Ex 14.3

~~3, 5, 7, 13, 15, 17, 23, 25, 27~~

3, 5, 17, 19, 21, 27, 31, 39, 47

(3) $\frac{d}{dy}\left(\frac{y}{x+y}\right)$

$\frac{y(1)-(x+y)1}{(x+y)^2}$

$= \frac{y-x-y}{(x+y)^2}$

(3) $\frac{d}{dy}\left(\frac{y}{x+y}\right)$

$\frac{(x+y).1-y}{(x+y)^2}$

$= \frac{x}{(x+y)^2}$

(5) $f_2(2,3,1)$

$f(x,y,z) = xyz$

$f_2(x,y,z) = (xy)$

$f_2(2,3,1) = 6$

(17) $z = \frac{x}{y}$.

$\frac{dz}{dx} = \frac{1}{y}$

$\frac{dz}{dy} = -\frac{x}{y^2}$

$6x$

$\frac{6}{y}$

~~(19) $z = \sqrt{9-x^2-y^2}$~~

~~$\frac{dz}{dx} = \sqrt{-2x}$~~

4

(19) $z = \sqrt{9-x^2-y^2}$

$z = \frac{-\cancel{2}x}{\cancel{2}\sqrt{9-x^2-y^2}}$

$= \frac{-x}{\sqrt{9-x^2-y^2}}$

(21) $z = (\sin x)(\sin y)$

$dz/dx = \sin x \cos y(0) + \sin y \cos x$

$\frac{dz}{dx} = \sin y \cos x$

$dz/dy = \sin x \cos y$

(27) $W = e^{r+s}$

$\frac{dw}{dr} = e^{r+s}$.

$\frac{dw}{ds} = e^{r+s}$

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(31) $z = e^{-x^2-y^2}$

$\frac{dz}{dx} = e^{-x^2-y^2} \cdot (-2x)$

$\frac{dz}{dy} = e^{-x-y^2} \cdot (-2y)$

(37) $Q = \frac{L}{M} e^{-Lt/M}$

$\frac{dQ}{dt} = \frac{L}{M} e^{-Lt/M} \cdot \frac{-L}{M}$

$$\boxed{\frac{dQ}{dt} = \frac{-L^2}{M^2} e^{-Lt/M}}$$

$\frac{dQ}{dL} = \frac{1}{M} e^{-Lt/M} \left(-\frac{t}{M}\right)$

$\frac{dQ}{dL} = \frac{-t}{M^2} e^{-Lt/M}$

$\frac{dQ}{dM} = \frac{-L}{M^2} e^{-Lt/M} \left(\frac{Lt}{M^2}\right)$

$$\boxed{\frac{dQ}{dM} = \frac{-L(Lt)}{M^4} e^{-Lt/M}}$$

(47) $\frac{dI}{dt} = 0.6845 - 2(0.00365T) - (0.1565H) + (0.001 \times 2 \times H)$

* Then, substitute the values.

$-\frac{2t}{3}$

$\frac{x}{4}$

$-\frac{(x) \cancel{t} 4}{\cancel{2} 3}$

$\frac{1}{M} - \frac{Lt}{M^2}$

$\left(\frac{-Lt}{M}\right)'$

$= \frac{1}{M^2}(Lt)$

(13) $f(x,y) = x^2y^3$

$(a,b) = (2,1)$

$f(2.01, 1.02)$ and $f(1.97, 1.01)$

$f_x(x,y) = 2xy^3 = 4$

$f_y(x,y) = \cancel{3x}\ 3y^2x^2 = 3\times 4 = 12$

$L(x) = \cancel{lnt}\ 4 + 4(x-2) + 12(y-1)$

$= 4 + 4(2.01-2) + 12(1.02-1)$

$= \underline{\underline{4.28}}$

(15) $f(x,y) = x^3y^{-4}$

$\boxed{\Delta f = f(2.03, 0.9) - f(2,1)}$

$(a,b) = (2,1)$

$f(2.03, 0.9)$

$f_x = 3x^2y^{-4} = 12$

$f_y = -4\cancel{y}y^{-5}x^3 = -4(8) = -32$

$L(x) = 8 + 12(x-2) - 32(x-1)$

$L(x) = 8 + 12(2.03-2) - 32(0.9-1)$

$= \cancel{8+}\ 11.56$

$\boxed{= 3.56}$

12.75 (17) $f(x,y) = e^{x^2+y}$ at $(0,0)$ ....

$f(0.01, -0.02) = ?$

$(a,b) = 0,0$

$f_x = e^{x^2+y}(2x) = e^0(0) = 0$

$f_y = e^{x^2+y}\cancel{4} = e^0 = 1$

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~~$L(x) = 1 + (y)$~~

$L(x) = 1 + 0(x-0) + 1(y-0)$

$= 1 \cancel{+} -0.02$

$= 0.98$

(19) $f(x,y,z) = z\sqrt{x+y}$ at $(8,4,5)$

$f\_x = 6\sqrt{x+5}$

$= z \dfrac{1}{2\sqrt{x+y}} \quad \cancel{x} = \dfrac{5}{2\times3}$

$f\_y = \dfrac{z}{2\sqrt{x+y}} = 5$

$f\_z = \sqrt{x+y}$.

