12(.03) = .36$
$f_y(x,y) = -\frac{4x^3}{y^5} \quad f_y(2,1) = -32(-.01) = .32$

$\boxed{\Delta f = 3.56}$

(17) $f(x,y) = e^{x^2+y} \quad (x_0, y_0) = (0,0) \quad f(0,0) = 1$
$f_x = e^{x^2+y} \cdot 2x \quad f_x(0,0) = 0$
$f_y = e^{x^2+y} \quad f_y(0,0) = 1$

$z = (y-0) + 1 = y + 1$

$L(.01, -.02) = -.02 + 1 = \boxed{.98}$

14.4 HW

Due 10/4

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14.4 - #23, 25, 27:

(23) $(2.01)^3(1.02)^2$ $\quad (x)^3(y)^2 \quad (x_0, y_0) = (2,1)$

$$z_0 = (2,1) = (2)^3(1)^2 = 8$$

$$f_x = 3y^2x^2 \quad f_x(2,1) = 3(1)^2(2)^2 = 12$$

$$f_y = 2x^3y \quad f_y(2,1) = 2(2)^3(1) = 16$$

$$z = 12(x-2) + 16(y-1) + 8$$

$$L(2.01, 1.02) = 12(.01) + 16(.02) + 8 = \boxed{8.44}$$ comp to 8.44867

(25) $\sqrt{3.01^2 + 3.99^2} = \sqrt{x^2 + y^2} \quad (x_0, y_0) = (3,4)$

$$z_0(3,4) = \sqrt{3^2+4^2} = 5$$

$$f_x = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}} = \frac{x}{\sqrt{x^2+y^2}} \quad f(3,4) = \frac{3}{\sqrt{3^2+4^2}} = \frac{3}{5}$$

$$f_y = \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2}} = \frac{y}{\sqrt{x^2+y^2}} \quad f(3,4) = \frac{4}{\sqrt{3^2+4^2}} = \frac{4}{5}$$

$$L = z = \frac{3}{5}(x-3) + \frac{4}{5}(y-4) + 5$$

$$L(3.01, 3.99) = \boxed{4.998}$$ compared to 4.9989

14.5 HW

Due 10/4

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14.5 - # 7, 11, 13, 19, 27, 31, 33, 37, 39, 41, 43:

(7) $h(x,y,z) = xyz^{-3}$

$\nabla f = (f_x, f_y, f_z)$ $\quad f_x = yz^{-3}$ $\quad f_y = xz^{-3}$ $\quad f_z = -3xyz^{-4}$

$= (yz^{-3}, xz^{-3}, -3xyz^{-4})$

(11) $\frac{dz}{dt} = \frac{dz}{dx} \cdot \frac{dx}{dt} + \frac{dz}{dy} \cdot \frac{dy}{dt}$

$\frac{dz}{dx} = 2x - 3y$ $\quad \frac{dx}{dt} = -\sin t$

$\frac{dz}{dy} = -3x$ $\quad \frac{dy}{dt} = \cos t$

$t = 0$ @ $r(t) =$ $\cos(0) = 1 = x$

$\sin(0) = 0 = y$

$\frac{dz}{dx} = 2(1) - 3(0) = 2$

$\frac{dx}{dt}$ at $t=0$ $= -\sin(0) = 0$

$\frac{dz}{dy} = -3(1) = -3$

$\frac{dy}{dt} = \cos(0) = 1$ $\quad 2 \cdot 0 + (-3 \cdot 1) = \boxed{-3}$

(13) $f(x,y) = \sin(xy)$ $\quad r(t) = (e^{2t}, e^{3t})$ $\quad t = 0$

$\frac{dz}{dx} = y\cos(xy)$ $\quad \frac{dx}{dt} = 2e^{2t}$ $\quad \frac{dz}{dy} = x\cos(xy)$ $\quad \frac{dy}{dt} = 3e^{3t}$

$t = 0$ in $r(t)$ $\quad x(t) = 1$ $\quad y(t) = 1$

dz/dx @ $t=1 = \cos(1)$ $\quad dx/dt = 2$ $\quad \frac{dz}{dy} = \cos(1)$ $\quad \frac{dy}{dt} = 3$

$\cos(1) \cdot 2 + \cos(1) \cdot 3 = \boxed{5\cos(1)}$

(19) $g(x,y,z) = xyz^{-1}$ $r(t) = \langle e^t, t, t^2 \rangle$ $t=1$

$\frac{dg}{dx} = yz^{-1}$ $\frac{dx}{dt} = e^t$ @ $t=1 \rightarrow x(t) = e$

$\frac{dg}{dy} = xz^{-1}$ $\frac{dy}{dt} = 1$ $y(1) = 1$

$z(1) = 1$

$\frac{dg}{dz} = \frac{-xy}{z^2}$ $\frac{dz}{dt} = 2t$

$\frac{dg}{dx}$ @ $t=1 = 1$ $\frac{dx}{dt}$ @ $t=1 = e$

$\frac{dg}{dy} = e$ $\frac{dy}{dt} = 1$

$\frac{dg}{dz} = -e$ $\frac{dz}{dt} = 2$

$1 \cdot e + 1 \cdot e - e \cdot 2 = 2e - 2e = \boxed{0}$

(27) $f(x,y) = \ln(x^2+y^2)$ $v = 3i - 2j$, $P = (1,0)$

$Df = \langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \rangle$ @ $(1,0) = \langle 2, 0 \rangle$

