14.3 - # 3, 5, 17, 19, 21, 27, 31, 39, 47

3. Quotient Rule: $\frac{f'g - fg'}{g^2}$

$$\frac{d}{dy}\frac{y}{x+y} = \frac{(x+y)-(y)}{(x+y)^2}$$

$$= \frac{x}{(x+y)^2}$$

5. Calculate $f_z(2,3,1)$

$f(x,y,z) = xyz$

$f_z = xy$

$f_z(2,3,1) = (2)(3) = 6$

17. $z = \frac{x}{y} = xy^{-1}$

$$\frac{dz}{dx} = \frac{1}{y}$$

$$\frac{dz}{dy} = \frac{-x}{y^2}$$

19. $z = \sqrt{9-x^2-y^2} = (9-x^2-y^2)^{1/2}$

$$f_x = \frac{1}{2}(9-x^2-y^2)^{-1/2}\cdot(-2x)$$

$$= \frac{-x}{\sqrt{9-x^2-y^2}}$$

$$f_y = \frac{1}{2}(9-x^2-y^2)^{-1/2}\cdot(-2y)$$

$$= \frac{-y}{\sqrt{9-x^2-y^2}}$$

21. $z = \sin x \sin y$

$$\frac{dz}{dx} = \cos x \sin y$$

$$\frac{dz}{dy} = \sin x \cos y$$

27. $W = e^{r+s}$

$$\frac{dW}{dr} = e^{r+s}\cdot 1 \qquad \frac{dw}{ds} = e^{r+s}\cdot 1$$

31. $z = e^{(-x^2-y^2)}$

$$\frac{dz}{dx} = e^{(-x^2-y^2)}\cdot(-2x)$$

$$\frac{dz}{dy} = e^{(-x^2-y^2)}\cdot(-2y)$$

39. $Q = \frac{L}{M}\cdot e^{-\frac{Lt}{m}}$ $\quad LM^{-1}$

$$\frac{dQ}{dL} = \left(\frac{1}{M}\right)\left(e^{-\frac{Lt}{m}}\right) + \left(\frac{L}{M}\right)\left(e^{-\frac{Lt}{m}}\right)\left(\frac{-t}{m}\right)$$

47.

ⓐ $I(95,50) = 45.33 + 0.6845(95) + 5.758(50)$
$-0.00365(95)^2 - 0.1565(95)(50)$
$+ 0.001(50)(95)^2$

ⓑ $\frac{dI}{dT} = 0.6845 - (2)(0.00365)T - .1565H$
$+ .001(2)HT$

$\frac{dI}{dT}(95,50) = 0.6845 - 2(0.00365)(95) - .1565(50)$
$+ .001(2)(95)(50)$

14.4- #3, 5, 7, 13, 15, 17, 23, 25, 27

3. Find equation of tangent plane:

$f(x,y) = x^2y + xy^3$ @ $(2,1)$

Recall: Equation of tangent plane to the surface $f(x,y,z) = K$ at (x_0, y_0, z_0) is...

$f_x(x_0,y_0,z_0)(x-x_0) + f_y(x_0,y_0,z_0)(y-y_0) + f_z(x_0,y_0,z_0)(z-z_0) = 0$

$f_x = 2xy + y^3 \to f_x(2,1) = 4+1 = 5$

$f_y = x^2 + 3xy^2 \to f_y(2,1) = 4+6 = 10$

tangent plane:

$f_x(2,1)(x-2) + f_y(2,1)(y-1) = 0$

$5(x-2) + 10(y-1) = 0$

$5x - 10 + 10y - 10 = 0$

$5x - 10y = 20$

$5(x-2y) = 20$

$x - 2y = 4$

5. $f(x,y) = x^2 + y^{-2}$ @ $(4,1)$

$f_x = 2x \to f_x(4,1) = 8$

$f_y = -2y^{-3} = \frac{-2}{y^3} \to f_y(4,1) = -2$

tangent plane:

$8(x-4) + (-2)(y-1) = 0$

$8x - 32 - 2y + 1 = 0$

$8x - 2y = 31$

$2(4x - y) = 31$

$4x - y = \frac{31}{2}$

7. $F(r,s) = r^2 s^{-1/2} + s^{-3}$ @ $(2,1)$

$\frac{dF}{dr} = 2rs^{-1/2} + s^{-3} \to \frac{dF}{dr}(2,1) = 2(2)(1)^{-1/2} + 1^{-3}$

