14.3 HW

$\frac{f'g - fg'}{g^2}$

3. $\frac{d}{dy} \frac{y}{x+y} = \frac{x}{(x+y)^2}$

5. $f_z = xy \, |_{(2,3,1)} = 6$

17. $z = \frac{x}{y}$ $\quad \frac{d}{dx}\left(\frac{x}{y}\right) = \frac{1}{y}$ $\quad \frac{d}{dy}\left(\frac{x}{y}\right) = -\frac{x}{y^2}$

19. $z = \sqrt{9-x^2-y^2}$ $\quad \frac{d}{dx} = \frac{-x}{\sqrt{9-x^2-y^2}}$ $\quad \frac{d}{dy} = \frac{-y}{\sqrt{9-x^2-y^2}}$

21. $z = (\sin x)(\sin y)$ $\quad \frac{d}{dx} = \sin y \cos x$ $\quad \frac{d}{dy} = \sin x \cos y$

27. $w = e^{r+s}$ $\quad \frac{d}{dr} = e^{r+s}$ $\quad \frac{d}{ds} = e^{r+s}$

31. $e^{-x^2-y^2}$ $\quad \frac{d}{dx} = -2xe^{-x^2-y^2}$ $\quad \frac{d}{dy} = -2ye^{-x^2-y^2}$

39. $Q = \frac{L}{M} e^{-Lt/M}$ $\quad \frac{dQ}{dL} = \frac{M-Lt}{M^2} e^{-Lt/M}$ $\quad \frac{dQ}{dM} = \frac{L(Lt-M)}{M^3} e^{-Lt/M}$

47. $I(95, 50) = 73.1913$ $\quad$ b) $\frac{dL}{dt} : 1.66$

14.4 HW

3. $z = f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1)$ $f(x,y) = x^2y + xy^3$ $(2,1)$

$4+2$

$z = 6 + 5(x-2) + 10(y-1)$

$\boxed{z = -14 + 5x + 10y}$

5. $f(x,y) = x^2 + y^{-2}$, $(4,1)$ $z = 17 + 8(x-4) - 2(y-1)$

$z = 17 + 8x - 32 - 2y + 2$

$\boxed{z = -13 + 8x - 2y}$

7. $F(r,s) = r^2 s^{-1/2} + s^{-3}$, $(2,1)$ $z = 5 + 4(r-2) - 5(s-1)$ $\boxed{z = 2 + 4r - 5s}$

13. $f(x,y) = x^2y^3$ $(a,b) = (2,1)$ $L(x,y) = 4 + 4(x-2) + 12(y-1)$ $= \boxed{4x + 12y - 16}$

15. $f(x,y) = x^3y^{-4}$ $\Delta f = f(2.03, 0.9) - f(2,1)$ $\Delta f = 3.56$

17. $f(x,y) = e^{x^2+y}$ at $(0,0)$ $f(x,y) = f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0)$

$f(\Delta x, \Delta y) \approx 1 + 0\Delta x + 1\Delta y = 1 + 0 + \Delta y$ $f(0.01, -0.02) = \boxed{0.98}$

23. $(2.01)^3(1.02)^2$ $f(x,y) = x^3y^3$ $f(x,y) = f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1)$

$f(2+\Delta x, 1+\Delta y) \approx 8 + 12\Delta x + 16\Delta y$ $f(2+0.01, 1+0.02) = f(2.01, 1.02) = \boxed{8.44}$

25. $\sqrt{3.01^2 + 3.99^2}$ $f(x,y) = \sqrt{x^2+y^2}$ $f(x,y) = f(3,4) + f_x(3,4)(x-3) + f_y(3,4)(y-4)$

$f(3+\Delta x, 4+\Delta y) = 5 + \frac{3}{5}\Delta x + \frac{4}{5}\Delta y$ $f(3.01, 3.99) = \boxed{4.998}$

27. $\sqrt{(1.9)(2.02)(4.05)}$ $f(x,y,z) = \sqrt{xyz}$ $f(x,y,z) = f(2,2,4) + f_x(2,2,4)(x-2) + f_y(2,2,4)(y-2$

$f_z(2,2,4)(z-4)$

$f(1.9, 2.02, 4.05) = \boxed{3.945}$

14.5 HW

7. $\nabla h = (yz^{-3}, xz^{-3}, -3xyz^{-4})$ $h(x,y,z) = xyz^{-3}$

11. $f(x,y) = x^2 - 3xy$, $r(t) = \langle \cos t, \sin t \rangle$ $t=0$ $f_x = 2x - 3y$ $f_y = -3x$ $\nabla f_{(1,0)} = \langle 2, -3 \rangle$ $r'(0) = \langle 0, 1 \rangle$ $f(r(t)) = \boxed{-3}$

13. $f(x,y) = \sin(xy)$, $r(t) = \langle e^{2t}, e^{3t} \rangle$, $t = \ln 3$ $\nabla f = \langle y\cos xy, x\cos xy \rangle_{(1,1)} = \langle \cos 1, \cos 1 \rangle$
$r'(t) = \langle 2e^{2t}, 3e^{3t} \rangle$ $t=0$ $= \langle 2, 3 \rangle$ $\nabla f \cdot r'(0) = \boxed{5\cos 1}$

19. $\nabla g = \langle yz^{-1}, xz^{-1}, -xyz^{-2} \rangle_{(e,1,1)} = \langle 1, e, -e \rangle$ $r'(t) = \langle e^t, 1, 2t \rangle$ $r'(1) = \langle e, 1, 2 \rangle$
$\nabla g \cdot r(1) = \boxed{0}$

27. $f(x,y) = \ln(x^2+y^2)$, $v = 3i - 2j$, $P = (1,0)$ $\nabla f = \langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \rangle$ $(1,0) = \langle 2, 0 \rangle$
$\frac{3-2}{\sqrt{13}}$ $D = \frac{6}{\sqrt{13}}$

31. $f(x,y) = x^2 + 4y^2$ at $P = (3,2)$ $v = \langle -3, -2 \rangle$ $\nabla f = \langle 2x, 8y \rangle$ $\nabla f(3,2) = \langle 6, 16 \rangle$
Direction $= \boxed{\frac{-50}{\sqrt{13}}}$

33. $(3,9,4)$ $(5,7,3)$ $T(x,y,z) = xe^{y-z}$ $\boxed{-49.47°C \text{ per meter}}$

37. $\nabla f_p = (2,-4,4)$ $v = \langle 2,1,3 \rangle$ $\nabla f \cdot v = 12 > 0$, $\boxed{\text{increasing}}$

39. $f(x,y,z) = \sin(xy+z)$ $P = (0,-1,\pi)$ $\theta = 30°$ $\nabla f_{(0,-1,\pi)} = \langle 1, 0, -1 \rangle$ $\|\nabla f_p\| = \sqrt{2}$
$D_u f(0,-1,\pi) = \boxed{\frac{\sqrt{6}}{2}}$

41. $x^2 + y^2 - z^2 = 6$ at $P = (3,1,2)$ $\nabla f = \langle 2x, 2y, -2z \rangle \big|_{(3,1,2)} = \boxed{\langle 6, 2, -4 \rangle}$

43. $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$ $\nabla f_p = \langle \frac{x}{2}, \frac{2y}{9}, 2z \rangle$, $(x,y,z) = \boxed{\pm\left(\frac{4}{\sqrt{17}}, \frac{9}{\sqrt{17}}, \frac{-2}{\sqrt{17}}\right)}$