Calc 251 HW: 14.3

low d high - high d low

3. $\frac{\partial}{\partial y}\frac{y}{x+y} = \frac{(x+y)y' - (x+y)'y}{(x+y)^2}$

$\frac{\partial}{\partial y}\frac{y}{x+y} = \frac{x+\cancel{y}-\cancel{y}}{(x+y)^2}$

$\boxed{\frac{\partial}{\partial y}\frac{y}{x+y} = \frac{1}{(x+y)^2}}$

5. $f_z(2,3,1) = (2)(3)(1)$

$\boxed{f_z(2,3,1) = 6}$

17. $z = \frac{x}{y} = xy^{-1}$

$f_x = \frac{\partial}{\partial x}(xy^{-1}) = y^{-1}$

$f_y = \frac{\partial}{\partial y}(xy^{-1}) = 0 - xy^{-2}$

$\boxed{f_x = y^{-1},\ f_y = -xy^{-2}}$

19. $z = \sqrt{9-x^2-y^2}$

$f_x = \frac{\partial}{\partial x}\left[(9-x^2-y^2)^{\frac{1}{2}}\right] = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}(9-x^2-y^2)'$

$f_x = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}(0-2x-0)$

$\boxed{f_x = -x(9-x^2-y^2)^{-\frac{1}{2}}}$

$f_y = \frac{\partial}{\partial y}\left[(9-x^2-y^2)^{\frac{1}{2}}\right] = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}(9-x^2-y^2)'$

$f_y = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}(0-0-2y)$

$\boxed{f_y = -y(9-x^2-y^2)^{-\frac{1}{2}}}$

21. $z=(\sin x)(\sin y)$

$f_x = \frac{\partial}{\partial x}[(\sin x)(\sin y)] = \cos x \sin y$

$f_y = \frac{\partial}{\partial y}[(\sin x)(\sin y)] = \sin x \cos y$

$\boxed{f_x = \cos x \sin y,\ f_y = \sin x \cos y}$

27. $W = e^{r+s}$

$f_r = \frac{\partial}{\partial r}(e^{r+s}) = e^{r+s}(r+s)'$

$\boxed{f_r = e^{r+s},\ f_s = e^{r+s}}$

31. $z = e^{-x^2-y^2}$

$f_x = \frac{\partial}{\partial x}(e^{-x^2-y^2}) = e^{-x^2-y^2}(-x^2-y^2)'$

$f_x = e^{-x^2-y^2}(-2x)$

$f_y = \frac{\partial}{\partial y}(e^{-x^2-y^2})$

$f_y = e^{-x^2-y^2}(-2y)$

$\boxed{f_x = -2xe^{-x^2-y^2},\ f_y = -2ye^{-x^2-y^2}}$

39. $Q = \frac{L}{M}e^{-Lt/M}$

$\frac{\partial Q}{\partial L} = \left[\frac{L}{M}e^{-Lt/M}\right]' = \frac{1}{M}e^{-Lt/M} + \frac{L}{M}e^{-Lt/M}\left(\frac{1}{M}(-Lt)'\right)$

$\frac{\partial Q}{\partial L} = \frac{1}{M}e^{-Lt/M} - \frac{Lt}{M^2}e^{-Lt/M}$

$\boxed{\frac{\partial Q}{\partial L} = \frac{M-Lt}{M^2}e^{-Lt/M}}$

$\frac{\partial Q}{\partial M} = \left[\frac{L}{M}e^{-Lt/M}\right]' = -LM^{-2}e^{-Lt/M} + \frac{L}{M}e^{-Lt/M}(LtM^{-2})$

$\frac{\partial Q}{\partial M} = -\frac{L}{M^2}e^{-Lt/M} + \frac{L^2t}{M^3}e^{-Lt/M}$

$\frac{\partial Q}{\partial M} = -\frac{LM}{M^3}e^{-Lt/M} + \frac{L^2t}{M^3}e^{-Lt/M}$

$\boxed{\frac{\partial Q}{\partial M} = \frac{L^2t - LM}{M^3}e^{-Lt/M}}$

$$\frac{\partial Q}{\partial t} = \left[\frac{L}{M} e^{-Lt/M}\right]' = \frac{L}{M} e^{-Lt/M}\left(-\frac{L}{M}\right)$$

$$\boxed{\frac{\partial Q}{\partial t} = -\frac{L^2}{M^2} e^{-Lt/M}}$$

47. $I(95,50) = 45.33 + 0.6845(95) + 5.758(50) - 0.00365(95)^2 - 0.1565(50)(95) + 0.001(50)(95)^2$

$$\frac{\partial I}{\partial T} = 0.6845 + 5.758H - 2(0.00365)T^2 - 0.1565H + 2(0.001)HT$$

$$\frac{\partial I}{\partial T} = 0.6845 + 5.758(50) - 2(0.00365)(95^2) - 0.1565(50) + 2(0.001)(50)(95)$$

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3. $f(x,y) = x^2y + xy^3$, $(2,1) \to f(2,1) = 4 + 2 = 6$

$f_x(x,y) = 2xy + y^3 \to f_x(2,1) = 4 + 1 = 5$.