$= 4$

$\frac{dF}{ds} = -\frac{1}{2}r^2 s^{-3/2} + (-3)s^{-4}$

$\hookrightarrow \frac{dF}{s}(2,1) = \frac{-1(2)^2}{2(1)^{3/2}} + \frac{-1}{3(1)^4} = \frac{-4}{2} - \frac{1}{3}$

$= \frac{-12}{6} - \frac{2}{6} = \frac{-14}{6}$

tangent line:

$4(x-2) - \frac{14}{6}(y-1) = 0$

$4x - 8 - \frac{14}{6}y + \frac{14}{6}$

$4x - \frac{14}{6}y = 8 + \frac{14}{6}$

13. Find linearization $L(x,y)$ of $f(x,y) = x^2y^3$ @ $(a,b) = (2,1)$. Use it to est. $f(2.01, 1.02)$ and $f(1.97, 1.01)$

Linear Approximation:

$L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$

$f_x = 2xy^3 \to f_x(2,1) = 2(2)(1)^3 = 4$

$f_y = x^2(3y^2) \to f_y(2,1) = (2)^2(3(1)^2) = 12$

$L(2,1) = (2)^2(1)^3 + 4(x-2) + 12(y-1)$

$L(2,1) = 4 + 4x - 8 + 12y - 12$

$L(2,1) = 4x + 12y - 16$

$F(2.01, 1.02) = 4(2.01) + 12(1.02) - 16$

$F(1.97, 1.01) = 4(1.97) + 12(1.01) - 16$

14.4 cont'd.

15. $f(x,y) = x^3 y^{-4}$

$f(u) = u^3$

$f'(u) = 3u^2$

- $u = 2.03$
- $a = 2.00$

$f(u) = f(a) + f'(a)(u-a)$

$f(2.03) = (2)^3 + 3(2)^2(u-2)$

$= 8 + 12(u-2)$

$= 12u - 16 = 12(2.03) - 16 = 8.36$

$f(2) = 12(2) - 16 = 8$

$\Delta f = f(2.03, 0.9) - f(2,1)$

$f_x = 3x^2 y^{-4}$

$f_y = x^3(-4y^{-5})$

$f(u) = u^{-4}$

$f'(u) = -4u^{-5}$

- $u = 0.9$
- $a = 1$

$f(u) = f(a) + f'(a)(u-a)$

$f(0.9) = (1)^{-4} + -4(1)^{-5}(u-1)$

$= 1 - 4(u-1) = 1 - 4u + 4$

$= 5 - 4(u) = 5 - 4(.9)$

$= 1.4$

$f(1) = 5 - 4 = 1$

$\Delta f = (8.36)(1.4) - (8)(1)$

$\Delta f = 3.704$

17. **Find Linearization:**

$f(x,y,z) = z\sqrt{x+y}$ @ $(8,4,5)$

$f_x = \frac{1}{2}(x+y)^{-1/2} \times z \rightarrow f_x(8,4,5) = \frac{1}{2}(8+4)^{-1/2} \times (5) = \frac{5}{2\sqrt{12}} = \frac{5}{4\sqrt{3}}$

$f_y = \frac{1}{2}(x+y)^{-1/2} \times z \rightarrow f_y = \frac{1}{2}(8+4)^{-1/2} \times (5) = \frac{5}{4\sqrt{3}}$

$f_z = (x+y)^{1/2} \rightarrow f_z = (8+4)^{1/2} = 2\sqrt{3}$

$L(x,y) = 5\sqrt{12} + \frac{5}{4\sqrt{3}}(x-8) + \frac{5}{4\sqrt{3}}(y-4) + 2\sqrt{3}(z-5)$

