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Sec 23

14.3

#3 $\frac{\partial}{\partial y} \frac{y}{x+y} = \frac{1(x+y)-(y(0+1))}{(x+y)^2} = \frac{x+y-y}{x^2+2xy+y^2} = \boxed{\frac{x}{x^2+2xy+y^2}}$

#5 $f_z(2,3,1)$ where $f(x,y,z) = xyz$ $\quad \frac{\partial}{\partial z} = xy \quad f_z(2,3,1) = (2)(3) = \boxed{6}$

#17 $z = \frac{x}{y} \quad \frac{\partial}{\partial x} = \frac{1}{y} \quad \frac{\partial}{\partial y} = -xy^{-2} = -\frac{x}{y^2}$

#19 $z = \sqrt{9-x^2-y^2} = (9-x^2-y^2)^{\frac{1}{2}}$

$\frac{\partial}{\partial y} = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}} \cdot (0-0-2y) = -\frac{2y}{2\sqrt{9-x^2-y^2}} = -\frac{y}{\sqrt{9-x^2-y^2}}$

$\frac{\partial}{\partial x} = \frac{1}{2}(9-x^2-y^2)^{-\frac{1}{2}} \cdot (0-2x-0) = -\frac{x}{\sqrt{9-x^2-y^2}}$

#21 $z = (\sin x)(\sin y) \quad \frac{\partial}{\partial x} = (\sin y)(\cos x) \quad \frac{\partial}{\partial y} = (\sin x)(\cos y).$

#27 $w = e^{r+s} \quad \frac{\partial}{\partial r} = e^{r+s} \quad \frac{\partial}{\partial s} = e^{r+s}$

#31 $z = e^{-x^2-y^2} \quad \frac{\partial}{\partial x} = e^{-x^2-y^2} \cdot (-2x) = -2xe^{-x^2-y^2} \quad \frac{\partial}{\partial y} = -2ye^{-x^2-y^2}$

#39 $Q = \frac{L}{M}e^{-Lt/M} \quad \frac{\partial}{\partial L} = \frac{1}{M}e^{-Lt/M} + \frac{L}{M}e^{-Lt/M} \cdot -\frac{t}{M} = \frac{e^{-Lt/M}}{M} - \frac{Le^{-Lt/M}-t}{M}$

#47 a) $I(95,50) = 45.33 + 0.6845(95) + 5.758(50) - 0.00365(95)^2 - 0.1565(50,95) + 0.001(50)(95)^2$

$I(95,50) = 73.1913$

b) T. $\quad \frac{\partial}{\partial T} = 0 + 0.6845 + 0 - 0.0073T - 0.1565H + 0.002HT$

$\frac{\partial}{\partial T}(95,50) = 1.66$

14.4

#3 $f(x,y) = x^2y + xy^3 \quad (2,1)$

$\frac{\partial}{\partial x} = 2yx + y^3 \qquad \frac{\partial}{\partial x}(2,1) = 2(1)(2) + (1)^3 = 5$

$\frac{\partial}{\partial y} = x^2 + 3xy^2 \qquad \frac{\partial}{\partial y}(2,1) = (2)^2 + 3(2)(1)^2 = 10$

$(x,y)(5,10) = (2,1)(5,10)$

$5x + 10y = 10 + 10 \longrightarrow \boxed{f(x,y) = 5x + 10y - 20}$

#5 $f(x,y) = x^2 + y^{-2} \quad (4,1)$

$\frac{\partial}{\partial x} = 2x \rightarrow 8$

$\frac{\partial}{\partial y} = -2y^{-3} = -\frac{2}{y^3} \rightarrow -2$

$(x,y)(8,-2) = (4,1)(8,-2)$

$8x - 2y = 32 - 2$

$f(x,y) = 8x - 2y - 30$

#7 $F(r,s) = r^2 s^{-1/2} + s^{-3}, \ (2,1)$

$\frac{\partial}{\partial r} = 2s^{-1/2} r \rightarrow 2$

$\frac{\partial}{\partial s} = -\frac{r^2}{2} s^{-3/2} - 3s^{-4} \rightarrow -\frac{4}{2} \cdot 1 - 3 = -5$

$(r,s)(2,-5) = (2,-5)(2,1)$

$2r - 5s = 4 - 5$

$F(r,s) = 2r - 5s + 1$

#15 $f(x,y) = x^3 y^{-4}$

#13 $f(x,y) = x^2 y^3$ at $(a,b) = (2,1)$

$\frac{\partial}{\partial x} = 2y^3 x \qquad \frac{\partial}{\partial y} = 3x^2 y^2$