$3\times8 = 24 \cancel{x}\ 5\sqrt{12}$

~~$L(x) = 24 + \dfrac{5}{6}$~~

$L(x) = 5\sqrt{12} + \dfrac{5}{2\sqrt{12}}(x-8) + \dfrac{5}{2\sqrt{12}}(x-4) +$

$\sqrt{12}(x-5)$

(23) $(2.01)^3(1.02)^2$

$f(x,y) = x^3y^2 \quad (2,1)$

$f(2,1) = 8$

$f\_x = 3x^2y^2 = 12$

$f\_y = 2yx^3 = 16$

$\cancel{f}\ L(x) = 8 + 12(x-2) + 16(\cancel{y}-1)$

$= 8 + 12(2.01-2) + 16(1.02-1)$

$= 8 + 0.12 + 0.32$

$= 8.44$

(25) $\sqrt{(3.01)^2+(3.99)^2}$

$(x,y) = (3,4)$

$f(x,y) = \sqrt{x^2+y^2}$, $f(3,4) = 5$

$f_x(x,y) = \frac{1}{\cancel{2}\sqrt{x^2+y^2}}(\cancel{2}x) = \frac{x}{\sqrt{x^2+y^2}} = \frac{3}{\sqrt{9+16}} = \frac{3}{5}$

$f_y(x,y) = \frac{1}{\cancel{2}\sqrt{x^2+y^2}}(\cancel{2}y) = \frac{y}{\sqrt{x^2+y^2}} = \frac{4}{\sqrt{9+16}} = \frac{4}{5}$

$L(x) = 5 + \frac{3}{5}(x-3) + \frac{4}{5}(x-4)$

$= 5 + \frac{3}{5}(3.01-3) + \frac{4}{5}(3.99-4)$

$= \underline{4.998}$

(27) $\sqrt{(1.9)(2.02)(4.05)}$ $(a,b,c) = (2,2,4)$

$f(x,y,z) = \sqrt{xyz}$ $f(2,2,4) = \sqrt{16} = 4$

$f_x = \frac{yz}{2\sqrt{xyz}} = \frac{8}{2\sqrt{16}} = \frac{8}{2\times4} = \frac{8}{8} = 1$

$f_y = \frac{xz}{2\sqrt{xyz}} = \frac{8}{2\sqrt{16}} = \frac{8}{8} = 1$

$f_z = \frac{xy}{2\sqrt{xyz}} = \frac{4}{8} = \frac{1}{2}$

$L(x) = 4 + 1(x-2) + 1(y-2) + \frac{1}{2}(z-4)$

$L(x) = 4 + 1(1.9-2) + (2.02-2) + \frac{1}{2}(4.05-4)$

$= \cancel{3.0{}45}\ 3.945$

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$x^{2t}+x^{4}$

Ex 14.5

~~7~~, ~~11~~, ~~13~~, ~~19~~, ~~27~~, ~~31~~, ~~33~~, ~~37~~, ~~39~~, ~~41~~, 43

(7) $h(x,y,z) = xyz^{-3}$

$\nabla h(x,y,z) = \langle yz^{-3}, xz^{-3}, xy \rangle$

(11) $f(x,y) = x^2 - 3xy \quad r(t) = \langle \cos t, \sin t \rangle, \; t = 0.$

$\frac{d}{dt} f(r(t))$

$f(r(t)) = \cos^2 t - 3\cos t \sin t.$

$\frac{d}{dt}(\cos^2 t - 3\cos t \sin t) = 2\cos t(-\sin t) - [3(\cos t \cdot \cancel{\sin t}\,\cos t) + \sin t(-\sin t)]$

$= \cancel{2\cos} -2\cos t \sin t - 3\cos^2 t + 3\sin^2 t$

$= -2\cos 0 \sin 0 - 3\cos^2 0 + 3\sin^2 0$

$= 0 - 3 + 0$

$= \underline{-3}$

(13) $f(x,y) = \sin(xy) \qquad r(t) = \langle e^{2t}, e^{3t} \rangle$ e

$|r(t)| = \frac{e^{2t}}{\sqrt{e^{4t^2}+e^{9t^2}}}, \frac{e^{3t}}{\sqrt{e^{4t^2}+e^{9t^2}}}$

