14.3-14.5 Homework

14.3

3. $\frac{(x+y) - y}{(x+y)^2} = \frac{x}{(x+y)^2}$

5. $f_z = xy$ $\boxed{6}$

17. $f_x = \frac{1}{y}$ $f_y = \frac{-x}{y^2}$

19. $z = \sqrt{9-x^2-y^2} = (9-x^2-y^2)^{1/2}$

$f_x = \frac{-2x}{2\sqrt{9-x^2-y^2}}$ $f_y = \frac{-2y}{2\sqrt{9-x^2-y^2}}$

21. $f_x = \cos x \sin y$ $f_y = \sin x \cos y$

27. $w = e^{r+s}$

$f_r = e^{r+s}$ $f_s = e^{r+s}$

31.

$$f_x = -2x e^{-x^2-y^2}$$

$$f_y = -2y e^{-x^2-y^2}$$

39. $$\boxed{f_t \; \frac{-L^2}{M^2} e^{-Lt/M}}$$

$$f_L = \frac{L}{M}\left(-\frac{t}{M}\right) e^{-Lt/M} + \frac{1}{M} e^{-Lt/M}$$

$$\left(\frac{tL}{M^2} + \frac{1}{M}\right) e^{-Lt/M}$$

$$\boxed{\left(\frac{M - tL}{M^2}\right) e^{-Lt/M}}$$

$$f_M = \left(\frac{L}{M}\right)\left(\frac{Lt}{M^2}\right) e^{-Lt/M} + \frac{-L}{M^2} e^{-Lt/M}$$

$$\left(\frac{L^2t}{M^3} - \frac{L}{M^2}\right) e^{-Lt/M}$$

$$\boxed{\left(\frac{L^2t - ML}{M^3}\right) e^{-Lt/M}}$$

47. a. $T(95, 50) = 73.19125$

b. Partial Derivative of T

$$\frac{\partial T}{\partial I} = .6845 - (2).00365T - .1565H + (2).0014T$$

$$\boxed{= 1.669}$$

14.4

3. $f_x = 2xy + y^3$ $\qquad z = x^2y + xy^3$

$f_y = x^2 + 3xy^2$ $\qquad f(2,1) = 6$

$f_z = 1$ $\qquad \boxed{F(x,y,z) = f(x,y) - z}$

$f_x(2,1)(x - x_0) + f_y(2,1)(y - y_0) +$

$f_z(2,1)(z - z_0) = 0$

$5(x-2) + 10(y-1) + 1(-z+6) = 0$

$5x - 10 + 10y - 10 + z - 6 = 0$

$$\boxed{5x + 10y - z = 14}$$

5. $F(x,y,z) = f(x,y) - z$

$F_x = 2x$ (4, 1)

$F_y = -2y^{-3}$

$F_z = -1$

$F_x(x,y,z)(x-x_0) + F_y(x,y,z)(y-y_0)$
$+ F_z(x,y,z)(z-z_0) = 0$

$8(x-4) + -2(y-1) - (z-17) = 0$
$8x - 32 \quad -2y + 2 \quad -z + 17 = 0$

$\boxed{8x - 2y - z = 13}$

7. $f_r = 2rs^{-1/2}$ (2, 1)

$f_s = \dfrac{-r^2 s^{-3/2}}{2}$ $\quad -3s^{-4}$

$f_z = -1$

$4(r-2) \quad -5(s-1) \quad -(z-5) = 0$
$4r - 8 \quad -5 \quad +5s \quad -z + 5 = 0$
$\boxed{4r - 5s - z = -2}$

