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Dr Z - Calc 3

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HW 4: 14.3-5

14.3

3) $\frac{\partial}{\partial y}\left(\frac{y}{x+y}\right) = \frac{1(1)-1(y)}{(x+y)^2} = \frac{1-y}{(x+y)^2}$

5) $f = xyz,\ f_z = xy,\ f(2,3,1) = 6$

17) $\frac{\partial}{\partial x}\left(\frac{x}{y}\right) = \frac{1}{y}x' = \frac{1}{y}$

$\frac{\partial}{\partial y}\left(\frac{x}{y}\right) = x(y^{-1})' = -\frac{x}{y^2}$

19) $\frac{\partial}{\partial x}(9-x^2-y^2)^{1/2} = (9-x^2-y^2)^{1/2}\cdot(-x^2)'$

$= -\frac{2x}{\sqrt{9-x^2-y^2}}$

$\frac{\partial}{\partial y} = -\frac{2y}{\sqrt{9-x^2-y^2}}$ (same as x)

21) $\frac{\partial}{\partial x}((\sin x)(\sin y)) = \sin y\cos x$

$\frac{\partial}{\partial y}((\sin x)(\sin y)) = \sin x\cos y$

27) $\frac{\partial}{\partial r}(e^{r+s}) = \frac{\partial}{\partial s}(e^{r+s}) = e^{r+s}$

31) $\frac{\partial}{\partial x}(e^{-x^2-y^2}) = -2xe^{-x^2-y^2}$

$\frac{\partial}{\partial y}(e^{-x^2-y^2}) = -2ye^{-x^2-y^2}$

39) $\frac{\partial}{\partial L}\left(\frac{L}{M}e^{-Lt/M}\right) = \frac{1}{M}e^{-Lt/M} + \frac{L}{M}\left(-\frac{t}{M}e^{-Lt/M}\right)$

$= \frac{1}{M}e^{-Lt/M}\left(1-\frac{Lt}{M}e^{-Lt/M}\right)$

$\frac{\partial}{\partial M}\left(\frac{L}{M}e^{-Lt/M}\right) = \frac{-\frac{L^2t}{M^2}e^{-Lt/M}(M) - Le^{-Lt/M}}{M^2}$

$= \frac{Le^{-Lt}\left(-\frac{Lt}{M^2}-1\right)}{M^2}$

$\frac{\partial}{\partial t}\left(\frac{L}{M}e^{-Lt/M}\right) = -\frac{L^2}{M^2}e^{-Lt/M}$

47) $I(95,50) = 73.19125$

$\frac{\partial}{\partial T} = 0.6545 - 0.00783T - 0.1565H + 0.002HT$

$f_T(95,50) = 1.666$

14.5 CONTINUED

37) $\langle 2,-4,4\rangle\cdot\left\langle\frac{2}{\sqrt{14}},\frac{1}{\sqrt{14}},\frac{3}{\sqrt{14}}\right\rangle = \frac{4-4+12}{\sqrt{14}} = \frac{12}{\sqrt{14}}$

INCREASING positive slope

39) $f_x = y\cos(xy+z),\ f_y = x\cos(xy+z),\ f_z = \cos(xy+z)$

$\nabla f(0,-1,\pi) = \langle -\cos(\pi), 0, \cos(\pi)\rangle = \langle 1,0,1\rangle$

$|\nabla f||\cos 30| = \sqrt{2}\cdot\frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

