14.3 Homework

(3) $\frac{d}{dy}\left[\frac{y}{x+y}\right]$

$\rightarrow \frac{(x+y)-y}{(x+y)^2}$

$\rightarrow \boxed{\frac{x}{(x+y)^2}}$

---

(5) $f_z(2,3,1)$, where $f(x,y,z) = xyz$

$\rightarrow \frac{f}{dz} = xy$

$\rightarrow \boxed{\frac{f}{dz} = 6}$

---

(17) $z = \frac{x}{y}$

$\rightarrow \boxed{\frac{dz}{dx} = \frac{1}{y}}$

$\rightarrow \boxed{\frac{dz}{dy} = -\frac{x}{y^2}}$

---

(19) $z = \sqrt{9-x^2-y^2}$

$\rightarrow \frac{dz}{dx} = \frac{1}{\cancel{2}}(9-x^2-y^2)^{-1/2} \cdot (-\cancel{2}x)$

$\rightarrow \boxed{\frac{dz}{dx} = \frac{-x}{\sqrt{9-x^2-y^2}}}$

$\rightarrow \frac{dz}{dy} = \frac{1}{\cancel{2}}(9-x^2-y^2)^{-1/2} \cdot (-\cancel{2}y)$

$\rightarrow \frac{dz}{dy} = \boxed{\frac{-y}{\sqrt{9-x^2-y^2}}}$

---

(21) $z = (\sin x)(\sin y)$

$\rightarrow \boxed{\frac{dz}{dx} = (\sin y)(\cos x)}$

$\rightarrow \boxed{\frac{dz}{dy} = (\sin x)(\cos y)}$

---

(27) $W = e^{r+s}$

$\rightarrow \boxed{\frac{d}{dr} = e^{r+s}}$

$\rightarrow \boxed{\frac{dW}{ds} = e^{r+s}}$

---

(31) $z = e^{-x^2-y^2}$

$\rightarrow \boxed{\frac{dz}{dx} = -2xe^{-x^2-y^2}}$

$\rightarrow \boxed{\frac{dz}{dy} = -2ye^{-x^2-y^2}}$

---

(39) $Q = \frac{L}{M} e^{-Lt/M}$

$\rightarrow \boxed{\frac{dQ}{dL} = \frac{M-Lt}{M^2} \cdot e^{-Lt/M}}$

$\rightarrow \boxed{\frac{dQ}{dM} = \frac{L(Lt-M)}{M^3} \cdot e^{-Lt/M}}$

---

(47) (a) $T=95, \quad H=50$

$\rightarrow I(95,50) = 45.33 + 0.6845(95) + (5.758)(50) - (0.00365)(95)^2 - 0.1565(95)(50)$

$+ (0.001)(50)(95)^2$

$\rightarrow \boxed{I(95,50) = 73.1913}$

(b) $\frac{dI}{dt} = 0.6845 - 0.00730T - 0.1565H + 0.002HT$

$\rightarrow \boxed{1.666}$

14.4 Homework

③ $f(x,y) = x^2y + xy^3, \ (2,1) \rightsquigarrow (2,1,6)$

$\to \frac{df}{dx} = 2xy + y^3 \rightsquigarrow @(2,1) = 5$

$\to \frac{df}{dy} = x^2+3xy^2 \rightsquigarrow @(2,1) = 10$

$\to z-6 = 5(x-2) + 10(y-1)$

$\to z-6 = 5x - 10 + 10y - 10$

$\to \boxed{z = 5x + 10y - 14}$

---

⑤ $f(x,y) = x^2 + y^{-2}, \ (4,1) \rightsquigarrow (4,1,17)$

$\to \frac{df}{dx} = 2x \rightsquigarrow @(4,1) = 8$

$\to \frac{df}{dy} = -2y^{-3} \rightsquigarrow @(4,1) = -2$

$\to z - 17 = 8(x-4) - 2(y-1)$

$\to z - 17 = 8x - 32 - 2y + 2$

$\to \boxed{z = 8x - 2y - 13}$

---

⑦ $F(r,s) = r^2s^{-1/2} + s^{-3}, \ (2,1) \rightsquigarrow (2,1,5)$

$\to \frac{dF}{dr} = 2rs^{-1/2} \rightsquigarrow @(2,1) = 4$

$\to \frac{dF}{ds} = -\frac{1}{2}r^2s^{-3/2} - 3s^{-4} \rightsquigarrow @(2,1) = -5$