23. $(2.01)^3(1.02)^2 \longrightarrow (8.12)(1.04) = 8.445$

$f(u) = u^3$

$f'(u) = 3u^2$

$f(u) = f(a) + f'(a)(u-a)$

- $u = 2.01$
- $a = 2.00$

$f(2.01) = (2)^3 + 3(2)^2(u-2)$

$f(2.01) = 8 + 12(u-2)$

$f(2.01) = 8 + 12u - 24$

$f(2.01) = 12u - 16 = 12(2.01) - 16 = 8.12$

$f(u) = u^2$

$f'(u) = 2u$

$f(u) = f(a) + f'(a)(u-a)$

- $u = 1.02$
- $a = 1.00$

$f(1.02) = (1)^2 + 2(1)(u-1)$

$= 1 + 2(u-1) = 1 + 2u - 2 = 2u - 1$

$= 2(1.02) - 1 = 1.04$

25. $\sqrt{3.01^2 + 3.99^2} \rightarrow \sqrt{9.06 + 23.92} = \sqrt{32.98}$

$f(u) = u^2$

$f'(u) = 2u$

- $u = 3.01$
- $a = 3.00$

$f(u) = f(a) + f'(a)(u-a)$

$f(u) = (3)^2 + 2(3)(u-3)$

$f(3.01) = 9 + 6(u-3)$

$= 9 + 6u - 18$

$= 6u - 9$

$= 6(3.01) - 9$

$= 9.06$

- $u = 3.99$
- $a = 4.00$

$f(3.99) = f(a) + f'(a)(u-a)$

$= (4)^2 + (2 \times 4)(u-3)$

$= 16 + 8(u-3)$

$= 16 + 8u - 24$

$= 8u - 8 = 8(3.99) - 8 = 31.92 - 8 = 23.92$

27. $\sqrt{(1.9)(2.02)(4.05)}$

$f(u) = \sqrt{u}$

$f'(u) = \frac{1}{2}u^{-1/2}$

- $u = 1.9$
- $a = 2$

$f(u) = \sqrt{2} + \frac{1}{2}(2)^{-1/2}(u-2)$

$f(u) = \sqrt{2} + \frac{1}{2\sqrt{2}}(u-2)$

$f(1.9) = \sqrt{2} + \frac{1}{2\sqrt{2}}(1.9-2)$

$f(2.02) = \sqrt{2} + \frac{1}{2\sqrt{2}}(2.02-2)$

$f(4.05) = \sqrt{2} + \frac{1}{2\sqrt{2}}(4.05-2)$

14.5 - # 7, 11, 13, 19, 27, 31, 33, 37, 39, 41, 43

7. Calculate ∇f

$\nabla f(x,y) = \langle f_x, f_y \rangle$

$h(x,y,z) = xyz^{-3}$

$f_x = yz^{-3}$

$f_y = xz^{-3}$

$f_z = xy(-3z^{-4})$

$\nabla f = (yz^{-3}, xz^{-3}, 3xyz^{-4})$

11. $f(x,y) = x^2 - 3xy$

$r(t) = \langle \cos t, \sin t \rangle \quad t=0$

$r(0) = \langle \cos(0), \sin(0) \rangle$

$r(0) = (1, 0)$

$f(r(0)) = f(1,0) = (1)^2 - 3(1)(0)$

$f(1,0) = 1 - 3 = -2$

$f(\cos t, \sin t) = \cos^2 t - 3\cos t \sin t$

$\frac{d}{dt} = -\sin^2 t - (-3\sin t \sin t + 3\cos t \cos t)$

$\frac{d}{dt} = -\sin^2 t + 3\sin^2 t - 3\cos^2 t$

$\frac{d}{dt} = 2\sin^2 t - 3\cos^2 t$

13. $f(x,y) = \sin(xy) \quad r(t) = \langle e^{2t}, e^{3t} \rangle \quad t=0$

$f(e^{2t}, e^{3t}) = \sin(e^{2t} \cdot e^{3t})$

$\frac{d}{dt} = \cos(e^{5t}) \times 5 = 5\cos(e^{5t})$

19. $g(x,y,z) = xyz^{-1} \quad r(t) = \langle e^t, t, t^2 \rangle \quad t=1$

$g(e^t, t, t^2) = e^t \cdot t \cdot t^{-2} = e^t \cdot t^{-1}$

$\frac{d}{dt} = e^t t^{-1} + e^t(-t^{-2}) = \frac{e^t}{t} - \frac{e^t}{t^2}$

27. $f(x,y) = \ln(x^2+y^2) \quad v = 3i - 2j \quad P(1,0)$

$u = \frac{\langle 3, -2 \rangle}{\sqrt{9+4}} = \left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$

