9/23/20 14.3 Partial Derivatives

14.3 # 3, 5, 17, 19, 21, 27, 31, 39, 47

3) $\frac{d}{dy} \quad \frac{y}{x+y} = \frac{(x+y)\cdot y' - y(x+y)'}{(x+y)^2}$

$= x + y - y = x \qquad \frac{x}{(x+y)^2}$

5) $f_z \; (2,3,1)$ where $f(x,y,z) = xyz$

$xyz \quad f_z = xy = 2\cdot 3 = \boxed{6}$

17) $z = \frac{x}{y}$

$z = xy^{-1}$

$\frac{dz}{dx} = y^{-1}$

$\frac{dz}{dy} = -xy^{-2}$

19) $z = \sqrt{9-x^2-y^2}$

$\frac{dz}{dx} = \frac{1}{2\sqrt{9-x^2-y}}, -2x \cdot 2$

$= \frac{-2x}{\sqrt{9-x^2-y^2}}$

$\frac{dz}{dy}$ follows the same pattern

$= \frac{-2y}{\sqrt{9-x^2-y^2}}$

$-x^2 = -2x \cdot -2$

31, 39, 47

21) $z = (\sin x)(\sin y)$

$\frac{dz}{dx} = \cos x \sin y$

$\frac{dz}{dy} = \cos y \sin x$

27) $W = e^{r+s}$

$\frac{dW}{dr} = e^{r+s}$

$\frac{dW}{ds} = e^{r+s}$

31) $Z = e^{-x^2-y^2}$

$\frac{dZ}{dx} = e^{-x^2-y^2} \cdot (-2x) = -2xe^{-x^2-y^2}$

$\frac{dZ}{dy} = e^{-x^2-y^2} \cdot -2y = -2ye^{-x^2-y^2}$

47) $I(95, 50) = 73.19$ ⓐ

ⓑ. Increase in I perchange in T = $\frac{dI}{dT}$

$= 0.0684 - 0.073T - 0.1565H + 0.002HT.$

at T=95, this is 1.66.

1/25/20 14.4 Differentiability & Tangent Planes

14.4 # 3, 5, 7, 13, 15, 17, 23, 25, 27

3| $f(x,y) = x^2 + xy^3$ at $(2,1)$ $f(2,1) = 6.$

$\frac{\partial}{dx} = 2x + y^3$, $(2,1)$: $2(2) + 1 = 5$

$\frac{\partial}{dy} = x^2 + 3y^2x$ @ $(2,1) = 4 + 3(2) = 10$

$z = 5(x-2) + 10(y-1)$

$z = 6 + 5(x-2) + 10(y-1)$

$z = -14 + 5x + 10y$

7) $F(r,s) = r^2 s^{-1/2} + s^{-3}$, @ $2,1$

$F(2,1) = 4 + 1 = 5$

$\frac{d}{dr} = 2rs^{-\frac{1}{2}} = 4$

$\frac{d}{ds} = -\frac{1}{2} s^{-3/2} r^2 - 3s^{-4} = -\frac{1}{2}(4)^2 - 3$

$= \boxed{-5.}$

$z = 5 + 4(x-2) - 5(y-1)$

13) $L(x,y)$ of $f(x,y) = x^2y^3$ at $(2,1)$

$\frac{d}{dx} = 2xy^3 = 4$

$\frac{d}{dy} = 3x^2y^2 = 12.$

$f(x,y) = 4.$

$L(x,y) = 4 + 4(x-2) + 12(y-1).$

$= 4x + 12y - 16$

$L(2.01, 1.02) = 4.28, \quad f(2.01, 1.02) = 5.10!$

$L(1.97, 1.01) = 4, \quad f(1.97, 1.01) = 3.998.$

15) $f(x,y) = x^3y^{-4} \quad f(2,1) = 8$

$\frac{d}{dx} = 3x^2y^{-4} \quad 2(3)(1) = 6.$

$\frac{d}{dy} = x^3 \cdot -4y^{-5} \quad = -32$

$z = 8 + 6(x-2) - 32(y-1)$

15) $f(x, y) = x^3 y^{-4} = 8$

$f_x(x, y) = 3x^2 y^{-4} = 12$

$f_y(x, y) = -4x^3 y^{-5} = -32$

$L(x, y) = 8 + 12(x-2) - 32(y-1)$

$L(2.03, 0.9) = 11.56$

$11.56 - 8 = \boxed{3.56}$

17) $f(x, y) = e^{x^2+y}$ at $(0,0) = 1$

$f_x(x, y) = e^{x^2+y} \cdot 2x = 0$

$f_y(x, y) = e^{x^2+y} = 1$

$L(x, y) = 1 + y$

$L(0.01, -0.02) = 1 - 0.02 = 0.98.$

$f(0.01, -0.02) = 0.9802966..$

23) $(2.01)^3 (1.02)^2$

Let $f(x, y) = x^3 y^2 = 8.448$

$f_x(x, y) = 3x^2 y^2 = 12.609$

$f_y(x, y) = 2x^3 y = 16.897.$

$L(x, y) = 8.44 - 12.609(x - 2.01) + 16.897(y - 1.02).$

$x(yz)$

25) $\sqrt{3.01^2 + 3.99^2}$

$f(x,y) = \sqrt{x^2 + y^2}$ let $x = 3$, $y = 4$

$f_x(x,y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot \cancel{2}x = \frac{x}{\sqrt{x^2+y^2}} = \frac{3}{5}$