$\to z-5 = 4(r-2) - 5(s-1)$

$\to z-5 = 4r - 8 - 5s + 5$

$\to \boxed{z = 4r - 5s + 2}$

---

⑬ $L(x,y)$ of $f(x,y) = x^2y^3$ at $(a,b) = (2,1)$ to estimate $f(2.01, 1.02)$ and $f(1.97, 1.01)$

$\to L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$\to \frac{df}{dx} = 2xy^3 \rightsquigarrow @(2,1) = 4$

$\to \frac{df}{dy} = 3x^2y^2 \rightsquigarrow @(2,1) = 12$

$\to L(2.01, 1.02) = 4 + 4(2.01-2) + 12(1.02-1) \approx \boxed{4.28}$

$\to L(1.97, 1.01) = 4 + 4(1.97-2) + 12(1.01-1) \approx \boxed{4}$

---

(15) $f(x,y) = x^3y^{-4}$

$\to \Delta f = f(2.03, 0.9) - f(2,1)$

$\to \frac{df}{dx} = 3x^2y^{-4} \rightsquigarrow @ (2,1) = 12$

$\to \frac{df}{dy} = -4x^3y^{-5} \rightsquigarrow @ (2,1) = -32$

$\to L(2.03, 0.9) = 8 + 12(2.03-2) - 32(0.9-1) = 11.56$

$\to 11.56 - 8 = \boxed{3.56}$

---

(17) $f(x,y) = e^{x^2+y}$ @ $(0,0)$ to estimate $f(0.01, -0.02)$

$\to \frac{df}{dx} = 2xe^{x^2+y}$ @ $(0,0) = 0$

$\to \frac{df}{dy} = e^{x^2+y}$ @ $(0,0) = 1$

$\to L(0.01, -0.02) = 1 + \cancel{0(0.01-0)} + 1(0.02-0) \approx \boxed{0.98}$

---

(23) $(2.01)^3(1.02)^2 \rightsquigarrow f(x,y) = x^3y^2$ @ $(2,1)$

$\to \frac{df}{dx} = 3x^2y^2$ @ $(2,1) = 12$

$\to \frac{df}{dy} = 2x^3y$ @ $(2,1) = 16$

$\to L(2.01, 1.02) = 8 + 12(2.01-2) + 16(1.02-1) = \boxed{8.44}$

---

(25) $\sqrt{3.01^2 + 3.99^2} \rightsquigarrow f(x,y) = \sqrt{x^2+y^2}$ @ $(3,4)$

$\to \frac{df}{dx} = x(x^2+y^2)^{-1/2}$ @ $(3,4) = \frac{3}{5}$

$\to \frac{df}{dy} = y(x^2+y^2)^{-1/2}$ @ $(3,4) = \frac{4}{5}$

$\to L(3.01, 3.99) = 5 + \frac{3}{5}(3.01-3) + \frac{4}{5}(3.99-4) = \boxed{4.998}$

---

(27) $\sqrt{(1.9)(2.02)(4.05)} \longrightarrow f(x,y) = \sqrt{xyz}$ @ $(2,2,4)$

$\to \frac{df}{dx} = \frac{yz}{2}(xyz)^{-1/2}$ @ $(2,2,4) = 1$

$\to \frac{df}{dy} = \frac{xz}{2}(xyz)^{-1/2}$ @ $(2,2,4) = 1$

$\to \frac{df}{dz} = \frac{xy}{2}(xyz)^{-1/2}$ @ $(2,2,4) = \frac{1}{2}$

$\to L(1.9, 2.02, 4.05) = 4 + 1(1.9-2) + 1(2.02-2) + \frac{1}{2}(4.05-4) = \boxed{3.945}$

