homework 4

14.3

3.) $\frac{\partial}{\partial y} : \frac{y}{x+y} \rightarrow \frac{\frac{\partial}{\partial y}(y)(x+y) - \frac{\partial}{\partial y}(x+y)(y)}{(x+y)^2}$

$= \frac{(x+y) - y}{(x+y)^2} \rightarrow \boxed{\frac{x}{(x+y)^2}}$

5.) $f_z(2,3,1) \rightarrow f(x,y,z) = xyz$

$f_z(2,3,1) \rightarrow f_z = xy$

$f_z(2,3,1) = (2)(3) = \boxed{6}$

17.) $z = \frac{x}{y}$

$\boxed{f_x = \frac{1}{y}}$ $\quad f_y = \frac{\partial}{\partial y}\left(\frac{x}{y}\right) \rightarrow x\frac{\partial}{\partial y}\left(\frac{1}{y}\right) \rightarrow x\left(-1 \cdot \frac{1}{y^2}\right) = \boxed{f_y = -\frac{x}{y^2}}$

M

19.) $z = \sqrt{9-x^2-y^2} \rightarrow$

$f_x = \frac{\partial}{\partial x}\left(\sqrt{9-x^2-y^2}\right) = -\frac{x}{\sqrt{9-x^2-y^2}} \rightarrow \frac{1}{2\sqrt{9-x^2-y^2}}\frac{\partial}{\partial x}(9-x^2-y^2)$

$\frac{\partial}{\partial x} : (9-x^2-y^2) = -2x$

$\frac{1}{2\sqrt{9-x^2-y^2}}(-2x) = -\frac{x}{\sqrt{9-x^2-y^2}} = \boxed{f_x - \frac{x}{\sqrt{9-x^2-y^2}}}$

$f_y : \frac{\partial}{\partial y} : \left(\sqrt{9-x^2-y^2}\right) = -\frac{y}{\sqrt{9-x^2-y^2}} \rightarrow \frac{1}{2\sqrt{9-x^2-y^2}}\frac{\partial}{\partial y}(9-x^2-y^2) =$

$\boxed{f_y = -\frac{y}{\sqrt{9-x^2-y^2}}}$

21.) $z = (\sin x)(\sin y)$

$f_x = \frac{\partial}{\partial x}(\sin(x)\sin(y)) \rightarrow \sin(y)\frac{\partial}{\partial y}(\sin(x)) \rightarrow \cos(x)$

$\boxed{f_x = \sin(y)\cos(x)}$

$\boxed{f_y = \sin(x)(\cos(y))}$

27.) $w = e^{r+s}$

$\boxed{f_x = e^{r+s}}$ $\boxed{f_y = e^{r+s}}$

31.) $z = e^{-x^2-y^2} \rightarrow \boxed{f_x = -2xe^{-x^2-y^2}}$ $\boxed{f_y = -2ye^{-x^2-y^2}}$

39.) $Q = \frac{L}{M}e^{-Lt/M}$

$\boxed{\frac{\partial Q}{\partial L} = \frac{M-Lt}{M^2}e^{-Lt/M}}$ $\quad \frac{\partial Q}{\partial M} = \frac{L(Lt-M)}{M^3}e^{-Lt/M} \rightarrow \boxed{\frac{\partial Q}{\partial M} = -\frac{L^2}{M^2}e^{-Lt/M}}$

41.) $f(x,y) = 3x^2y + 4x^3y^2 - 7xy^5$

$= 6xy,\ 12x^2y^2,\ 7y^5 \rightarrow f_x = 6xy + 12x^2y^2 - 7y^5$

$f(1,2) = 6(1)(2) + 12(1)^2(2)^2 - 7(2)^5 = 12 + 24 - 224 = 36 - 224 = \boxed{-188}$

