10/4/20

14.3 Homework

3. $\frac{\partial}{\partial y} \frac{y}{x+y} = \frac{(x+y)1-[y(1)]}{(x+y)^2} = \frac{x+y-y}{(x+y)^2} = \frac{x}{(x+y)^2}$

5. $f_z(2,3,1)$ where $f(x,y,z) = xyz$

$f_z = xy \qquad f_z(2,3,1) = (2)(3) = 6$

17. $z = \frac{x}{y} \qquad \frac{\partial z}{\partial x} : z' = \frac{y \cdot 1 - [x \cdot 0]}{y^2} = \frac{y}{y^2} = \frac{1}{y} = \frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial y} = z' = \frac{y \cdot 0 - [x \cdot 1]}{y^2} = \frac{-x}{y^2} = \frac{\partial z}{\partial x}$

19. $z = \sqrt{9-x^2-y^2}$

$\frac{\partial z}{\partial x} = (9-x^2-y^2)^{1/2} = \frac{1}{2} \cdot (9-x^2-y^2)^{-1/2} \cdot -2x$

$\frac{-2x}{2\sqrt{9-x^2-y^2}} = \frac{-x}{\sqrt{9-x^2-y^2}} = \frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial y} = (9-x^2-y^2)^{1/2} = \frac{1}{2} \cdot (9-x^2-y^2)^{-1/2} \cdot -2y = \frac{-y}{\sqrt{9-x^2-y^2}} = \frac{\partial z}{\partial y}$

21. $z = (\sin x)(\sin y) \qquad \frac{\partial z}{\partial x} = ((\cos x)(\sin y) + (\sin x)(0) = \cos x \sin y$

$\frac{\partial y}{\partial x} = (0)(\sin y) + (\sin x)(\cos y) = \sin x \cos y$

27. $W = e^{v+s} \qquad \frac{\partial w}{\partial v} = e^{v+s} \cdot 1 = e^{v+s}$

$\frac{\partial w}{\partial s} = e^{v+s} \cdot 1 = e^{v+s}$

39. $Q = \frac{L}{M} e^{-Lt/M}$ $\qquad \frac{dQ}{dL} = \frac{L}{M} \cdot \frac{\partial Q}{\partial L}\left(e^{-Lt/m}\right) + e^{-Lt/m} \cdot \frac{\partial Q}{\partial L}\left(\frac{L}{M}\right)$

$$= \frac{L}{M} e^{-Lt/m} \cdot \left(-\frac{t}{m}\right) + e^{-Lt/m} \cdot \left(\frac{1}{m}\right)$$

$$= \frac{e^{-Lt/m}}{M} \cdot \left(1 - \frac{Lt}{M}\right)$$

47. $\frac{\partial Q}{\partial M} = \frac{L}{m} \cdot \frac{\partial}{\partial m}\left(e^{-Lt/m}\right) + e^{-Lt/m} \cdot \frac{\partial}{\partial M}\left(\frac{L}{M}\right)$

47. I at $(95, 50) = 45.33 + 0.6485(95) + 5.758(50)$
$- 0.00365(90)^2 - 0.1565(95)(50) + 0.001(50)(90)^2$

It would be $\frac{\partial I}{\partial T} = 0.6845 - 2(0.00365)T - 0.1565H + 2(0.001)HT$

14.4 Homework

3. $f(2,1) = (2)^2(1) + (2)(1)^3 = 4+2 = 6$

$f_x : 2x + y^3$ $\quad f_x(2,1) = (2)(2) + (1)^3 = 5$

$f_y = x^2 + 3xy^2$ $\quad f_y(2,1) = (2)^2 + 3(2)(1)^3 = 10$

$z - 6 = 5(x-2) + 10(y-1)$

$z - 6 = 4x - 8 + 10y - 10 \rightarrow z - 6 = 4x + 10y - 18$

$z = 5x + 10y - 12$

5. $f(4,1) = (4)^2 + (1)^{-2} = 16 + 1 = 17$

$f_x = 2x$ $\quad f_y = \frac{-2}{y}$ $\quad f_x(4,1) = 8$ $\quad f_y(4,1) = -2$

$z - 17 = 8(x-4) - 2(y-1)$

$z = 8x - 32 - 2y + 2 + 17$

$z = 8x - 2y - 13$

7. $F(2,1) = (2)^2(1)^{-1/2} + (1)^{-3} = 5$

$F_r = 2rs^{-1/2}$ $\quad F_s = \frac{-1}{2}s^{-3/2} - 3s^{-4}$ $\quad F_r(2,1) = 4$

