14.3: 3 5 17 19 21 27 31 39 47
14.4: 3 5 7 13 15 17 23 25 27
14.5: 7 11 13 19 27 31 33 37 39 41 43

Chapter 14 HW

Orion Kress-Sanfilippo

14.3

3) $\frac{\partial}{\partial y}\left(\frac{y}{x+y}\right) = \boxed{\frac{1(x+y) - 1(y)}{(x+y)^2}}$

5) $f(x,y,z) = xyz \quad f_z = xy \quad f_z(2,3,1) = 6$

17) $z = \frac{x}{y} \quad z'_x = \frac{1}{y} \quad z'_y = \frac{-x}{y^2}$

19) $z = \sqrt{9-x^2-y^2} \quad z_x = \frac{1}{\cancel{2}} \frac{(-\cancel{2}x)}{\sqrt{9-x^2-y^2}} = \boxed{\frac{-x}{\sqrt{9-x^2-y^2}}}$

$\boxed{z_y = \frac{-y}{\sqrt{9-x^2-y^2}}}$

21) $z = (\sin x)(\sin y) \quad z_x = \cos x \sin y \quad z_y = \cos y \sin x$

27) $W = e^{r+s} = e^r e^s \quad W_r = e^{r+s} \quad W_s = e^{r+s}$

31) $z = e^{(-x^2-y^2)} \quad z_x = -2xe^{-x^2-y^2} \quad z_y = -2ye^{-x^2-y^2}$

39) $Q = \frac{L}{M} e^{-Lt/M} \quad \boxed{Q_L = \frac{1}{M}\left(e^{-Lt/M} + L\left(\frac{-t}{M} e^{-Lt/M}\right)\right)}$

$\boxed{Q_M = L\left(-M^{-2} e^{-Lt/M} + \frac{1}{M}\left(+\frac{Lt}{M^2}\right) e^{-\frac{Lt}{M}}\right)}$

$\boxed{Q_t = -\left(\frac{L}{M}\right)^2 e^{-\frac{Lt}{M}}}$

14.3 Cont

47) $I(T,H) = 45.33 + 0.6845T + 5.758H$
$- 3.65E{-3}T^2 - 0.1565HT$
$+ 1E{-3}HT^2$

a) $I(95,50) = 45.33 + 65.0275 + 352.9275$
$- 32.94 - 743.375$
$+ 451.25 = \boxed{138.22}$

b) $\boxed{I_T = 0.6845 - 7.3E{-3}T - 0.1565H + 2E{-3}HT}$

$I_T(95,50) = 0.6845 - 0.665 - 7.825 + 9.5 = \boxed{1.6945}$

14.4 HW

3) $f(x,y) = x^2y + xy^3$ @ $(2,1) = 6$

$f_x = 2xy + y^3$ @ $(2,1)$ $\boxed{f_x = 5}$

$f_y = x^2 + 3xy^2$ $\boxed{f_y(2,1) = 10}$

$\boxed{p_T(x,y) = 5(x-2) + 10(y-1) + 6}$

5) $f(x,y) = x^2 + y^{-2}$ @ $(4,1) = 17$

$f_x(4,1) = 2(4) = \boxed{8}$ $f_y(x,y) = -2\left(\frac{1}{1}\right) = \boxed{-2}$

$p_T(x,y) = 8(x-4) - 2(y-1) + 17$

14.4 Cont

7) $F(r,s) = \frac{r^2}{\sqrt{s}} + \frac{1}{s^3}$ @ $(2,1) = 5$

$F_r(r,s) = \frac{1}{\sqrt{s}}(2r) \Rightarrow \boxed{4}$

$F_s(r,s) = \frac{-1}{2} \cdot \frac{r^2}{s^{3/2}} - \frac{-3}{s^4} \Rightarrow \boxed{-5}$

$\boxed{P_1(r,s) = 4(r-2) - 5(s-1) + 5}$

13) $L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$f(x,y) = x^2y^3 \quad (a,b) = (2,1)$

$L(x,y) = 4 + 4(x-2) + 12(y-1)$

@ $(2.01, 1.02) \Rightarrow \boxed{4.28} < f(2,1) = 5.101$

@ $(1.97, 1.01) \Rightarrow \boxed{4}$ ✓

15) $f(x,y) = x^3y^{-4}$ @ $(2,1)$

$L(x,y) = 8 + 12(x-2) - 32(y-1)$

$f(2.03, 0.9) \approx 8 + 12(0.03) - 32(-0.1) = \boxed{11.56}$

$\Delta f = 11.56 - 8 = \boxed{3.56}$

14.4 Cont

17) $f(x,y) = \sqrt{\dfrac{x}{y}}$ @ $(9,4)$ $\boxed{f_x = \frac{1}{2}\sqrt{\frac{1}{xy}} \quad f_y = \frac{-1}{2}\sqrt{\frac{x}{y^3}}}$

$$L(x,y) = \frac{3}{2} + \frac{1}{12}(x-9) - \frac{3}{16}(y-4)$$

$$L(9.1, 3.9) = \frac{3}{2} + \frac{0.1}{12} + \frac{0.3}{16} = 1.53 \approx f(9.1, 3.9) = 1.5275$$