$f_y(x,y) = \frac{y}{\sqrt{x^2+y^2}} = \frac{4}{5}$

$5 + \frac{3}{5}(x-3) + \frac{4}{5}(y-4) = L(x,y)$

$L(3.01, 3.99) = 5 + \frac{3}{5}(0.01) + \frac{4}{5}(-0.01)$

$= 4.998$

Calculator: 4.998 ✓

27) $\sqrt{(1.9)(2.02)(4.05)}$

$f(w) = \sqrt{xyz}$ let $x, y, z = 2, 2, 4$

$f_x f(w) = \frac{1}{2\sqrt{xyz}} \cdot (yz) = \frac{8}{8} = 1$

$f_y(fw) = \frac{1}{2\sqrt{xyz}} \cdot xz = \frac{8}{8}$

$f_z(fw) = \frac{1}{2\sqrt{xyz}} \cdot xy = \frac{4}{8} = 0.5$

$L(x,y,z) = 4 + 1(x-2) + (y-2) + 0.5(z-4)$

$L(1.9, 2.02, 4.05) = 3.945$

$f(w) = f(x,y,z) = 3.94257$

9/27/20 14.5 Gradients & Directional Derivatives

#14.5, 7, 11, 13, 19, 27, 31, 33, 37, 3

7) $h(x,y,z) = xyz^{-3}$

$\nabla h = \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle$

11) $f(x,y) = x^2 3xy.$ $r(t) = \langle \cos t, \sin t \rangle$

$t = 0.$

find $\frac{d}{dt} f(r(t)))$ $r(0) = \langle 1, 0 \rangle$

$r'(0) = \langle 0, 1 \rangle$

$\nabla f = \langle 2x-3, -3x \rangle$

$\nabla f_{\langle 1,0 \rangle} = \langle 2, -3 \rangle$

$\nabla f_{r(0)} \cdot r'(0) = \langle 2,-3 \rangle \cdot \langle 0,1 \rangle = \boxed{-3}$

13) $f(x,y) = \sin(xy)$

$\nabla f = \langle y\cos(xy), x\cos(xy) \rangle$

$r(t) = e^{2t}, e^{3t} \rangle$

$r(0) = \langle 1, 1 \rangle$

$r'(0) = \langle 2e^{2t}, 3e^{3t} \rangle = \langle 2, 3 \rangle$

$\langle 1\cos(1), 1\,\overset{\cos}{\cancel{\sin}}(1) \rangle \cdot \langle 2, 3 \rangle$

$= 5\cos(1)$

19) $g(x, y, z) = xyz^{-1}$

$\nabla g = \langle yz^{-1}, xz^{-1}, -xyz^{-2} \rangle$

$r(t) = \langle e^{t}, t, t^{2} \rangle$

$r(1) = \langle e, 1, 1 \rangle$

$\nabla g_{r(1)} = \langle 1, e, -e \rangle$

$r'(1) = \langle e, 1, 1 \rangle$

$\boxed{e}$

27) $f(x, y) = \ln(x^{2} + y^{2})$ $\boxed{v = 3, -2}$, $P = (1, 0)$

$\nabla f = \langle \frac{2x}{x^{2}+y^{2}}, \frac{2y}{x^{2}+y^{2}} \rangle$

$\| \langle 3, -2 \rangle \| = 9 + 4 = 13$

$\nabla f_{1} = \langle \frac{2}{1}, 0 \rangle$

$\boxed{\frac{1}{\sqrt{13}} \langle 3, -2 \rangle}$

$\langle 2, 0 \rangle \cdot \frac{1}{\sqrt{13}} \langle 3, -2 \rangle$

$= \boxed{2 + \frac{3}{\sqrt{13}}}$

31) $f(x, y) = x^{2} + 4y^{2}$

$\nabla f = \langle 2x, 8y \rangle$

$\nabla f_{(3,2)} = \langle 6, 16 \rangle$

$-\frac{1}{\sqrt{13}} \langle 6, 16 \rangle \cdot \langle -3, -2 \rangle$

$= -\frac{50}{\sqrt{13}}$

$a \cdot b : a \, b \cos\theta$

37) $\nabla f = \langle 2, -4, 4 \rangle$. $v = \langle 2, 1, 3 \rangle$

$\langle 2, -4, 4 \rangle \cdot \langle 2, 1, 3 \rangle = 12$

39) $f(x, y, z) = \sin(xy + z)$, $P = (0, -1, \pi)$.

$\nabla f = \langle y\cos(xy+z), x\cos(xy+z), \cos(xy) \rangle$

$\nabla f_p = \langle 1, 0, 1 \rangle$

$\|\nabla f_p\| = \sqrt{2}$.

$\sqrt{2} \cdot \frac{\sqrt{3}}{2} = \boxed{\frac{\sqrt{6}}{2}}$

41) $x^2 + y^2 - z^2 = 6$,

$P = (1, -1, 1)$

$\nabla f = \langle 2x, 2y, -2z \rangle$

$\nabla f_p = \langle 6, 2, -4 \rangle$

$\boxed{\langle 6, 2, -4 \rangle}$

43) $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$

$\nabla f \ \langle \frac{2x}{2}, \frac{2y}{9}, 2z \rangle$

$\nabla f_p = N$

$$\frac{x}{2} = \lambda$$

$$\frac{2y}{9} = \lambda, \qquad 2z = -2\lambda$$

$$\frac{(2\lambda)^2}{4} + \frac{(9\lambda/2)^2}{9} + (\lambda)^2 = 1$$

$$\lambda = \pm \frac{2}{\sqrt{17}}$$

$$= \left(\frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, \frac{-2}{\sqrt{17}} \right)$$