41) $f_x = x(x^2+y^2-6)^{-1/2},\ f_y = y(x^2+y^2-6)^{-1/2}$

$f_x(3,1) = \frac{3}{2},\ f_y(3,1) = \frac{1}{2}$

$z-2 = \frac{3}{2}(x-3) + \frac{1}{2}(y-1)$

$z = \frac{3}{2}x + \frac{1}{2}y - 3 \Rightarrow \langle 3,1,-2\rangle$

14.4 $f_x = 2xy + y^3,\ f_y = x^2 + 3xy^2$

$f_x(2,1) = 5,\ f_y(2,1) = 10$

$z = 5(x-2) + 10(x-1) + 6$

$= 5x + 10x - 14$

5) $f_x = 2x,\ f_y = -2y$

$f_x(4,1) = 8,\ f_y(4,1) = -2$

$z = 8(x-4) - 2(y-1) + 17$

$= 8x - 2y - 13$

7) $f_r = 2rs^{-1/2},\ f_s = -\frac{1}{2}r^2s^{-3/2} 3s^{-4}$

$f_r(2,1) = 4,\ f_s(2,1) = -5$

$z = 4(r-2) - 5(s-1) + 5$

$= 4r - 5s + 2$

13) $f_x = 2xy^3,\ f_y = 3x^2y^2$

$f_x(2,1) = 4,\ f_y(2,1) = 12$

$z = 4(x-2) + 12(y-1) + 4$

$= 4x + 12y - 16$

approx $= 4 + 4(0.01) + 12(0.02)$

$= 4.28$

approx $= 4 + 4(-0.03) + 12(0.01)$

$= 4$

15) $f_x = \frac{3x^2}{y^4},\ f_y = \frac{-4x^3}{y^{-5}}$

$f_x(2,1) = 12,\ f_y(2,1) = -32$

approx: $12(0.03) - 32(-0.1) = 3.56$

17) $f_x = 2xe^{x^2+y},\ f_y = e^{x^2+y}$

$f_x(0,0) = 0,\ f_y(0,0) = 1$

approx $= 1 + 0(0.01) + 1(-0.02)$

$= -0.98$

Calculator: ≈ 0.98029

23) $f(x,y) = x^3y^2,\ f_x = 3x^2y^2,\ f_y = 2x^3y$

$f_x(2,1) = 12,\ f_y(2,1) = 16$

approx $= 8 + 12(0.01) + 16(0.02) = 8.44$

calc: 8.44864

25) $f(x,y) = \sqrt{x^2+y^2},\ f_x = \frac{2x}{\sqrt{x^2+y^2}},\ f_y = \frac{2y}{\sqrt{x^2+y^2}}$

$f_x(3,4) = \frac{6}{5},\ f_y(3,4) = \frac{8}{5}$

approx $= 5 + \frac{6}{5}(0.01) + \frac{8}{5}(-0.01) = 4.998$

calc: 4.998019

27) $f(x,y,z) = \sqrt{xyz},\ f_x = f_y = f_z = \frac{1}{2\sqrt{xyz}}$

approx $= 4 + \frac{1}{8}(-0.1 + 0.02 + 0.05)$

$= 3.99625\ldots?$

calc: 3.94257

14.5 7) $f_x = \frac{y}{z^3},\ f_y = \frac{x}{z^3},\ f_z = \frac{-3xy}{z^4}$

$\nabla h = \left\langle\frac{y}{z^3}, \frac{x}{z^3}, -\frac{3xy}{z^4}\right\rangle$

11) $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

$\frac{\partial f}{\partial x} = 2x - 3y,\ \frac{dx}{dt} = -\sin t$

$\frac{\partial f}{\partial y} = -3x,\ \frac{dy}{dt} = \cos t$

$x=1, y=0, t=0$

$2(-0) - 3(1) = -3$

13) $\frac{df}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

$\frac{\partial f}{\partial x} = \cos(xy),\ \frac{dx}{dt} = 2e^{2t}$

$\frac{\partial f}{\partial y} = \cos(xy),\ \frac{dy}{dt} = 3e^{3t}$

$x=1, y=1, t=0$

$2\cos 1 + 3\cos 1 = 5\cos 1$

19) $\frac{dg}{dt} = \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt} + \frac{\partial g}{\partial z}\frac{dz}{dt}$

$\frac{\partial g}{\partial x} = \frac{y}{z},\ \frac{dx}{dt} = e^t$

$\frac{\partial g}{\partial y} = \frac{x}{z},\ \frac{dy}{dt} = 1$

$\frac{\partial g}{\partial z} = -\frac{xy}{z^2},\ \frac{dz}{dt} = 2t$

$x=e, y=1, z=1, t=1$

$1\cdot e + e - 2e = 0$

27) $f_x = \frac{2x}{x^2+y^2},\ f_y = \frac{2y}{x^2+y^2}$

$\nabla f(1,0) = \langle 2,0\rangle$

$|\langle 3,-2\rangle| = \sqrt{3^2+2^2} = \sqrt{13} \Rightarrow \frac{1}{\sqrt{13}}$

$u = \frac{1}{\sqrt{13}}\langle 3,-2\rangle = \left\langle\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\right\rangle$

$\nabla f\cdot u = \frac{6}{\sqrt{13}} + 0 = \frac{6}{\sqrt{13}}$

31) $f_x = 2x,\ f_y = 8y$

$\nabla f(3,2) = \langle 6,16\rangle$

$|\langle -3,-2\rangle| = \sqrt{3^2+2^2} = \sqrt{13} \Rightarrow \frac{1}{\sqrt{13}}$

$u = \frac{1}{\sqrt{13}}\langle -3,-2\rangle = \left\langle -\frac{3}{\sqrt{13}}, -\frac{2}{\sqrt{13}}\right\rangle$

$\nabla f\cdot u = -\frac{18}{\sqrt{13}} - \frac{32}{\sqrt{13}} = -\frac{50}{\sqrt{13}}$

33) $f_x = e^{y-z},\ f_y = xe^{y-z},\ f_z = -xe^{y-z}$

$\nabla f(3,9,4) = \langle e^5, 3e^5, -3e^5\rangle$

$|\langle 2,-2,-1\rangle| = \sqrt{4+4+1} = 3 \Rightarrow \frac{1}{3}$

$u = \frac{1}{3}\langle 2,-2,-1\rangle = \left\langle\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}\right\rangle$

$\nabla f\cdot u = \frac{2}{3}e^5 - \frac{6}{3}e^5 + \frac{3}{3}e^5 = -\frac{1}{3}e^5$