23) $f(x,y) = x^3y^2$ @ $(2,1)$

$$L(x,y) = 8 + 8(x-2) + 16(y-1)$$

$$L(2.01, 1.02) = 8(1 + 1(0.01) + 2(0.02)) = 8.4 \approx 8.449$$

25) $f(x,y) = \sqrt{x^2+y^2}$ @ $(3,4)$ $f_x = \frac{1}{\cancel{2}}\frac{\cancel{2}x}{\sqrt{x^2+y^2}}$ $f_y = \frac{y}{\sqrt{x^2+y^2}}$

$$L(x,y) = 5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4)$$

$$L(3.01, 3.99) = 5 + \frac{3}{5}(0.01) + \frac{4}{5}(-0.01) = \boxed{4.998} = f(3.01, 3.99)$$

27) $\sqrt{(1.9)(2.02)(4.05)}$ $f(x,y,z) = \sqrt{xyz}$ $(2,2,4)$

$$L(x,y,z) = 4 + 1(x-2) + 1(y-2) + \frac{1}{2}(z-4)$$

$$L(1.9, 2.02, 4.05) = 4 + (-0.1) + (0.02) + (0.025) = \boxed{3.945}$$

$$\approx 3.94257$$

14.5 HW

7) $h(x,y,z) = xyz^{-3}$ $\boxed{\nabla h = \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle}$

11) $f(x,y) = x^2 - 3xy$ $r(t) = \langle \overset{x(t)}{\cos t}, \overset{y(t)}{\sin t} \rangle$ $t = 0$

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt}(0) + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}(0)$$

$$= \cancel{(2x - 3y)(\sin(0))} + (-3x)(\cos(0))$$

$$\boxed{\frac{df}{dt} = -3x}$$

13) $f(x,y) = \sin(xy)$ $r(t) = \langle e^{2t}, e^{3t} \rangle$ $t = 0$

$$\frac{df}{dt} = y\cos(xy) \cdot (2e^0) + x\cos(xy) \cdot (3e^0)$$

$$\boxed{= \cos(xy)(2y + 3x)}$$

18) $g(x,y,z) = xyz^{-1}$ $r(t) = \langle t^2, t^3, t-1 \rangle$ $t = 1$

$$\frac{dg}{dt} = \frac{y}{z}(2(1)) + \frac{x}{z}(3(1)^2) + \frac{xy}{z^2}(1)$$

$$\boxed{= \frac{1}{2}\left(2y + 3x + \frac{xy}{z^2}\right)}$$

14.5 Cont

27) $f(x, y) = \ln(x^2 + y^2)$ $\vec{v} = \langle 3, -2 \rangle$ $\vec{u} = \langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle$
$@\ P = (1, 0)$

$$\nabla f \cdot \vec{u} = \frac{2}{(1^2 + 0^2)} \cdot \frac{3}{\sqrt{13}} + 0 = \boxed{\frac{6}{\sqrt{13}}}$$

31) $f(x, y) = x^2 + 4y^2 ; @\ P = (3, 2)$ $\vec{u} = \langle \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle$

$$\nabla f \cdot \vec{u} = 2(3) \cdot \frac{-3}{\sqrt{13}} + 8(2)\left(\frac{-2}{\sqrt{13}}\right) = \boxed{\frac{-50}{\sqrt{13}}}$$

33) $\vec{PQ} = \langle (5-3), (7-9), (3-4) \rangle = \langle 2, -2, -1 \rangle$

$T(x, y, z) = xe^{y-z}$ $\nabla T = \langle e^{y-z}, xe^{y-z}, -xe^{y-z} \rangle$

$$\nabla T(3, 9, 4) \cdot \vec{u} = e^5 \cdot \frac{2}{3} - \cancel{3}e^5 \cdot \frac{2}{\cancel{3}} + \frac{\cancel{3}e^5 1}{\cancel{3}} = \boxed{\frac{-e^5}{3}}$$

37) $\nabla f_p = \langle 2, -4, 4 \rangle$ $\vec{v} = \langle 2, 1, 3 \rangle$

$$\nabla f_p \cdot \vec{u} = \frac{2 \cdot 2}{\sqrt{14}} - \frac{4 \cdot 1}{\sqrt{14}} + \frac{3 \cdot 4}{\sqrt{14}} = \boxed{\frac{12}{\sqrt{14}}} > 0$$

$\boxed{\therefore \text{ Increasing in the dir of } \vec{v}}$

14.5 Cont

39) $f(x, y, z) = \sin(xy + z) \quad P = (0, 1, -\pi)$

$$\theta_{\vec{u}-f_p} = 30^\circ ; \cos\theta = \frac{\nabla f_p \cdot \vec{u}}{|\nabla f_p| \cdot \underbrace{|\vec{u}|}_{\boxed{1}}} \Rightarrow \frac{\sqrt{3}}{2} = \frac{\nabla f_p \cdot \vec{u}}{|\nabla f_p|}$$

$$\nabla f_p = \langle \underline{1\cos(0 + (-\pi))},\ 0,\ \cos(0 - \pi) \rangle$$

$$= \langle -1,\ 0,\ -1 \rangle$$

$$|f_p| \cdot \frac{\sqrt{3}}{2} = D_{\vec{u}} f(P) = \boxed{\sqrt{\frac{3}{2}}}$$

41) $\vec{v}_N \quad (x^2 + y^2 - z^2 = 6) \quad @ \ P = (3, 1, 2)$

$$\langle 2x,\ 2y,\ -2z \rangle \Rightarrow \boxed{\vec{v}_N = \langle 6, 2, -4 \rangle}$$

43) $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1 \quad \vec{v} = \langle 1, 1, -2 \rangle$

$$\nabla f_p = \left\langle \frac{x}{2}, \frac{2y}{9}, 2z \right\rangle$$

$$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = \frac{(k)^2}{2} + \frac{2(k)^2}{9} + (k^2 \quad ??$$