$D_u f(x,y) = f_x a + f_y b$

$f_x = \frac{2x}{x^2+y^2} \quad f_y = \frac{2y}{x^2+y^2}$

$D_u f(x,y) = \frac{2}{x^2+y^2}\left(\frac{3}{\sqrt{13}}\right) - \left(\frac{2}{\sqrt{13}}\right)\left(\frac{2y}{x^2+y^2}\right)$

$D_u f(1,0) = \frac{2}{1}\left(\frac{3}{\sqrt{13}}\right) - \frac{2}{\sqrt{13}}(0) = \frac{6}{\sqrt{13}}$

31. $f(x,y) = x^2 + 4y^2$

$P = (3,2)$ and $Q = (0,0)$

$D_u f(x,y) = f_x a + f_y b$

v = head - tail

$v = (0-3, 0-2) = \langle -3, -2 \rangle$

$|v| = \sqrt{9+4} = \sqrt{13}$

$u = \left\langle \frac{-3}{\sqrt{13}}, \frac{-2}{\sqrt{13}} \right\rangle$

$f_x = 2x \quad f_y = 8y$

$D_u f(x,y) = 2x\left(\frac{-3}{\sqrt{13}}\right) + 8y\left(\frac{-2}{\sqrt{13}}\right)$

$D_u f(3,2) = \frac{-6(3)}{\sqrt{13}} + \frac{-16(2)}{\sqrt{13}}$

$D_u f(3,2) = 50/\sqrt{13}$

33. $T(x,y,z) = xe^{y-z}$

vector = head - tail

$v = (5-3, 7-9, 3-4)$

$v = \langle 2, -2, -1 \rangle$

$|v| = \sqrt{9} = 3$

$u = \left\langle \frac{2}{3}, \frac{-2}{3}, \frac{-1}{3} \right\rangle$

$D_u f(x,y,z) = f_x a + f_y b + f_z c$

$f_x = e^{y-z}$

$f_y = xe^{y-z}$

$f_z = -xe^{y-z}$

$D_u f(x,y,z) = \frac{2e^{y-z}}{3} - \frac{2xe^{y-z}}{3} + \frac{xe^{y-z}}{3}$

$D_u f(x,y,z) = \frac{e^{y-z}}{3}(2 - 2x + x)$

37. $\nabla f_p = \langle 2, -4, 4 \rangle$

$v = \langle 2, 1, 3 \rangle$

$D_u f(x,y,z) = \nabla f_p \cdot u$

$|v| = \sqrt{9+4+1} = \sqrt{14}$

$D_u f(x,y,z) = \langle 2, -4, 4 \rangle \cdot \left\langle \frac{2}{\sqrt{14}}, \frac{1}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$

$= \frac{4}{\sqrt{14}} + \frac{-4}{\sqrt{14}} + \frac{12}{\sqrt{14}} = \frac{12}{\sqrt{14}}$

increasing

39. $f(x,y,z) = \sin(xy+z)$ $P = (0, -1, \pi)$

$u = \langle \cos\theta, \sin\theta \rangle$

$u = \langle \cos(30°), \sin(30°) \rangle = \left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle$

$f_x = \cos(xy+z) \times y$

$f_y = \cos(xy+z) \times x$

$f_z = \cos(xy+z)$

$\nabla f = \langle y\cos(xy+z), x\cos(xy+z), \cos(xy+z) \rangle$

$\cancel{D_u f(x,y,z) = \nabla f_p \cdot u}$

$\cancel{= \langle y\cos(xy+z), x\cos(xy+z), \cos(xy+z) \rangle}$

$\nabla f_p(0,-1,\pi) = \langle -\cos(\pi), 0, \cos(\pi) \rangle$

$\nabla f_p = \langle 1, 0, -1 \rangle$

$D_u f(x,y,z) = |\nabla f_p||u|\cos\theta$

$= \sqrt{2} \times 1 \times \cos(30°)$

$= 0.218$

41. vector $\perp$ to $x^2 + y^2 - z^2 = 6$

@ $P = (3, 1, 2)$

↳ find ∇f

$\nabla f = \langle f_x, f_y, f_z \rangle$

$\nabla f = \langle 2x, 2y, -2z \rangle$

$\nabla f_p(3, 1, 2) = \langle 6, 2, -4 \rangle$

43. $f(x,y,z) = \frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$

tangent plane:

$V_{normal} \cdot (x - x_p, y - y_p, z - z_p) = 0$