14.4

3.) $f(x,y) = x^2y + xy^3$, $(2,1)$

$f_x = 2xy + y^3$ $\quad f_y = x^2 + 3xy^2$

$f(2,1) = (2)^2(1) + (2)(1)^3 = 6$

$f_x(2,1) = 2(2)(1) + (1)^3 = 5$

$f_y(2,1) = (2)^2 + 3(2)(1)^2 = 10$

$z = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x,y)(y-y_0)$

$z = 6 + 5(x-2) + 10(1)$

$z = 6 + 5x - 10 + 10$

$\boxed{z = 5x + 10y - 14}$

5.) $f(x,y) = x^2 + y^{-2}$ $\quad (4,1)$

$f_x = 2x$ $\quad f_y = \frac{\partial}{\partial y}(y^{-2}) = -\frac{2}{y^3}$ $\boxed{f_y = -\frac{2}{y^3}}$

$f(4,1) = (4)^2 + (-1)^{-2} = 16$

$f_x(4,1) = 2(4) = 8$

$f_y(4,1) = -\frac{2}{(1)^3} = -2$

$z = 16 + 8(x-4) + -2(y-1)$

$z = 16 + 8x - 32 - 2y + 2$

$\boxed{z = 8x - 2y - 13}$

7.) $F(r,s) = r^2 s^{-1/2} + s^{-3}$ $\quad (2,1)$

$F_r = \frac{\partial}{\partial r}(r^2 s^{-1/2}) + \frac{\partial}{\partial r}(s^{-3})$ $\quad F_s = (r^2 s^{-1/2})\frac{\partial}{\partial s}(s^{-3})$

$= 2rs^{-1/2}$ $\quad = \frac{-r^2}{2s^{3/2}} - \frac{3}{s^4}$

$F(2,1) = \cancel{4}(2)^2(1) + (1) = 5$

$F_r(2,1) = 2(2)(1) = 4$

$F_s(2,1) = -\frac{(2)^2}{2(1)} - \frac{3}{1} = -2 - 3 = -5$

$z = 5 + 4(x-2) + -5(y-1)$

$= 5 + 4x - 8 - 5y + 5$

$z = \boxed{4x - 5y + 2}$

13.) $L(x,y)$ of $f(x,y) = x^2y^3$ @ $(a,b) = (2,1)$

$L(x,y) = f(a,b) + (x-a)f_x(a,b) + (y-b)f_y(a,b)$

$f_x = 2xy^3$ $\quad$ $f_y = 3x^2y^2$

$f(2,1) = (2)^2(1)^3 = 4$

$f_x(2,1) = 2(2)^2(1)^3 = 8$

$f_y(2,1) = 3(2)^2(1)^2 = 12$

$L(x,y) = 4 + 8(x-2) + 12(y-1)$

$= 4 + 8x - 16 + 12y - 12$

$= 8x + 12y + 4 - 16 - 12$

$=$

$f(x,y) = 4x + 12y - 16$

$f(2.01, 1.02) = 4(2.01) + 12(1.02) - 16 = 4.28$

$f(1.97, 1.01) = 4(1.97) + 12(1.01) - 16 = 4$

15.) $f(x,y) = x^3y^{-4}$

$\Delta f = f(2.03, 0.9) - f(2,1)$

$f_x = 3x^2y^{-4}$ $\quad$ $f_y = x^3 \cdot \frac{\cancel{34}}{y^5} = -\frac{4x^3}{y^5}$

