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14.3 : 3, 5, 17, 19, 21, 27, 31, 39, 47 (14.4 continued).

(3) $\frac{d}{dy}\left(\frac{y}{x+y}\right) = \frac{(x+y)-(y)}{(x+y)^2} = \frac{x}{(x+y)^2}$

(5) $f_z(2,3,1)$ and $f(x,y,z) = xyz$

$f_z = xy \Rightarrow f_z(2,3,1) = 6$

(17) $z = \frac{x}{y} \longrightarrow \frac{d}{dx} = \frac{1}{y} \quad \frac{d}{dy} = -\frac{x}{y^2}$

(19) $z = \sqrt{9-x^2-y^2} = (9-x^2-y^2)^{\frac{1}{2}}$

$\frac{d}{dx} = \frac{1}{\cancel{2}}(9-x^2-y^2)^{-\frac{1}{2}}(-\cancel{2}x) = \frac{-x}{\sqrt{9-x^2-y^2}}$

$\frac{d}{dy} = \frac{-y}{\sqrt{9-x^2-y^2}}$

(21) $z = (\sin x)(\sin y) \quad \frac{d}{dz} = \cos y \sin x$

$\frac{d}{dx} = \cos x \sin y$

(27) $w = e^{r+s} \quad \frac{d}{dr} = e^{r+s} \quad \frac{d}{ds} = e^{r+s}$

(31) $z = e^{-x^2-y^2} \quad \frac{d}{dy} = -2ye^{-x^2-y^2}$

$\frac{d}{dx} = -2xe^{-x^2-y^2}$

(39) $Q = \frac{L}{M}e^{-Lt/M} \quad \frac{d}{dL} = \frac{M-Lt}{m^2}(e)^{-Lt/M}$

$\frac{d}{dM} = \left(\frac{Lt-M}{M^2}\right)\left(\frac{L}{M}\right)e^{-Lt/M} = \frac{L^2t-LM}{M^3}e^{-Lt/M}$

$\frac{d}{dt} = \frac{-L^2}{M^2}e^{-Lt/M}$

(47) $I(T,H) = 45.33 + 0.6845T + 5.758H - 0.00365T^2 - 0.1565HT + 0.001HT^2$

a. $I(95,50) = 75.19$ (heat index)

b. The partial derivative that tells us this is $\frac{d}{dt}$ which is $\frac{d}{dt} = 0.6845 - 2(0.00365)T - 0.1565H + 2(0.001)HT$

$\left.\frac{d}{dt}\right|_{t=90, H=40} = 1.66$. ($\frac{d}{dt}$ = degree increase in I per degree increase in T).