$f_y(x,y) = x^2 + 3xy^2 \to f_y(2,1) = 4 + 6 = 10$

$z = f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1)$

$z = 6 + 5(x-2) + 10(y-1)$

$z = 6 + 5x - 10 + 10y - 10$

$\boxed{z = 5x + 10y - 14}$

5. $f(x,y) = x^2 + y^{-2}$, $(4,1) \to f(4,1) = 16 + 1 = 17$

$f_x(x,y) = 2x \to f_x(4,1) = 8$

$f_y(x,y) = -2y^{-3} \to f_y(4,1) = -2$

$z = 17 + 8(x-4) - 2(y-1)$

$z = 17 + 8x - 32 - 2y + 2$

$\boxed{z = 8x - 2y - 13}$

7. $F(r,s) = r^2 s^{-1/2} + s^{-3}, (2,1) \rightarrow F(2,1) = 4 + 1 = 5$

$F_r(r,s) = 2rs^{-1/2} \rightarrow F_r(2,1) = 2(2) = 4$

$F_s(r,s) = -\frac{1}{2}r^2 s^{-3/2} - 3s^{-4} \rightarrow F_s(2,1) = -\frac{1}{2}(4) - 3 = -5$

$t = 5 + 4(r-2) - 5(s-1)$

$t = 5 + 4r - 8 - 5s + 5$

$\boxed{t = 4r - 5s + 2}$

13. $f(x,y) = x^2 y^3, (2,1) \rightarrow f(2,1) = 4$

$f_x(x,y) = 2xy^3 \rightarrow f_x(2,1) = 4$

$f_y(x,y) = 3x^2y^2 \rightarrow f_y(2,1) = 12$

$L(x,y) = 4 + 4(x-2) + 12(y-1)$

$L(x,y) = 4 + 4x - 8 + 12y - 12$

$\boxed{L(x,y) = 4x + 12y - 16}$

$2 + \Delta x, \; 1 + \Delta y \rightarrow$ For $(2.01, 1.02)$ $\Delta x = .01, \Delta y = .02$

$f(2+\Delta x, 1+\Delta y) = 4(2+\Delta x) + 12(1+\Delta y) - 16$ $.24$

$f(2+\Delta x, 1+\Delta y) = 8 + 4\Delta x + 12 + 12\Delta y - 16$

$f(2+\Delta x, 1+\Delta y) = 4 + 4\Delta x + 12\Delta y$

$f(2.01, 1.02) = 4 + 4(.01) + 12(.02) = 4.28$

$\Delta x = -.03, \Delta y = .01$ for $(1.97, 1.01)$

$f(1.97, 1.01) = 4 + 4(-.03) + 12(.01) = 4$

$\boxed{f(2.01, 1.02) \approx 4.28, \; f(1.97, 1.01) \approx 4}$

15. $f(x,y) = x^3 y^{-4} \Rightarrow f(2,1) = 8$

$f_x(x,y) = 3x^2 y^{-4} \rightarrow f_x(2,1) = 12$

$f_y(x,y) = -4x^3 y^{-5} \rightarrow f_y(2,1) = -32$

$z = 8 + 12(x-2) - 32(y-1)$

$z = 8 + 12x - 24 - 32y + 32$

$z = 12x - 32y$

$\Delta x = .03, \ \Delta y = -.1$

$z = 12(2 + \Delta x) - 32(1 + \Delta y)$

$z = 24 + 12\Delta x - 32 - 32\Delta y$

$z = 8 + 12\Delta x - 32\Delta y$

$f(2.03, .90) \approx 8 + 12(.03) + 32(.10)$

$f(2.03, .90) \approx 11.56$

$\Delta f = f(2.03, .90) - f(2,1)$

$\boxed{\Delta f = 3.56}$

17. $f(x,y) = e^{x^2+y} \quad f(0,0) = 1$

$f_x(x,y) = 2xe^{x^2+y} \rightarrow f_x(0,0) = 0$

$f_y(x,y) = e^{x^2+y} \rightarrow f_y(0,0) = 1$

$\Delta x = .01, \ \Delta y = -.02$

$\Delta f \approx f_x(0,0)(.01) + f_y(0,0)(-.02)$

$\Delta f \approx -.02 \qquad 1 + x = -.02$

$f(.01, -.02) \approx -1.2$

23. $f(x,y) = x^3, y^2$, $(2,1) \to f(2,1) = 8$
$f(2+\Delta x, 1+\Delta y)$ $\Delta x = .01$, $\Delta y = .02$