14.5 Homework

⑦ $h(x,y,z) = xyz^{-3}$

$\rightarrow h_x = yz^{-3}$

$\rightarrow h_y = xz^{-3}$

$\rightarrow h_z = -3xyz^{-4}$

$\rightarrow \boxed{\nabla h = \langle yz^{-3},\ xz^{-3},\ -3xyz^{-4} \rangle}$

---

⑪ $f(x,y) = x^2 - 3xy, \quad r(t) = \langle \cos(t), \sin(t) \rangle, \quad t = 0$

$\rightarrow f_x = 2x - 3y$

$\rightarrow f_y = -3x$

$\rightarrow \nabla f = \langle 2x - 3y, -3x \rangle$

$\rightarrow r(0) = \langle 1, 0 \rangle$

$\rightarrow \nabla f(1,0) = \langle 2, -3 \rangle$

$\rightarrow r'(0) = \langle 0, 1 \rangle$

$\rightarrow \frac{d}{dt} f(r(t))\Big|_{t=0} = \nabla f_{r(0)} \cdot r'(0)$

$\rightarrow \langle 2, -3 \rangle \cdot \langle 0, 1 \rangle = -3$

$\rightarrow \boxed{\frac{d}{dt} f(r(t))\Big|_{t=0} = -3}$

---

⑬ $f(x,y) = \sin(xy), \quad r(t) = \langle e^{2t}, e^{3t} \rangle, \quad t = 0$

$\rightarrow f_x = y\cos(xy)$

$\rightarrow f_y = x\cos(xy)$

$\rightarrow \nabla f = \langle y\cos(xy), x\cos(xy) \rangle$

$\rightarrow r(0) = \langle 1, 1 \rangle$

$\rightarrow \nabla f(1,1) = \langle \cos(1), \cos(1) \rangle$

$\rightarrow r'(0) = \langle 2, 3 \rangle$

$\rightarrow \frac{d}{dt} f(r(t))\Big|_{t=0} = \nabla f_{r(0)} \cdot r'(0)$

$\rightarrow \langle \cos(1), \cos(1) \rangle \cdot \langle 2, 3 \rangle = 5\cos(1)$

$\rightarrow \boxed{\frac{d}{dt} f(r(t))\Big|_{t=0} = 5\cos(1)}$

---

(19) $g(x,y,z) = xyz^{-1}, \quad r(t) = \langle e^t, t, t^2 \rangle, \quad t=1$

$\rightarrow g_x = yz^{-1}$

$\rightarrow g_y = xz^{-1}$

$\rightarrow g_z = -xyz^{-2}$

$\rightarrow \nabla g = \langle yz^{-1}, xz^{-1}, -xyz^{-2} \rangle$

$\rightarrow r(1) = \langle e, 1, 1 \rangle$

$\rightarrow \nabla g(e,1,1) = \langle 1, e, -e \rangle$

$\rightarrow r'(1) = \langle e, 1, 2 \rangle$

$\rightarrow \frac{d}{dt} g(r(t))\Big|_{t=1} = \nabla g_{r(1)} \cdot r'(1)$

$\rightarrow \langle 1, e, -e \rangle \cdot \langle e, 1, 2 \rangle = e + e - 2e = 0$

$\rightarrow \boxed{\frac{d}{dt} g(r(t))\Big|_{t=1} = 0}$

---

(27) $f(x,y) = \ln(x^2+y^2), \quad v = \langle 3, -2 \rangle, \quad P = (1,0)$

$\rightarrow f_x = \frac{2x}{x^2+y^2}$

$\rightarrow f_y = \frac{2y}{x^2+y^2}$

$\rightarrow \nabla f = \langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \rangle$

$\rightarrow \nabla f(1,0) = \langle 2, 0 \rangle$

$\rightarrow |\langle 3, -2 \rangle| = \sqrt{13}$

$\rightarrow \frac{1}{\sqrt{13}} \langle 3, -2 \rangle = \langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle$