$F_s(2,1) = -5$

$z = 4(r-2) - 5(s-1) + 5$

$z = 2 + 4r - 5s$

13. $f(x,y) = x^2y^3$ $\quad f_x = 2xy^3$ $\quad f_y = 3x^2y^2$

All are continuous

$f(2,1) = 4$ $\quad f_x(2,1) = 4$ $\quad f_y(2,1) = 12$

$f(x,y) \approx 4 + 4(x-2) + 12(y-1)$

$= 4x + 12y - 16$

$f(2.01, 1.02) \approx 4 + 4(0.01) + 12(0.01)$

$= 4.28$

$f(1.97, 1.01) = 4 + 4(-.03) + 12(.01) = 4$

15. $f(x,y) = x^3y^{-4}$ $f_x = 3x^2y^{-4}$ $f_y = -4x^3y^{-5}$

$f(2,1) = 8$ $f_x(2,1) = 12$ $f_y(2,1) = -32$

$\Delta f = f(2+0.03, 1-0.1) - f(2,1)$

$12 \times 0.03 + (-32) \times (-0.1) = 3.56$

17. $f(x,y) = e^{x^2+y}$ $f_x = 2xe^{x^2+y}$ $f_y = e^{x^2+y}$

$f(0,0) = 1$ $f_x(0,0) = 0$ $f_y(0,0) = 1$

$f(0.01, -0.02) \approx 1 + 1 \times (-0.01)$

$= 0.99$

23. $f(x,y) = x^3y^2$ $f_x = 3x^2y^2$ $f_y = 2x^3y$

$f(2,1) = 8$ $f_x(2,1) = 12$ $f_y(2,1) = 16$

$f(2+0.01, 1+0.02) = 8 + 12(0.01) + 16(0.02)$

$= 8.44$

25. $f(x,y) = \sqrt{x^2+y^2}$ $f_x = \dfrac{x}{\sqrt{x^2+y^2}}$ $f_y = \dfrac{y}{\sqrt{x^2+y^2}}$

$f(3,4) = 5$ $f_x(3,4) = 3/5$ $f_y(3,4) = 4/5$

$f(3.01, 3.99) \approx 5 + 3/5\,(0.01) + 4/5\,(-.01) = 4.998$

27. $f(x,y,z) = \sqrt{xyz}$ $f_x = \dfrac{yz}{2\sqrt{xyz}}$ $f_y = \dfrac{xz}{2\sqrt{xyz}}$

$f_z = \dfrac{xy}{2\sqrt{xyz}}$

$f(2,2,4) = 4$ $f_x = 1$ $f_y = 1$ $f_z = 1/2$

$f(1.9, 2.02, 4.05) = 4 + 1 \times (-0.1) + 1\,(0.02) + 1/2\,(0.05)$

14.5 Homework

7. $h(x,y,z) = xyz^{-3}$

$h_x = yz^{-3}$ $\quad h_y = xz^{-3}$ $\quad h_z = -3xyz^{-4}$

$\nabla f = \left\langle yz^{-3},\ xz^{-3},\ -3xyz^{-4} \right\rangle$

11. $f(x,y) = x^2 - 3xy$ $\quad r(t) = \langle \cos t, \sin t \rangle,\ t=0$

$r(0) = \langle 1, 0 \rangle$

$f_x = 2x - 3y$ $\quad f_y = -3x$ $\quad \nabla f = \langle 2x - 3y, -3x \rangle$

$\nabla f_{1,0} = \langle 2, -3 \rangle$

$r'(t) = \langle -\sin t, \cos t \rangle$

$r'(0) = \langle 0, 1 \rangle$

$\nabla f_{r(0)} \cdot r'(0) = (2)(0) + (-3)(1) = -3$

13. $f(x,y) = \sin(xy),\ r(t) = \langle e^{2t}, e^{3t} \rangle$ $\quad t=0$

$r(0) = \langle 1, 1 \rangle$

$f_x = y\cos(xy)$ $\quad f_y = x\cos(xy)$ $\quad \nabla f = \langle y\cos(xy), x\cos(xy) \rangle$