$f(r(t)) = \sin e^{5t}$

$= 5\cos e^{5t}$

$= 5\cos 1$

≈ 2.702

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(19) $g(x,y,z) = xyz^{-1}$ $\quad r(t) = \langle e^t, t, t^2 \rangle,\ t=1$

$$f(r(t)) = \frac{e^t \cdot t}{t^2}$$

$$f(r(t)) = \frac{e^t}{t}$$

$$\frac{d}{dt}(f(r(t))) = \frac{t e^t - e^t (1)}{(t)^2}$$

$$= \frac{te^t - e^t}{t^2}$$

$$= \frac{e-e}{1}$$

$$= 0$$

(27) $f(x,y) = \ln(x^2+y^2)$ $\quad V = 3i - 2\hat{j}$ $\quad P = (1,0)$

$$\nabla f(x,y) = \left\langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \right\rangle$$

$$v = 3i - 2j$$

$$|v| = \frac{3}{\sqrt{13}} i - \frac{2}{\sqrt{13}} \hat{j}$$

$$\nabla f(x,y) \cdot |v| = \left\langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \right\rangle \cdot \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$$

$$= \left\langle \frac{2}{1}, 0 \right\rangle \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$$

$$= \frac{6}{\sqrt{13}}$$

$x \cdot e(\cancel{x})$

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(31) $f(x,y) = x^2 + 4y^2 \quad P = (\cancel{x}\,3, 2) \quad \cancel{v = \{0, 0, 0\}}$

$\nabla f(x,y) = \langle 2x, 8y \rangle \qquad [0,0]\cancel{,0,1} - [3,2]$

$= \langle 6, 16 \rangle \qquad \underline{3} \qquad = \underline{\underline{[-3, -2]}}$

$v = -\dfrac{18}{\sqrt{13}} \mp \dfrac{32}{\sqrt{13}}$

$= \dfrac{-50}{\sqrt{13}}$

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33

$B - A = \langle 5, 7, 3\rangle - \langle 3, 9, 4\rangle$

$= \langle 2, -2, -1\rangle$

Unit vector in that direction $= \langle \frac{2}{\sqrt{9}}, \frac{-2}{\sqrt{9}}, \frac{-1}{\sqrt{9}} \rangle$

$|u| = \langle \frac{2}{3}, \frac{-2}{3}, \frac{-1}{3} \rangle$

$\text{grad}(T) = \langle xe^{y-z} + e^{y-z}, xe^{y-z}, xe^{y-z} \rangle$

$= \langle 3e^5 + e^5, 3e^5, 3e^5 \rangle$

$= \langle 4e^5, 3e^5, 3e^5 \rangle$

$\langle 4e^5, 3e^5, 3e^5 \rangle \langle \frac{2}{3}, \frac{-2}{3}, \frac{-1}{3} \rangle$

$= \frac{8e^5}{3} - \frac{\cancel{6}^{2} e^5}{\cancel{3}} - e^5$

$= \frac{8e^5}{3} - 2e^5 - e^5$

$= \frac{8e^5}{3} - 3e^5$

$= \frac{-e^5}{3}.$

$\cancel{\langle e^4, 3e^5}$,

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(37) $\nabla f_P = \langle 2, -4, 4 \rangle$

$v = \langle 2, 1, 3 \rangle$

$\nabla f \cdot v = \langle 2, -4, 4 \rangle \cdot \langle 2, 1, 3 \rangle$

$= \cancel{\langle 4, -4, 12 \rangle}$ $\quad 4 - 4 + 12$

$= \underline{12}$ increasing

(39) $f(x, y, z) = \sin(xy + z)$ $\qquad \frac{\sqrt{3}}{2}$

$P = (0, 1, \pi)$

$V = \langle 1, 1, 1 \rangle \quad \theta = 30^\circ$

$\nabla f(x, y, z) = \langle \cancel{\sin}\ \cos(xy+z)(y), \cos(xy+z)\,x, \cos(xy+z) \rangle$

$= \langle \cos(\pi), 0, \cos\pi \rangle \cdot \langle 1, 1, 1 \rangle \cos 30^\circ$

$= \langle 1, 0, -1 \rangle \langle 1, 1, 1 \rangle \left(\frac{\sqrt{3}}{2}\right)$

$\cancel{\frac{\sqrt{3}}{2}} + \cancel{0} - \cancel{\frac{\sqrt{3}}{2}}$ $\quad \boxed{\frac{\sqrt{3}}{2} \cdot \frac{-\sqrt{3}}{2}}$

$= \underline{0}$

$\perp = \cos 90$

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(41) $x^2 + y^2 - z^2 = 6$ at $P = (3, 1, 2)$

Find vector normal.

$$\nabla f(x, y, z) = \langle 2x, 2y, -2z \rangle$$

Normal $= \langle 6, 2, -4 \rangle$

Unit normal vector:

$$\langle 6, 2, -4 \rangle / \sqrt{56}$$

36
4
16
56

(43) $\boxed{\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1}$

~~Et~~ tangent plane is normal to $V = \langle 1, 1, -2 \rangle$

$$\nabla f(x, y, z) = \left\langle \frac{1}{4}(2x), \frac{1}{9}(2y), 2z \right\rangle$$

$$= \left\langle \frac{x}{2}, \frac{2y}{9}, 2z \right\rangle$$

$$\left\langle \frac{x}{2}, \frac{2}{9}y, 2z \right\rangle \cdot \langle 1, 1, -2 \rangle = 0$$

$$\boxed{\frac{x}{2} + \frac{2y}{9} - 4z = 0}$$