14.3

3. $\frac{\partial}{\partial y}\frac{y}{x+y}$

$=\frac{(x+y)-y}{(x+y)^2}$

$=\frac{x}{(x+y)^2}$

5. $f_z = xy$

$f_z(2,3,1)=2\times 3=6$

17. $\frac{\partial z}{\partial x}=\frac{1}{y}$

$\frac{\partial z}{\partial y}=-xy^{-2}$

19. $\frac{\partial z}{\partial x}=\frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}\cdot(-2x)$

$=-x(9-x^2-y^2)^{-\frac{1}{2}}$

$\frac{\partial z}{\partial y}=\frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}}\cdot(-2y)$

$=-y(9-x^2-y^2)^{-\frac{1}{2}}$.

21. $\frac{\partial z}{\partial x}=\sin y\cdot\cos x$.

$\frac{\partial z}{\partial y}=\sin x\cdot\cos y$.

27. $\frac{\partial w}{\partial r}=e^{r+s}$

$\frac{\partial w}{\partial s}=e^{r+s}$.

31. $\frac{\partial z}{\partial x}=-2x\cdot e^{-x^2-y^2}$

$\frac{\partial z}{\partial y}=-2y\cdot e^{-x^2-y^2}$

39. $\frac{\partial Q}{\partial L}=\cancel{e^{-Lt/M}}\,\frac{1}{M}\left(e^{-Lt/M}-\frac{Lt}{M}\cdot e^{-Lt/M}\right)$

$=\frac{M-Lt}{M^2}\cdot e^{-Lt/M}$

$\frac{\partial Q}{\partial M}=\frac{1}{M^2}\left(Le^{-Lt/M}\cdot\frac{Lt}{M^2}\cdot M-Le^{-Lt/M}\right)$

$=\frac{L^2t-LM}{M^3}\cdot e^{-Lt/M}$

$\frac{\partial Q}{\partial t}=\frac{L}{M}\left(\left[-\frac{L}{M}\right]\cdot e^{-Lt/M}\right)$

$\cancel{=\frac{L^2}{M^2}\cdot e^{Lt/M}}$

$=-\frac{L^2}{M^2}\cdot e^{Lt/M}$.

47. (a) $I(95,50)=45.33+0.6845\times 95$
$+5.758\times 50-0.00365\times 95^2$
$-0.1565\times 95\times 50+0.01\times 50\times 95^2$
$=73.19125$.

(b) $\frac{\partial I}{\partial T}=0.6845-0.0073T$
$-0.1565H+0.002HT$

$\frac{\partial I}{\partial T}(95,50)=0.6845-0.0073\times 95$
$-0.1565\times 50+0.002\times 95\times 50$

$=1.666$.

14.4.

3. $f(2,1)=2^2+2=6.$

$f_x(2,1)=2xy+y^3=5.$

$f_y(2,1)=x^2+3xy^2=10.$

$z-6=5(x-2)+10(y-1)$

$z=5x+10y-14$

5. $f(4,1)=16+1=17$

$f_x(4,1)=2x=8$

$f_y(4,1)=-2y^{-3}=-2$

$z-17=8(x-4)-2(y-1)$

$z=8x-2y-13$

7. $F(2,1)=4+1=5.$

$F_r(2,1)=2s^{-\frac{1}{2}}r=\cancel{8}\,4.$

$F_s(2,1)=-\frac{1}{2}r^2s^{-\frac{3}{2}}-3s^{-4}=-5$

$z-5=4(x-2)-5(y-1)$

$z=4x-5y+2$

13 $f(2,1)=2^2=\cancel{8}\,4.$

$f_x(2,1)=2y^3x=4$

$f_y(2,1)=3x^2y^2=12$

$z-4=4(x-2)+12(y-1)$

$L(x,y)=\cancel{f(x,y)}=z=4x+12y-16$

$f(2.01,1.02)=8.04+12.24-16=4.28$

$f(1.97,1.01)=7.88+12.12-16=4.$

15.