$\nabla_{1,1} = \langle \cos 1, \cos 1 \rangle$

$r'(t) = \langle 2e^{2t}, 3e^{3t} \rangle$

$r'(0) = \langle 2, 3 \rangle$

$\nabla f_{1,1} \cdot r'(0) = \cos 1 \cdot 2 + \cos 1 \cdot 3 = 5\cos 1$

19. $g(x,y,z) = xyz^{-1},\ r(t) = \langle e^t, t, t^2 \rangle$ $\quad t=1$

$r(1) = \langle e, 1, 1 \rangle$

$f_x = yz^{-1}$ $\quad f_y = xz^{-1}$ $\quad f_z = -xyz^{-2}$ $\quad \nabla f = \langle yz^{-1}, xz^{-1}, xyz^{-2} \rangle$

$\nabla f_{e,1,1} = \langle 1, e, -e \rangle$

$r'(t) = \langle e^t, 1, 2t \rangle$

$r'(1) = \langle e, 1, 2 \rangle$

$\nabla f_{e,1,1} \cdot r'(1) = e + e - 2e = 0$

27. $f(x,y) = \ln(x^2+y^2)$, $v = 3\hat{i} - 2\hat{j}$ $P = (1,0)$

$f_x = \dfrac{2x}{x^2+y^2}$ $f_y = \dfrac{2y}{x^2+y^2}$

$\nabla f = \left\langle \dfrac{2x}{x^2+y^2}, \dfrac{2y}{x^2+y^2} \right\rangle$

$v = \langle 3, -2 \rangle$

$\|v\| = \sqrt{9+4} = \sqrt{13}$

$u = \dfrac{1}{\sqrt{13}} \langle 3, -2 \rangle$

$\nabla f_{1,0} = \langle 2, 0 \rangle$

$\nabla f_{1,0} \cdot u = 2 \cdot \dfrac{3}{\sqrt{13}} + 0 \cdot \dfrac{-2}{\sqrt{13}} = \dfrac{6}{\sqrt{13}}$

31. $f(x,y) = x^2 + 4y^2$ $P = (3,2)$ $v = \langle 0-3, 0-2 \rangle = \langle -3, -2 \rangle$

$f_x = 2x$ $f_y = 8y$

$\nabla f = \langle 2x, 8y \rangle$

$\|v\| = \sqrt{9+4} = \sqrt{13}$

$u = \dfrac{1}{\sqrt{13}} \langle -3, -2 \rangle$

$\nabla f_{3,2} = \langle 6, 16 \rangle$

$\nabla f_{3,2} \cdot u = 6 \cdot \dfrac{-3}{\sqrt{13}} + 16 \cdot \dfrac{-2}{\sqrt{13}} = \dfrac{-18}{\sqrt{13}} \; \dfrac{-32}{\sqrt{13}}$

33. $T(x,y,z) = xe^{y-z}$ $P(3,9,4)$ $Q = (5,7,3)$

$v = \langle 2, -2, 1 \rangle$

$T_x = e^{y-z}$ $T_y = xe^{y-z}$ $T_z = -xe^{y-z}$

$\nabla T = \langle e^{y-z}, xe^{y-z}, -xe^{y-z} \rangle$

$\|v\| = \sqrt{4+4+1} = 3$

$u = \dfrac{1}{3} \langle 2, -2, 1 \rangle$

$\nabla T_{3,9,4} = \langle e^5, 3e^5, -3e^5 \rangle$

$\nabla T_{3,9,4} \cdot u = \dfrac{2e^5}{3} + \dfrac{-6e^5}{3} + \dfrac{-3e^5}{3}$

37. $\nabla f_P = \langle 2,-4,4\rangle \quad v = \langle 2,1,3\rangle$

$\nabla f_P \cdot v = 4 - 4 + 12 = 12$

So it is INCREASING

39. $f(x,y,z) = \sin(xy+z) \quad P = (0,-1,\pi) \quad \theta = 30^\circ$ with ∇f_P

$\nabla f = \langle y\cos(xy+z),\ x\cos(xy+z),\ \cos(xy+z)\rangle$

$\nabla f_{0,-1,\pi} = \langle 1,0,-1\rangle$

$\|\nabla f_P\| = \sqrt{2}$

$\sqrt{2}\cdot\cos 30 = \sqrt{6}/2$

41. $x^2+y^2-z^2 = 6 \quad P = (3,1,2)$

$\nabla f = \langle 2x, 2y, -2z\rangle$

$\nabla f_{3,1,2} = \langle 6,2,-4\rangle$

$\langle +6,2,-4\rangle$

43. $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1 \quad v = \langle 1,1,-2\rangle$

Not sure how to approach this...

$\nabla f = \left\langle \frac{2x}{4}, \frac{2y}{9}, 2z \right\rangle$