$f_x(x,y) = 3x^2y^2 \to f_x(2,1) = 12$
$f_y(x,y) = 2x^3y \to f_y(2,1) = 16$

$\Delta f \approx 12(.01) + 16(.02)$
$\Delta f \approx 0.44$
$\Delta f = f(x,y) - f(2,1)$
$\boxed{(2.01)^3(1.02)^2 \approx 8.44}$

25. $f(x,y) = \sqrt{x^2+y^2}$, $(3,4) \to f(3,4) = \sqrt{9+16} = 5$
$f(3+\Delta x, 4+\Delta y)$ $\Delta x = .01$, $\Delta y = -.01$

$f_x(x,y) = x(x^2+y^2)^{-1/2} \to f_x(3,4) = \frac{3}{5}$
$f_y(x,y) = y(x^2+y^2)^{-1/2} \to f_y(3,4) = \frac{4}{5}$

$\Delta f \approx \frac{3}{5}(.01) + \frac{4}{5}(-.01)$
$\Delta f \approx -\frac{1}{500}$
$\boxed{\sqrt{3.01^2 + 3.99^2} \approx 4.998}$

27. $f(x,y,z) = \sqrt{xyz}$, $(2,2,4) \to f(2,2,4) = 4$
$f(2+\Delta x, 2+\Delta y, 4+\Delta z)$ $\Delta x = -.1$, $\Delta y = .02$, $\Delta z = .05$
$f_x(x,y,z) = \frac{1}{2}yz(xyz)^{-1/2} \to f_x(2,2,4) = 4(\frac{1}{4}) = 1$
$f_y(x,y,z) = \frac{1}{2}xz(xyz)^{-1/2} \to f_y(2,2,4) = 4(\frac{1}{4}) = 1$
$f_z(x,y,z) = \frac{1}{2}xy(xyz)^{-1/2} \to f_z(2,2,4) = 2(\frac{1}{4}) = \frac{1}{2}$

$\Delta f \approx 1(-.1) + 1(.02) + \frac{1}{2}(.05)$

$\Delta f \approx -0.055$

$\boxed{\sqrt{(1.9)(2.02)(4.05)} \approx 3.945}$

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7. $h(x,y,z) = xyz^{-3}$

$h_x = yz^{-3}$, $h_y = xz^{-3}$, $h_z = -3yz^{-4}$

$\boxed{\nabla h = \langle yz^{-3}, xz^{-3}, -3yz^{-4}\rangle}$

11. $f(x,y) = x^2 - 3xy$, $r(t) = \langle \cos t, \sin t\rangle$, $t=0$

$f_x = 2x - 3y$, $f_y = -3x$

$\nabla f = \langle 2x-3y, -3x\rangle$

$r(0) = \langle \cos 0, \sin 0\rangle = \langle 1,0\rangle$

$\nabla f(1,0) = \langle 2,-3\rangle$

$r'(0) = \langle -\sin 0, \cos 0\rangle = \langle 0,1\rangle$

$\frac{d}{dt} f(r(t))\Big|_{t=0} = \nabla f_{r(0)} \cdot r'(0)$

$\langle 2,-3\rangle \cdot \langle 0,1\rangle = -3$

$\boxed{\frac{d}{dt} f(r(t))\Big|_{t=0} = -3}$

13. $f(x,y) = \sin(xy)$, $r(t) = \langle e^{2t}, e^{3t}\rangle$, $t=0$

$f_x = y\cos(xy)$, $f_y = x\cos(xy)$

$\nabla f = \langle y\cos(xy), x\cos(xy)\rangle$

$r(0) = \langle 1,1\rangle$

$\nabla f_{(1,1)} = \langle \cos(1), \cos(1)\rangle$

$r'(t) = \langle 2e^{2t}, 3e^{3t} \rangle$

$r'(0) = \langle 2, 3 \rangle$

$\frac{d}{dt} f(r(t)) \Big|_{t=0} = \langle \cos(1), \cos(1) \rangle \cdot \langle 2, 3 \rangle$