$f(2,1) = 4$

$L(x,y) = 4 + 4(x-2) + 12(y-1)$

$= 4 + 4x - 8 + 12y - 12$

$= 4x + 12y - 16$

$f(2.01, 1.02) = 4(2.01) + 12(1.02) - 16 = 4.28$

$f(1.97, 1.01) = 4.$

#19 $f(x,y,z) = z\sqrt{x+y}$ at $(8,4,5)$

$\frac{\partial}{\partial x} = \frac{z}{2}(1)^{-\frac{1}{2}}$

$\frac{\partial}{\partial y} = \frac{z}{2}(1)^{-1/2}$

$\frac{\partial}{\partial z} = 1$

$L(x,y,z) = 5\sqrt{12} + \frac{5}{2}(x-8) + \frac{5}{2}(y-4) + (z-5)$

$= 5\sqrt{12} + \frac{1}{2}5x - 20 + \frac{1}{2}5y - 10 + z - 5$

14.5

#7 $h(x,y,z) = xyz^{-3}$

$\frac{\partial}{\partial x} = yz^{-3}$

$\frac{\partial}{\partial y} = xz^{-3}$

$\frac{\partial}{\partial z} = -3xyz^{-4}$

$\Delta h = \langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle$

#11 $f(x,y) = x^2 - 3xy, \ r(t) = \langle \cos t, \sin t \rangle, \ t = 0$

$\frac{\partial}{\partial x} = 2x - 3y$

$\frac{\partial}{\partial y} = -3x$

$\Delta f = \langle 2x - 3y, -3x \rangle \cdot \langle -\sin t, \cos t \rangle$

$\frac{df}{dt} f(r(t)) = -2x\sin t - 3y\sin t - 3x\cos t$

$\frac{df}{dt} f(r(0)) = 0 - 0 - 3 = \boxed{-3}$

#13

$f(x,y) = \sin(xy)$, $r(t) = \langle e^{2t}, e^{3t} \rangle$, $t=0$

$\frac{\partial}{\partial x} = y\cos(xy)$ $\quad \frac{\partial}{\partial y} = x\cos(xy)$

$\Delta f = \langle y\cos(xy), x\cos(xy) \rangle$

$\frac{df}{dt} f(r(t)) = \langle y\cos(xy), x\cos(xy) \rangle \cdot \langle 2e^{2t}, 3e^{3t} \rangle$

$= 2ye^{2t}\cos(xy) + 3xe^{3t}\cos(xy)$

$\frac{df}{dt} f(r(0)) = 2y\cos(xy) + 3x\cos(xy)$

#19 $g(x,y,z) = xyz^{-1}$

$r(t) = \langle e^t, t, t^2 \rangle$, $t=1$

$\frac{\partial}{\partial x} = yz^{-1}$ $\quad \frac{\partial}{\partial y} = xz^{-1}$

$\frac{\partial}{\partial z} = -xyz^{-2}$

$\Delta g = \langle yz^{-1}, xz^{-1}, -xyz^{-2} \rangle$

$\frac{dg}{dt} g(r(t)) = \langle yz^{-1}, xz^{-1}, -xyz^{-2} \rangle \langle e^t, 1, 2t \rangle$

$= \frac{ye^t}{z} + \frac{x}{z} - \frac{xyt}{z^2}$

$\frac{dg}{dt} g(r(1)) = \frac{ye}{z} + \frac{x}{z} - \frac{xy}{z^2}$

#27 $f(x,y) = \ln(x^2+y^2)$, $V = 3i - 2j$, $P = (1,0)$

$\frac{\partial}{\partial x} = \frac{2x}{x^2+y^2}$ $\quad \frac{\partial}{\partial y} = \frac{2y}{x^2+y^2}$

$\Delta f = \langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \rangle \rightarrow (2,0)$

$\|\vec{v}\| = \sqrt{3^2+(-2)^2} = \sqrt{9+4} = \sqrt{13}$

$v \rightarrow \langle \frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}} \rangle$

$(2,0) \cdot (\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}) = \boxed{\frac{6}{\sqrt{13}}}$

#31 $f(x,y) = x^2 + 4y^2$ $\quad V = \langle 0,0 \rangle$ $\quad P = (3,2)$

$\frac{\partial}{\partial x} = 2x$ $\quad \frac{\partial}{\partial y} = 8y$

$\Delta f = (2x, 8y) \rightarrow (6, 16)$

$\langle 0,0 \rangle \cdot \langle 6,16 \rangle = 0$

#33 $P = (3,9,4)$ $\quad u = \langle 5,7,3 \rangle$ $\quad T(x,y,z) = xe^{y-z}$

$\frac{\partial}{\partial x} = 1$ $\quad \frac{\partial}{\partial y} = xe^{y-z}$ $\quad \frac{\partial}{\partial z} = -xe^{y-z}$

$\Delta T = \langle 1, xe^{y-z}, -xe^{y-z} \rangle \rightarrow (1, 3e^5, -3e^5)$

$\|\Delta T\| = \sqrt{1^2 + (3e^5)^2 + (-3e^5)^2}$

$= 629.665$

#37 $\Delta f_P = \langle 2,-4,4 \rangle$ $\quad N = \langle 2,1,3 \rangle$

Increasing

#39 $f(x,y,z) = \sin(xy+z)$ $\quad \|\Delta f_P\| \cos\theta$ $\quad P(0,-1,\pi)$

$\frac{\partial}{\partial x} = y\cos(xy+z) \rightarrow -\cos(\pi) = 1$

$\frac{\partial}{\partial y} = x\cos(xy+z) \rightarrow \frac{\partial}{\partial z} = 0$

$\frac{\partial}{\partial z} = \cos(xy+z) \rightarrow \frac{\partial}{\partial z} = \cos(\pi) = -1$

$\Delta f_P = \langle 1,0,-1 \rangle$ $\quad \|\Delta f_P\| = \sqrt{1^2+0^2+(-1)^2} = \sqrt{2}$

$\sqrt{2}\cos 30^\circ = \boxed{1.2247}$