$f(2,1) = (2)^3(1)^{-4} = 8$

$f_x(2,1) = 3(2)^2(1)^{-4} = 24$

$f_y(2,1) = -\frac{4(2)^3}{1} = 32$

$= 8 + 24(x-2) + 32(y-1)$

$= 8 + 24x - 48 + 32y - 32$

$= 8 + 24x + 32y$

$= \Delta f \approx 3.56$

17.) $f(x,y) = e^{x^2+y}$ $(0,0)$

$f_x = 2xe^{x^2+y}$ $\quad$ $f_y = e^{x^2+y}$

$f(0,0) = e^{0^2+0} = 1$

$f_x(0,0) = 2(0)e^{(0)^2+0} = 0$

$f_y(0,0) = e^{0^2+0} = 1$

$= 1 + 0(x-0) + 1(y-0)$

$= 1 + 0 + y - 0$

$= 1+y$

$f(0.01,-0.02) \approx 1 + (-0.02) = \boxed{0.98}$

23.) $(2.01)^3(1.02)^2$

$f(x,y) = x^3y^2$

$f_x = 3x^2y^2$ $\quad$ $f_y = 2x^3y$

$\cancel{f(3,2) = (3)^3(2)^2 = 36}$ $\quad$ $f(2,1) = (2)^3(1)^2 = 8$

$\cancel{f_x(3,2) = 3(3)^2(2)^2 = 108}$ $\rightarrow$ $f_x(2,1) = 3(2)^2(1)^2 = 12$

$\cancel{f_y(3,2) = 2(3)^3(2) = 36}$ $\quad$ $f_y(2,1) = 2(2)^3(1) = 16$

$\cancel{36 + 108(x-3) + 36(y-2)}$

$f(2+0.01, 1+0.02) = (2.01)^3(1.02)^2 = 8 + 12(0.01) + 16(0.02) \boxed{= 8.44}$

calc val: 8.4487

25.) $f(x,y) = \sqrt{x^2+y^2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}} \frac{\partial}{\partial x}(x^2+y^2) = \frac{x}{\sqrt{x^2+y^2}}$ $\quad$ $f_y = \frac{y}{\sqrt{x^2+y^2}}$

$f(3,4) = \sqrt{16+9} = 5$

$f_y(3,4) = \frac{(4)}{\sqrt{25}} = \frac{4}{5}$ $\quad$ $f_x(3,4) = \frac{3}{\sqrt{25}} = \frac{3}{5}$

$= 5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4)$

$= 5 + \frac{3}{5}x - \frac{9}{5} + \frac{4}{5}y - \frac{16}{5}$

$= \frac{3}{5}x + \frac{4}{5}y + 5 - \frac{9}{5} - \frac{16}{5}$

$= \frac{3}{5}x + \frac{4}{5}y + 0$

$f(3.01, 3.99) = \boxed{4.998}$

27.) $\sqrt{(1.9)(2.02)(4.05)}$

$f(xyz)\sqrt{xyz} = f(2,2,4) = \sqrt{16} = 4$

$f_x = \frac{yz}{2\sqrt{xyz}}$ $f_y = \frac{xz}{2\sqrt{xyz}}$ $f_z = \frac{xy}{2\sqrt{xyz}}$

$x=2$ $y=2$ $z=4$

$f_x(2,2,4) = \frac{8}{2\sqrt{16}} = \frac{4}{\sqrt{16}} = 1$ $f_z(2,2,4)\ \frac{4}{2\sqrt{16}} = \frac{2}{4} = \frac{1}{2}$ $f_y = \frac{8}{2\sqrt{16}} = 1$

$\frac{1}{2} = 4 + (x-2) + (y-2)$

$= \Delta x = -0.1$

$\Delta y = 0.02$

$\Delta z = 0.05$

$\boxed{= 3.95}$

14.5

7.) $h(x,y,z) = xyz^{-3}$

$\nabla h = \langle h_x, h_y, h_z \rangle = \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle$