14.4: 3, 5, 7, 13, 15, 17, 23, 25, 27

(3) $f(x,y) = x^2y + xy^3 \quad (2,1)$

$f_x = 2xy + y^3 \quad f_y = x^2 + 3y^2x$

$f_x(2,1) = 4+1 = 5 \quad f_y(2,1) = 4+6 = 10$

$z - 0 = 5(x-2) + 10(y-1) + (z-0)(0) = 0$

$z = 5x - 10 + 10y - 10 \longrightarrow z = 5x + 10y - 20 + 6$

$z = 5x + 10y - 14$

(5) $f(x,y) = x^2 + y^{-2} \quad (4,1)$

$f_x = 2x \quad f_y = -2y^{-3}$

$f_x(4,1) = 8 \quad f_y(4,1) = -\frac{2}{1} = -2$

$z = 8(x-4) + -2(y-1)$

$= 8x - 32 - 2y + 2 + 17 \Rightarrow z = 8x - 2y - 13$

(7) $F(r,s) = r^2s^{-\frac{1}{2}} + s^{-3} \quad (2,1)$

$F_r = 2rs^{-\frac{1}{2}} \Rightarrow 4$

$F_s = -\frac{1}{2}r^2\frac{1}{s} + -3s^{-4} \Rightarrow -5$

$z = 4(r-2) + -5(s-1) + 5$

$= 4r - 8 - 5s + 5 + 5 \rightarrow 4r - 5s + 2 = z$

(13) $L(x,y)$ of $f(x,y) = x^2y^3 \quad (2,1)$

$f_x = 2xy^3 \quad f_y = 3y^2x^2$

$f_x(2,1) = 4 \quad f_y(2,1) = 12 \longrightarrow z = 4(x-2) + 12(y-1) + 4$

$z = 4x - 8 + 12y - 12 + 4$

$z = 4x + 12 - 16$

$f(2.01, 1.02) \approx 4.28$

$f(1.97, 1.01) \approx 4$

(15) $f(x,y) = x^3y^{-4}$

$f(2.03, 0.9) = 12.749$

$f(2,1) = 8$

$\Delta f = 4.75 \rightarrow ?$

(17) $f(x,y) = e^{x^2+y} \quad (0,0) \quad f(0.01, -0.02)$

$f_x = 2xe^{x^2+y} \Rightarrow 0$

$f_y = e^{x^2+y} \Rightarrow 1$

$\rightarrow z = y + 1 \quad f(0.01, -0.02) \approx 0.98$

$L(x) = f(a) + f'(a)(x-a)$

Linear approximation Formula

(23) $(2.01)^3(1.02)^2$

$f(x,y) = x^3y^2 \quad f_x = 3x^2y^2 \quad f_y = 2yx^3$

$f(x,y) \approx 8.44 + (12.61)(2.01 - 2.01) + f_y(x,y)(1.02 - 1.02)$

≈ 8.44

(25) $\sqrt{3.01^2 + 3.99^2} = \sqrt{x^2+y^2}$

$f(x,y) \approx 4.998$

$f_x = \frac{1}{2}2x(x^2)^{-\frac{1}{2}}$

$f_y = \frac{1}{2}2y(y^2)^{-\frac{1}{2}}$

(27) $\sqrt{(1.9)(2.02)(4.05)}$

(using linear approx. to estimate the values).

$f_x = yz\frac{1}{2}(xyz)^{-\frac{1}{2}}$

$f_y = xz\frac{1}{2}(xyz)^{-\frac{1}{2}}$

$f_z = xy\frac{1}{2}(xyz)^{-\frac{1}{2}}$

$f(x,y,z) \approx 3.945$

14.5: 7, 11, 13, 19, 27, 31, 33, 37, 39, 41, 43

(7) $h(x,y,z) = xyz^{-3}$ $h_x = yz^{-3}$ $h_z = -3xyz^{-4}$
$h_y = xz^{-3}$

$\nabla h = \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle$

(11) $\frac{d}{dt} f(r(t))$ $f(x,y) = x^2 - 3xy$ $r(t) = \langle \cos t, \sin t \rangle$ $t=0$
$r(0) = \langle 1, 0 \rangle$

$f_x = 2x - 3y$ $\frac{d}{dt} = f'(1,0) \cdot \langle -\sin t, \cos t \rangle$
$f_y = -3x$ $= \langle 2, -3 \rangle \cdot \langle 0, 1 \rangle$

$\nabla f(2x-3y, -3x)$ $= 0 - 3 = -3$

(13) $f(x,y) = \sin(xy)$ $r(t) = \langle e^{2t}, e^{3t} \rangle = 0$ $r'(t) = \langle 2e^{2t}, 3e^{3t} \rangle$
$r'(0) = \langle 2, 3 \rangle$

$f_x = y\cos(xy)$ $\nabla f = \langle y\cos(xy), x\cos(x,y) \rangle$

$f_y = x\cos(x,y)$ $\nabla f = \langle 3\cos 6, 2\cos 6 \rangle \cdot \langle 2, 3 \rangle$

?