$f(2,1)=2^3=8$

$f_x(2,1)=3y^{-4}x^2=12$

$f_y(2,1)=-4x^3y^{-5}=-32$

$z-8=12(x-2)-32(y-1)$

$z=12x-32y+16$

$f(2.03,0.9)=24.36-28.8+16=11.56$

$\Delta f=11.56-8=3.56$

~~17. $f(0,0)=1.$~~

~~$f_x(0,0)=2x\cdot e^{x^2+y}=0$~~

~~$f_y(0,0)=e^{x^2+y}=1.$~~

~~$z-1=y-1$~~

~~$z=y$~~

19. $f(8,4,5)=10\sqrt{3}$

$f_x(8,4,5)=\frac{1}{2}z(x+y)^{-\frac{1}{2}}=\frac{5}{12}\sqrt{3}$

$f_y(8,4,5)=\frac{1}{2}z(x+y)^{-\frac{1}{2}}=\frac{5}{12}\sqrt{3}$

$f_z(8,4,5)=\sqrt{x+y}=2\sqrt{3}$

$L-10\sqrt{3}=\frac{5}{12}\sqrt{3}(x-8)+\frac{5}{12}\sqrt{3}(y-4)+2\sqrt{3}(z-5)$

$L=\frac{5}{12}\sqrt{3}x+\frac{5}{12}\sqrt{3}y+2\sqrt{3}z-5\sqrt{3}$

23. $f(x,y)=x^3y^2$ at $(2,1)$.

$f(2,1)=8$

$f_x(2,1)=3y^2x^2=12$

$f_y(2,1)=2x^3y=16$

$z-8=12(x-2)+16(y-1)$

$z=12x+16y-32$

$f(2.01,1.02)=24.12+16.32-32$

$=8.44$

25. $f(x,y)=\sqrt{x^2+y^2}$ at $(3,4)$

$f(3,4)=5.$

$f_x(3,4)=\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}\cdot 2x$

$=x(x^2+y^2)^{-\frac{1}{2}}=\frac{3}{5}$

$f_y(3,4)=y(x^2+y^2)^{-\frac{1}{2}}=\frac{4}{5}$

$z-5=\frac{3}{5}(x-3)+\frac{4}{5}(y-4).$

$z=\frac{3}{5}x+\frac{4}{5}y$

$f(3.01,3.99)=3.01\times 0.6+3.99\times 0.8$

$=1.806+3.192$

$=4.998$

27. $f(x,y,z)=\sqrt{xyz}$ at $(2,2,4)$.

$f(2,2,4)=4.$

$f_x(2,2,4)=\frac{1}{2}(xyz)^{-\frac{1}{2}}=\frac{1}{8}$

$f_y(2,2,4)=\frac{1}{2}(xyz)^{-\frac{1}{2}}=\frac{1}{8}$

$f_z(2,2,4)=\frac{1}{2}(xyz)^{-\frac{1}{2}}=\frac{1}{8}$

$\cancel{z-4}$ $L-4=\frac{1}{8}(x-2)+\frac{1}{8}(y-2)+\frac{1}{8}(z-4)$

$L=\frac{1}{8}x+\frac{1}{8}y+\frac{1}{8}z+3$

$L(1.9,2.02,4.05)=3.997$

14.5.