$\rightarrow D_u f(1,0) = \langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle \cdot \langle 2, 0 \rangle = \frac{6}{\sqrt{13}}$

$\to \boxed{D_u f(1,0) = \frac{6}{\sqrt{13}}}$

---

(31) $f(x,y) = x^2 + 4y^2$, $v = \langle -3,-2 \rangle$, $P: (3,2)$

$\to f_x = 2x$

$\to f_y = 8y$

$\to \nabla f = \langle 2x, 8y \rangle$

$\to \nabla f(3,2) = \langle 6, 16 \rangle$

$\to |\langle -3,-2 \rangle| = \sqrt{13}$

$\to \frac{1}{\sqrt{13}} \langle -3,-2 \rangle = \langle \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle$

$\to D_u f(3,2) = \langle \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \rangle \cdot \langle 6, 16 \rangle = \frac{-18}{\sqrt{13}} - \frac{32}{\sqrt{13}} = \frac{-50}{\sqrt{13}}$

$\to \boxed{D_u f(3,2) = \frac{-50}{\sqrt{13}}}$

---

(33) $T(x,y,z) = xe^{y-z}$, $(3,9,4)$, $\langle 5-3, 7-9, 3-4 \rangle = \langle 2,-2,-1 \rangle$

$\to T_x = e^{y-z}$

$\to T_y = xe^{y-z}$

$\to T_z = -xe^{y-z}$

$\to \nabla T = \langle e^{y-z}, xe^{y-z}, -xe^{y-z} \rangle$

$\to \nabla T(3,9,4) = \langle e^5, 3e^5, -3e^5 \rangle$

$\to |\langle 2,-2,-1 \rangle| = 3$

$\to \frac{1}{3} \langle 2,-2,-1 \rangle = \langle \frac{2}{3}, \frac{-2}{3}, -\frac{1}{3} \rangle$

$\to D_u T(3,9,4) = \langle e^5, 3e^5, -3e^5 \rangle \cdot \langle \frac{2}{3}, \frac{-2}{3}, -\frac{1}{3} \rangle = \frac{-e^5}{3}$

$\to \boxed{D_u T(3,9,4) = \frac{-e^5}{3}}$

---

37 $\langle 2,-4,4\rangle \cdot \langle 2,1,3\rangle = 4-4+12$ in which $12>0$

$\to \boxed{f \text{ is increasing}}$

---

39 $f(x,y,z) = \sin(xy+z)$, $P=(0,-1,\pi)$

$\to f_x = y\cos(xy+z)$

$\to f_y = x\cos(xy+z)$

$\to f_z = \cos(xy+z)$

$\to \nabla f = \langle y\cos(xy+z), x\cos(xy+z), \cos(xy+z)\rangle$

$\to \nabla f(0,-1,\pi) = \langle 1,0,-1\rangle$

$\to |\langle 1,0,-1\rangle| = \sqrt{2}$

$\to D_u f(P) = \sqrt{2}\cos 30$

$\to \boxed{D_u f(0,-1,\pi) = \frac{\sqrt{6}}{2}}$

---

41 $x^2+y^2-z^2=6$, $P=(3,1,2)$

$\to f(x,y,z) = x^2+y^2-z^2=6$

$\to f_x = 2x$

$\to f_y = 2y$

$\to f_z = -2z$

$\to \nabla f = \langle 2x, 2y, -2z\rangle$

$\to \boxed{\nabla f(3,1,2) = \langle 6,2,-4\rangle}$

---

43 $\frac{x^2}{4}+\frac{y^2}{9}+z^2=1$, $v=\langle 1,1,-2\rangle$

$\to \nabla f_p = \langle \frac{x2}{2}, \frac{2y}{9}, 2z\rangle$

$\to \boxed{(x,y,z) = \pm\left(\frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, \frac{-2}{\sqrt{17}}\right)}$