$= 6\cos 6 + 6\cos 6$ X $= 5\cos 1$

(19) $g(x,y,z) = xyz^{-1}$ $r(t) = \langle e^t, t, t^2 \rangle$ $t=1$

① Find ∇g
② find $r(t)$ and plug in values to ∇g
③ $r'(t)$
④ $\nabla g \cdot r'(t)$

$\nabla g = \langle yz^{-1}, xz^{-1}, -1xyz^{-2} \rangle$ $r(1) = \langle e, 1, 1 \rangle$
$r'(t) = \langle e^t, 1, 2t \rangle$
$r'(1) = \langle e, 1, 2 \rangle$

$\nabla g(e,1,1) = \langle 1, e, -e \rangle \cdot \langle e, 1, 2 \rangle$

$= \langle e + e - 2e \rangle = 0$

(27) $f(x,y) = \ln(x^2+y^2)$ $v = 3i - 2j$ $P = (1,0)$

$f_x = \frac{2x}{x^2+y^2}$ $f_y = \frac{2y}{x^2+y^2}$ $\nabla f = \langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \rangle = \langle \frac{2}{1}, 0 \rangle$

$|\langle 3, -2 \rangle| = \sqrt{13} \rightarrow u = \langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle$

$\nabla f \cdot u \longrightarrow \langle 2, 0 \rangle \cdot$ $= \frac{6}{\sqrt{13}} + 0$

$= \frac{6}{\sqrt{13}}$ ***

(31) $f(x,y,z) = x^2 + 4y^2$ $P = (3,2)$

"in the direction pointing to the origin" $\rightarrow \langle -3, -2 \rangle$

$\nabla f = \langle 2x, 8y \rangle = \langle 6, 16 \rangle$

$u = |\langle -3, -2 \rangle| = \sqrt{13}$

going to origin so opposite?

$u = \langle \frac{3}{\sqrt{13}}, \frac{2}{\sqrt{13}} \rangle$ $u \cdot \nabla f = \frac{-18}{\sqrt{13}} + \frac{-32}{\sqrt{13}} = \frac{-50}{\sqrt{13}}$

(33) $(3,9,4)$ $(5,7,3)$ $\rightarrow |(2,-2,-1)| \rightarrow = 3$

$T(x,y,z) = xe^{4-z}$

$\nabla T = \langle e^{4-z}, xe^{4-z}, -xe^{4-z} \rangle \rightarrow \langle 1, 3, -3 \rangle$

$\nabla T(3,9,4) = e^{9-4} \langle 1, 3, -3 \rangle = e^5 \langle 1, 3, -3 \rangle$

$u = \frac{1}{3}\langle 2, -2, -1 \rangle$ $\nabla T \cdot u = \frac{-e^5}{3} \approx -49.47$

(37) f is increasing at P in the direction of v.
f is increasing at P in the direction $v = \langle 2, 1, 3 \rangle$.
* $(\nabla f = \langle 2, -4, 4 \rangle$ P in the direction

(39) $f(x,y,z) = \sin(xy + z)$ $\nabla f = \langle y\cos(xy+z), x\cos(xy+z), \cos(xy+z) \rangle$

$P = (0, -1, \pi)$ $u = \cos 30 i + \sin 30 j$

?

(41) $x^2 + y^2 - z^2 = 6$ $P = (3, 1, 2)$

Finding a vector normal to a point P

$\nabla f = \langle 2x, 2y, 2z \rangle$

$\nabla f = \langle 6, 2, -4 \rangle$

(43) $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$

Finding 2 points on the ellipsoid where the tangent plane is normal to $v = \langle 1, 1, -2 \rangle$.

$\nabla f = \langle \frac{x}{2}, \frac{2y}{9}, 2z \rangle$

$= k\langle 1, 1, -2 \rangle$

$x = 2k$
$y = 9k/2$
$z = -k$

$\frac{(2k)^2}{4} + \frac{(9k)^2}{9} z + k$

$k = \pm \frac{2}{\sqrt{17}}$

$(x, y, z) = (2k, \frac{9k}{2}, k)$

$= (\frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, \frac{-2}{\sqrt{17}})$

and

$(-\frac{4}{\sqrt{17}}, -\frac{9}{\sqrt{17}}, \frac{2}{\sqrt{17}})$