7. $\nabla h=\langle yz^{-3}, xz^{-3}, -3xyz^{-4}\rangle$

11. $t=0$, $r(0)=\langle 1,0\rangle$ $=\langle -\sin t,\cos t\rangle$

$x=1,\ y=0$ $r'(0)=\langle 0,1\rangle$

$\nabla f=\langle 2x-3y,-3x\rangle=\langle 2,-3\rangle$

$\frac{d}{dt}f(r(t))=f'(r(t))\cdot r'(t)$

$=\langle 2,-3\rangle\cdot\langle 0,1\rangle$

$=-3.$

13. $r'(t)=\langle 2e^{2t},3e^{3t}\rangle$

$r'(0)=\langle 2,3\rangle$

$x=y=1$

$\nabla f=\langle y\cos(xy), x\cos(xy)\rangle$

$\frac{d}{dt}f(r(t))=f'(r(t))\cdot r'(t)$

$=\langle\cos(1),\cos(1)\rangle\cdot\langle 2,3\rangle$

$=5\cos(1).$

19. $r(1)=\langle e,1,1\rangle$

$r'(t)=\langle e^t,1,2t\rangle$

$r'(1)=\langle e,1,2\rangle$

$\nabla f=\langle yz^{-1}, xz^{-1}, -xyz^{-2}\rangle.$

$\frac{d}{dt}f(r(t))=f'(r(t))\cdot r'(t)$

$=\langle 1,e,-e\rangle\cdot\langle e,1,2\rangle$

$=\cancel{e}\,e+e-2e$

$=0$

27. $f(1,0)=\ln 1=0$

$\nabla f=\langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2}\rangle$

$\sqrt{3^2+2^2}=\sqrt{13}$

$u=\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\rangle$

$\nabla f(1,0)=\langle 2,0\rangle$

$D_uf=\nabla f\cdot u=\frac{6}{\sqrt{13}}=\frac{6\sqrt{13}}{13}$

31. $f(3,2)=9+16=25$

$\nabla f=\langle 2x,8y\rangle$

$\nabla f(3,2)=\langle 6,16\rangle$

$\sqrt{3^2+2^2}=\sqrt{13}$

$u=\langle -\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\rangle$

$D_uf=\nabla f\cdot u=-\frac{50}{\sqrt{13}}=-\frac{50}{13}\sqrt{13}$

33. $T(3,9,4)=3e^5$

$\langle 5,7,3\rangle-\langle 3,9,4\rangle=\langle 2,-2,-1\rangle$

$\sqrt{2^2+2^2+1^2}=3$.

$u=\langle \frac{2}{3},-\frac{2}{3},-\frac{1}{3}\rangle$.

$\nabla f=\langle e^{y-z}, xe^{y-z}, -xe^{y-z}\rangle$

$\nabla f(3,9,4)=\langle e^5,3e^5,-3e^5\rangle$

$D_uf=\nabla f\cdot u=\frac{2}{3}e^5-2e^5+e^5=-\frac{1}{3}e^5$

37. $\sqrt{2^2+1^2+3^2}=\sqrt{14}$.

$u=\langle \frac{2}{\sqrt{14}},\frac{1}{\sqrt{14}},\frac{3}{\sqrt{14}}\rangle$.

$D_uf=\nabla f\cdot u=\frac{4-4+12}{\sqrt{14}}=\frac{6\sqrt{14}}{7}>0$

∴ It's increasing.

39

$\nabla f=\langle y\cos(xy+z), x\cos(xy+z), \cos(xy+z)\rangle$

$\nabla f(0,-1,\pi)=\langle 1,0,-1\rangle$.

$D_uf=|\nabla f|\cos\theta$

$=\sqrt{1+1}\cdot\frac{\sqrt{3}}{2}$

$=\frac{\sqrt{6}}{2}$

41. $\nabla f=\langle 2x,2y,-2z\rangle$

$\nabla f(3,1,2)=\langle 6,2,-4\rangle$.

∴ the vector is $\langle 6,2,-4\rangle$.

43. $\nabla f=\langle \frac{x}{2},\frac{2y}{9},2z\rangle$

$=k\langle 1,1,-2\rangle$.

$x=2k$, $y=\frac{9}{2}k$, $z=-k$.

$k^2+\frac{9}{4}k^2+k^2=1$

∴ the points are $k=\pm\frac{2}{\sqrt{17}}$

$\langle \frac{4}{17}\sqrt{17}, \frac{9}{17}\sqrt{17}, -\frac{2}{17}\sqrt{17}\rangle$.

And $\langle -\frac{4}{17}\sqrt{17}, -\frac{9}{17}\sqrt{17}, \frac{2}{17}\sqrt{17}\rangle$.