11.) $f(x,y) = x^2 - 3xy$, $r(t) = \langle \cos t, \sin t \rangle$ $t=0$

$r(0) = \langle 1, 0 \rangle$

$f(1,0) = (1)^2 - 3(1)(0)$

$\frac{d}{dt}(f(r(t))\Big|_{t=0} \boxed{= -3}$

13.) $f(x,y) = \sin(xy)$ $r(t) = \langle e^t, e^{2t} \rangle$ $t = \ln 3$

$r(t) = \langle 3, 9 \rangle$

$\frac{d}{dt}(f(r(t))\Big|_{t=u} \boxed{= 5\cos(1)}$

19.) $g(x,y,z) = xyz^{-1}$ $r(t) = \langle e^t, t, t^2 \rangle$ $t=1$

$r(t) = \langle e, 1, 1 \rangle$

$\frac{d}{dt} g(r(t)) = \Big|_{t=1}$ $= \boxed{0}$

27.) $f(x,y) = \ln(x^2+y^2)$ $\quad v = 3i - 2j \quad P=(1,0)$

$v = \langle 3, 2 \rangle \quad P = (1,0)$

$f_x = \dfrac{2x}{x^2+y^2} \qquad f_y = \dfrac{2y}{x^2+y^2}$

$\dfrac{2(1)}{1^2+0} = 2 \qquad \dfrac{0}{1+0} = 0$

$\nabla f_{(1,0)} = \langle 2, 0 \rangle$

$\sqrt{9+4} = \sqrt{13} \rightarrow \cancel{\dfrac{A}{\sqrt{13}}}, 0$

$\langle 2, 0 \rangle \langle 3, 2 \rangle = \langle 6, 0 \rangle$

$\cancel{\dfrac{\sqrt{39}}{\sqrt{13}}} \quad \boxed{\dfrac{6}{\sqrt{13}}}$

31.) $f(x,y) = x^2 + 4y^2 \qquad P = (3,2) \rightarrow (0,0)$

$\langle -3, -2 \rangle$

$f_x = 2x \qquad f_y = 8y$

$\nabla f(3,2) = \langle 6, 16 \rangle$

$\cancel{\sqrt{36+256} = \sqrt{292}} \qquad \sqrt{9+4} = \sqrt{13}$

$\langle -3, -2 \rangle \langle 6, 16 \rangle = \langle -18, -32 \rangle$

$\dfrac{-18 + -32}{\sqrt{292}} = \dfrac{-50}{\cancel{\sqrt{292}}} \quad \boxed{\dfrac{-50}{\sqrt{13}}}$

33.) $(3,9,4) \rightarrow (5,7,3)$

$\langle 2, -2, -1 \rangle$

$T(x,y,z) = xe^{y-z}$

$f_x = e^{y-z} \qquad f_y = xe^{y-z} \qquad f_z = -xe^{y-z}$

$e^{9-4} = e^5 \qquad 3 \cdot e^5 \qquad -3e^5$

$4+4+1 = \sqrt{9} = 3$

$\langle 2, -2, -1 \rangle \cdot \langle e^5, 3e^5, -3e^5 \rangle$

$\langle 2e^5, -6e^5, 3e^5 \rangle$

$\dfrac{-e^5}{\sqrt{9}} = \boxed{\dfrac{-e^5}{3}}$

37.) $\nabla f_P = \langle 2, -4, 4\rangle \to \langle 2, 1, 3\rangle$

It is increasing!

39.) $f(x,y,z) = \sin(xy+z)$ $P: (0,-1,\pi)$

$u = \theta = 30°$

$f_x = \cos(xy+z)y$ $f_y = \cos(xy+z)x$ $f_z = \cos(xy+z)$

$f_x = \cos(0+\pi) -1$ $\cos(0+\pi) 0$

$= 1$ $= 0$ $= -1$

$\langle 1, 0, -1\rangle$

$\boxed{\frac{6}{\sqrt{2}}}$ $\sqrt{2}$

41.) $x^2 + y^2 - z^2 = 6$ @ $P: (3,1,2)$

$f_x = 2x$ $f_y = 2y$ $f_z = -2z$

6 2 -4

$\boxed{\langle 6, 2, -4\rangle}$

43.) $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$

$f_x = \frac{1}{2}x$ $f_y = \frac{1}{18}y$ $f_z = 2z$

$\nabla f_P = \lambda u$

$x = 2\lambda,\ y\ \frac{9\lambda}{2},\ z = \lambda$

$\frac{2\lambda^2}{4} + \frac{(\frac{9\lambda}{2})^2}{9} + (-\lambda)^2 = 1$

$\lambda = \pm\frac{2}{\sqrt{17}}$

$\boxed{\text{points} = \pm\left(\frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, -\frac{2}{\sqrt{17}}\right)}$