$\boxed{\frac{d}{dt} f(r(t)) \Big|_{t=0} \; 5\cos(1)}$

19. $g(x,y,z) = xyz^{-1}$, $r(t) = \langle e^t, t, t^2 \rangle$, $t=1$

$g_x = yz^{-1}$, $g_y = xz^{-1}$, $g_z = -xyz^{-2}$

$\nabla g = \langle yz^{-1}, xz^{-1}, -xyz^{-2} \rangle$

$r(1) = \langle e, 1, 1 \rangle$

$\nabla g(e,1,1) = \langle 1, e, -e \rangle$

$r'(t) = \langle te^t, 1, 2t \rangle$

$r'(1) = \langle e, 1, 2 \rangle$

$$\frac{d}{dt} f(r(t)) \Big|_{t=1} = \langle 1, e, -e \rangle \cdot \langle e, 1, 2 \rangle$$

$$= e + e - 2e$$

$\boxed{\frac{d}{dt} f(r(t)) \Big|_{t=1} = 0}$

27. $f(x,y) = \ln(x^2+y^2)$, $v = \langle 3,-2\rangle$, $P = (1,0)$

$f_x = \frac{2x}{x^2+y^2}$, $f_y = \frac{2y}{x^2+y^2}$

$\nabla f = \langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}\rangle$

$\nabla f(1,0) = \langle 2, 0\rangle$

$|\langle 3,-2\rangle| = \sqrt{9+4} = \sqrt{13}$

$u = \frac{1}{\sqrt{13}}\langle 3,-2\rangle = \langle \frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\rangle$

$D_u f(1,0) = \langle 2,0\rangle \cdot \langle \frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\rangle = \frac{6}{\sqrt{13}}$

$\boxed{D_u f(1,0) = \frac{6}{\sqrt{13}}}$

31. $f(x,y) = x^2+4y^2$, $(3,2)$, $\langle 0-3, 0-2\rangle = \langle -3,-2\rangle$

$f_x = 2x$, $f_y = 8y$

$\nabla f = \langle 2x, 8y\rangle$

$\nabla f(3,2) = \langle 6, 16\rangle$

$|\langle -3,-2\rangle| = \sqrt{9+4} = \sqrt{13}$

$u = \frac{1}{\sqrt{13}}\langle -3,-2\rangle = \langle -\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\rangle$

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$D_u f(3,2) = \langle 6,16\rangle \cdot \langle -\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\rangle$

$= -\frac{18}{\sqrt{13}} - \frac{32}{\sqrt{13}}$

$\boxed{D_u f(3,2) = \frac{50}{\sqrt{13}}}$

$y-z\,c$

$1-z\,e^{y-z}$

33. $T(x,y,z) = xe^{y-z}$ $(3,9,4)$, $\langle 5-3, 7-9, 3-4\rangle = \langle 2,-2,-1\rangle$

$T_x = e^{y-z}$, $T_y = xe^{y-z}$, $T_z = -xe^{y-z}$

$\nabla T = \langle e^{y-z}, xe^{y-z}, -xe^{y-z}\rangle$

$\nabla T(3,9,4) = \langle e^5, 3e^5, -3e^5\rangle$

$|\langle 2,-2,-1\rangle| = \sqrt{4+4+1} = 3$

$u = \frac{1}{3}\langle 2,-2,-1\rangle = \langle \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\rangle$

$D_uT(3,9,4) = \langle e^5, 3e^5, -3e^5\rangle \cdot \langle \frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\rangle$

$= \frac{2}{3}e^5 - 2e^5 + e^5 = \frac{2}{3}e^5 - \frac{3}{3}e^5$

$\boxed{D_uT(3,9,4) = -\frac{e^5}{3}}$

37. $\langle 2,-4,4\rangle \cdot \langle 2,1,3\rangle = 4-4+12 = 12 > 0$

$\boxed{f \text{ is increasing}}$

39. $f(x,y,z) = \sin(xy+z)$, $P = (0,-1,\pi)$

$f_x = y\cos(xy+z)$, $f_y = x\cos(xy+z)$, $f_z = \cos(xy+z)$

$\nabla f = \langle y\cos(xy+z), x\cos(xy+z), \cos(xy+z)\rangle$

$\nabla f(0,-1,\pi) = \langle -\cos(\pi), 0, \cos(\pi)\rangle = \langle 1, 0, -1\rangle$

$|\langle 1,0,-1\rangle| = \sqrt{1+0+1} = \sqrt{2}$

$D_uf(P) = \sqrt{2}\cos 30$

$\boxed{D_uf(0,-1,\pi)\ \frac{\sqrt{6}}{2}}$

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