14.3

3. $\frac{\partial}{\partial x}\, xy^2 = y^2$ $\qquad$ $\frac{\partial}{\partial y}\, xy^2 = 2xy$

$\frac{\partial}{\partial y} = \frac{(x+y)\, y \frac{\partial}{\partial y} - y\,(x+y) \frac{\partial}{\partial y}}{[x+y]^2} = \frac{(x+y) - y}{x^2+y^2+2xy}$

5. $f_z(2,3,1)$ $\qquad$ $f(x,y,z) = xyz$

$= xy\,(z')$

$f_z(2,3,1) = 1$

17. $z = \frac{x}{y}$

$\frac{dz}{dx} = \frac{1}{y}(x') = \frac{1}{y}$ $\qquad$ $\frac{dz}{dy} = x\left(\frac{1}{y}\right)' = \frac{-x}{y^2}$

19. $z = \sqrt{9-x^2-y^2}$

$\frac{dz}{dx} = \frac{1}{2}(9-x^2-y^2)^{-1/2} \cdot (-2x)$ $\qquad$ $\frac{dz}{dy} = \frac{1}{2}(9-x^2-y^2)^{-1/2} \cdot (-2y)$

21. $z = (\sin x)(\sin y)$

$\frac{dz}{dx} = \sin x(\sin y)' + (\sin x)' \sin y = 0 + \cos x \sin y$ $\qquad$ $\frac{dz}{dy} = \cos y \sin x$

27. $W = e^{r+s}$

$\frac{dW}{dr} = (e^{r+s})'$ $\qquad$ $\frac{dW}{ds} = e^r + e^s$

$\frac{dW}{dr} = e^r + e^s$ $\qquad$ $\frac{dW}{ds} = se^s + 1$

$\frac{dW}{dr} = re^r + 1$

31. $z = e^{-x^2-y^2}$

$\frac{dz}{dx} = e^{-x^2} + e^{-y^2}$ $\qquad$ $\frac{dz}{dy} = e^{-x^2} + e^{-y^2}$

$\frac{dz}{dx} = -x^2 e^{-x^2} + 1$ $\qquad$ $\frac{dz}{dy} = e^{-y^2} + 1$

39. $Q = \frac{L}{M} e^{-Lt/M}$

$\frac{dQ}{dL} = \frac{1}{M}\left(\frac{-L^2 t}{M} e^{-Lt/M}\right)$ $\qquad$ $\frac{dQ}{dM} = L\left(\frac{-Lt}{M^2} e^{-Lt/M}\right)$

47. a. $I(95,50) = 45.33 + 0.6845(95) + 5.758(50) - 0.00365(95^2) - 0.1565(95)(50) + .001(50)(95^2)$

$= -372.4965$

b. $f_T = 0.6845 - 2(0.00365)T - 0.1565H + 2(0.001)HT = 1.666$

14.4

3. $f(x,y) = x^2y + xy^3 \quad (2,1)$

$\frac{df}{dx} = 2xy + 0 + 1y^3 + x(0)$

$\frac{df}{dx} = 2xy + y^3$

$\frac{df}{dx} = 2(2)(1) + 1^3 = 5$

$\frac{df}{dy} = 0 + x^2 + x3y^2 + 0y^3$

$\frac{df}{dy} = x^2 + x3y^2$

$\frac{df}{dy} = 2^2 + 2(3)(1^2) = 10$

$f = 5(x-2) + 10(y-1)$

5. $f(x,y) = x^2 + y^{-2}, \quad (4,1)$

$\frac{df}{dx} = 2x$

$= 8$

$\frac{df}{dy} = -2y^{-3}$

$= 2$

$f = 8(x-4) + 2(y-1)$

7. $F(r,s) = r^2s^{-1/2} + s^{-3} \quad (2,1)$

$\frac{dF}{dr} = 2rs^{-1/2} + r^2(0) + 0$

$\frac{dF}{dr} = 2(2)(1^{-1/2})$

$= 4$

$\frac{dF}{ds} = r^2 \frac{-1}{2} s^{-3/2} + 0 + -3s^{-4}$

$\frac{dF}{ds} = (2^2)\frac{-1}{2}(1)^{-3/2} + -3(1)^{-4}$

$= -2 - 3 = -5$

$f = 4(r-2) - 5(s-1)$

13. $f(x,y) = x^2y^3$ at $(a,b) = (2,1)$

$f_x = 2xy^3 = 4 \quad f_y = 3y^2x^2 = 12$

$L(x,y) = 2^2 1^3 + 4(x-2) + 12(y-1)$

$L(2.01, 1.02) = 4 + 4(2.01-2) + 12(1.02-1)$

$= 6.8$

$L(1.97, 1.01) = 4 + 4(1.97-2) + 12(1.01-1)$

$= 4$

15. $\Delta f = f_x(\Delta x) + f_y(\Delta y)$

$f_x = 3x^2y^{-4} \quad f_y = -4y^{-5}x^3$

$\Delta f = 13(0.3) \cdot (-.1^{-4}) + -4(-1)^{-5}(.3^3)$

$\Delta f = -10802700$

$(8,4,5)$

19. $f_x = 0\sqrt{x+y} + z\frac{1}{2}(x+y)^{-1/2}\cdot 1 = \frac{z}{2}(x+y)^{-1/2} = \frac{5}{2\sqrt{12}}$

$f_y = 0\sqrt{x+y} + z\frac{1}{2}(x+y)^{-1/2}\cdot 1 = \frac{z}{2}(x+y)^{-1/2} = \frac{5}{2\sqrt{12}}$

$f_z = 1\sqrt{x+y} + z\frac{1}{2}(x+y)^{-1/2}\cdot 0 = 1\sqrt{x+y} = \sqrt{12}$

$f = \frac{5}{2\sqrt{12}}(x-8) + \frac{5}{2\sqrt{12}}(x-4) + \sqrt{12}(z-5)$

23. $f(2,1) = x^3y^2$

$f_x = 3x^2y^2$ $\quad f_y = 2y\,x^3$

$f(2.01, 1.02) = 3(.1)^2(.2)^2 + 2(.2)(.1^3)$

$= 8.1204$

25. $f(3,4) = \sqrt{3^2+4^2} = 5$

$f_x = \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2x$ $\quad f_y = \frac{1}{2}(x^2+y^2)^{-1/2}\cdot 2y$

$f(3.01, 3.99) = 5 + \frac{1}{2}(.1^2 + -.1^2)^{-1/2}\cdot 2(.1) + \frac{1}{2}(.1^2+(-.1^2))^{-1/2}\cdot 2(-.1)$

$= 4.998$

27. $f(2,2,4) = 4$

$f_x = \frac{1}{2}(xyz)^{-1/2}\cdot xy$ $\quad f_y = \frac{1}{2}(xyz)^{-1/2}\cdot xz$ $\quad f_z = \frac{1}{2}(xyz)^{-1/2}\cdot xy$

$f(1.9, 2.02, 4.05) = 4 + \frac{1}{2}(-.1)(.2)(.05)^{-1/2}\cdot(-.1)(.2) + \frac{1}{2}(-.1\cdot .2\cdot .05)^{-1/2}\cdot(-.1\cdot .2$

$= 3.9425$