14.5

7. $\nabla h(x,y,z) = \left(\frac{y}{z^3}, \frac{x}{z^3}, -\frac{3xy}{z^4}\right)$

11. $\frac{d}{dt} f\big(r(t)\big) = -2\sin t\cos t + 3(\sin t)^2 - 3(\cos t)^2$

$which\ equal\ to - 3\ when\ t = 0$

13. $\frac{d}{dt} f\big(r(t)\big) = 5e^{5t}\cos e^{5t}$

$which\ equal\ to\ 5\cos 1$

19. $\frac{d}{dt} g\big(r(t)\big) = \frac{te^t - e^t}{t^2}\ which\ equal\ to\ 0\ when\ t = 1$

27. $\nabla f(x,y) = \left(\frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}\right)\ u = \left(\frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\right)$

$\nabla f(1,0) = (2,0)\ the\ directional\ derivative\ is\ \frac{6}{\sqrt{13}}$

31 $\nabla f(x,y) = (2x, 8y)\ u = \left(\frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\right)$

$\nabla f(3,2) = (6,16)\ the\ directional\ derivative\ is\ \frac{-50}{\sqrt{13}}$

33. $\nabla T(x,y,z) = (e^{y-z}, xe^{y-z}, -xe^{y-z})\ u = \left(\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right)$

$\nabla T(3,9,4) = (e^5, 3e^5, -3e^5)\ \ the\ directional\ derivative\ is - \frac{e^5}{3}\frac{℃}{m}$

37. $increasing$

39. $\nabla f(x,y,z) = (y\cos(xy+z),\ x\cos(xy+z), \cos(xy+z))$

$\nabla f(0,-1,\pi) = (1,0,-1)\ D_u f(p) = \frac{\sqrt{6}}{2}$

41. $f(x,y,z) = x^2 + y^2 - z^2$

$\nabla f(x,y,z) = (2x, 2y, -2z)$

$\nabla f(3,1,2) = (6,2,-4)$

43. $f(x,y,z) = \left(\frac{x^2}{4}, \frac{y^2}{9}, z^2\right), \nabla f(x,y,z) = \left(\frac{x}{2}, \frac{2y}{9}, 2z\right)$

$$\nabla f(x,y,z) = k\boldsymbol{v} \ \rightarrow \begin{cases} x = 2k \\ y = \dfrac{9k}{2} \\ z = -k \end{cases}$$

$plugging\ the\ equations\ into\ f(x,y,z), we\ can\ get\ k^2 + \frac{9k^2}{4} + k^2 = 1$

$k = \pm\frac{2}{\sqrt{17}},$

$so\ the\ points\ are\ \left(\frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, -\frac{2}{\sqrt{17}}\right) and\ \left(-\frac{4}{\sqrt{17}}, -\frac{9}{\sqrt{17}}, \frac{2}{\sqrt{17}}\right).$