Magnitude of $|v| = \sqrt{3^2+(-2)^2+0^2} = \sqrt{13}$

$\langle 2, 0 \rangle \cdot \langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle$

$= \frac{6}{\sqrt{13}} + 0 = \boxed{\frac{6}{\sqrt{13}}}$

(31) $v = \langle -3, -2 \rangle$ $f(x,y) = x^2+4y^2$ $P = (3,2)$

$Df = \langle 2x, 8y \rangle$ @ $(3,2) = \langle 6, 16 \rangle$

$|v| = \sqrt{(-3)^2+(-2)^2} = \sqrt{13}$

$\langle 6, 16 \rangle \cdot \langle \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle$

$\frac{-18}{\sqrt{13}} + \frac{-32}{\sqrt{13}} = \boxed{\frac{-50}{\sqrt{13}}}$

14.5 HW
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Rahul [illegible]

14.5: # 33, 37, 39, 41, 43;

33 $v = \langle 2,-2,-1\rangle$ $T(x,y,z) = xe^{y-z}$ $P = (3,9,4)$

$\nabla T = \langle e^{y-z}, xe^{y-z}, -xe^{y-z}\rangle$ @ $(3,9,4)$

$= \langle e^5, 3e^5, -3e^5\rangle$

$|v| = \sqrt{4+4+1} = 3$

$\langle e^5, 3e^5, -3e^5\rangle \cdot \langle \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\rangle$

$\frac{2}{3}e^5 - 2e^5 + e^5 = \frac{2}{3}e^5 - \frac{e^5}{1} = \boxed{\frac{-e^5}{3}}$

37 $\nabla f_P = \langle 2,-4,4\rangle$ $|v| = \sqrt{4+1+9} = \sqrt{14}$

$\langle 2,-4,4\rangle \cdot \langle \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}}\rangle$

$\frac{4}{\sqrt{14}} - \frac{4}{\sqrt{14}} + \frac{12}{\sqrt{14}} = \frac{12}{\sqrt{14}} \rightarrow$ Increasing Because Directional Derivative Is Positive

39 $\nabla f = \langle y\cos(xy+z), x\cos(xy+z), \cos(xy+z)\rangle$

@ $(0,-1,\pi) = \langle 1, 0, -1\rangle$

$|\langle 1,0,-1\rangle| = \sqrt{1^2+(-1)^2} = \sqrt{2}$ $\theta = 30°$

$\sqrt{2}\cos 30° = \sqrt{2}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\frac{\sqrt{6}}{2}}$

(41) $x^2+y^2-z^2=6$ @ $P=(3,1,2)$

$f(x,y,z)=x^2+y^2-z^2-6$

$Df=\langle 2x, 2y, -2z\rangle$ @ $(3,1,2)=\boxed{\langle 6, 2, -4\rangle}$

(43) $Df=\langle \frac{x}{2}, \frac{2y}{9}, 2z\rangle = c\cdot\langle 1,1,-2\rangle$

$\frac{x}{2}=c \quad \frac{2y}{9}=c \quad 2z=-2c$

$x=2c \quad y=\frac{9c}{2} \quad z=-c$

$(2c)^2+\left(\frac{9c}{2}\right)^2-(-c)^2=6$

$2c^2+\frac{81c^2}{4}-c^2=6$

$\left(\frac{4}{4}\right)\frac{c^2}{1} = \frac{4c^2}{4}+\frac{81c^2}{4} = \left(\frac{85c^2}{4}=6\right)4$

$c=\sqrt{\frac{24}{85}} \qquad \frac{85c^2}{85}=\frac{24}{85}$

Points: $\left\langle 2\sqrt{\frac{24}{85}}, \frac{9\left(\sqrt{\frac{24}{85}}\right)}{2}, -\sqrt{\frac{24}{85}}\right\rangle$

$\left\langle -2\sqrt{\frac{24}{85}}, \frac{-9\sqrt{\frac{24}{85}}}{2}, \sqrt{\frac{24}{85}}\right\rangle$

(27) $\sqrt{(1.9)(2.02)(4.05)}$ $\sqrt{xyz}$ $(x_0, y_0, z_0) = (2,2,4)$

$z_0(2,2,4) = \sqrt{2\cdot 2\cdot 4} = 4$

$f_x = \frac{yz}{2\sqrt{xyz}}$ $f_x(2,2,4) = \frac{8}{2\sqrt{16}} = \frac{8}{2(4)} = 1$

$f_y = \frac{xz}{2\sqrt{xyz}}$ $f_y(2,2,4) = \frac{8}{2\sqrt{16}} = 1$

$f_z = \frac{xy}{2\sqrt{xyz}}$ $f_z(2,2,4) = \frac{4}{2\sqrt{16}} = \frac{1}{2}$

$L_0 = 4 + (x-2) + (y-2) + \frac{1}{2}(z-4)$

$L_0(1.9, 2.02, 4.05) = 4 + (1.9-2) + (2.02-2) + \frac{1}{2}(4.05-4)$

$= \boxed{3.945}$ compared to 3.94257