13. $L(x,y) = f(a,b) + f_x(a,b)(x-a)$
$+ f_y(a,b)(y-b)$

$a=2$ $\quad f_x = 2xy^3 \quad f_y = 3x^2y^2$
$b=1$

$L(2.01, 1.02) = 4 + 4(2.01-2) +$
$12(1.02-1)$

$\boxed{L(2.01, 1.02) = 4.28}$

$L(1.97, 1.01) = 4 + 4(1.97-2) +$
$12(1.01-1)$

$\boxed{L(1.97, 1.01) = 4}$

15. $f(2,1) = 8$

$L(x,y) = f(a,b) + f_x(a,b)(x-a)$
$+ f_y(a,b)(y-b)$

$a=2 \quad f_x = 3x^2y^{-4}$
$b=1 \quad f_y = -4x^3y^{-5}$

$L(2.03, .9) = 8 + 12(.03) + -32(-.1)$
$= 11.56$

$\boxed{\Delta f = 11.56 - 8 = 3.56}$

17. $L(x,y) = f(a,b) + f_x(a,b)(x-a)$
$+ f_y(a,b)(y-b)$

$a = 0$ $\quad f_x = 2x e^{x^2+y}$
$b = 0$ $\quad f_y = e^{x^2+y}$

$L(.01, -.02) = 1 + 0(.01) +$
$1(-.02)$
$\boxed{= -.98}$

23. $f(x,y) = x^3 y^2$

$a = 2$ $\quad f_x = 3x^2 y^2$
$b = 1$ $\quad f_y = 2x^3 y$

$L(2.01, 1.02) = 8 + 12(.01) + 16(.02)$
$\boxed{= 8.44}$

25. $f(x,y) = \sqrt{x^2+y^2} = (x^2+y^2)^{1/2}$

$a = 3$ $\quad f_x = 1/2\,(x^2+y^2)^{-1/2}(2x)$
$b = 4$ $\quad f_y = 1/2\,(x^2+y^2)^{-1/2}(2y)$

$L(3.01, 3.99) = 5 + 3/5(.01)$
$+ 4/5(-.01)$
$\boxed{= 4.998}$

27. $f(x,y,z) = \sqrt{xyz} = (xyz)^{1/2}$

$a = 2$ $\quad c = 4$ $\quad f_x = 1/2\,(xyz)^{-1/2}(yz)$
$b = 2$
$f_y = 1/2\,(xyz)^{-1/2}(xz)$
$f_z = 1/2\,(xyz)^{-1/2}(xy)$

$L(1.9, 2.02, 4.05) = 4 + 1(-.1) + 1(.02)$
$+ 1/2(.05)$
$\boxed{= 3.945}$

14.5

7. $\langle yz^{-3}, xz^{-3}, -3xyz^{-4} \rangle$

11. $\left.\frac{d}{dt}\right|_{t=0} \cos^2 t - 3\cos t \sin t$

$= -2\cos t \sin t - (-3\cos^2 t + 3\sin^2 t)$

$= -2\cos t \sin t - 3\cos^2 t - 3\sin^2 t$

$= 0 - 3 - 0$

$\boxed{= -3}$

13. $\left.\frac{d}{dt}\right|_{t=0} \sin((e^{2t})(e^{3t}))$

$= \left.\frac{d}{dt}\right|_{t=0} \sin(e^{5t})$

$= (\cos e^{5t})(5e^{5t})$

$= 5 \cos 1 \boxed{= 2.7018}$

19. $\left.\frac{d}{dt}\right|_{t=1} e^t(t)(t^2)^{-1}$

$\left.\frac{d}{dt}\right|_{t=1} e^t t^{-1}$

$= -e^t t^{-2} + e^t t^{-1}$

$= -1 + 1$

$\boxed{= 0}$

27.

$D_u f = \langle \frac{2x}{x^2+y^2}, \frac{2y}{x^2+y^2} \rangle \cdot \langle 3/\sqrt{13}, -2/\sqrt{13} \rangle$

$= \langle 2, 0 \rangle \langle 3/\sqrt{13}, -2/\sqrt{13} \rangle$

$\boxed{= 6/\sqrt{13}}$

31. $D_u f = \langle 2x, 8y \rangle \cdot \langle -3/\sqrt{13}, -2/\sqrt{13} \rangle$

$\frac{\langle -3, -2 \rangle}{|\langle -3, -2 \rangle|}$

$= \langle 6, 16 \rangle \cdot \langle -3/\sqrt{13}, -2/\sqrt{-13} \rangle$

$\boxed{= \frac{-50}{\sqrt{13}}}$

$= \langle -3/\sqrt{13}, -2/\sqrt{13} \rangle$

33. $\langle 5-3, 7-9, 3-4 \rangle = \langle 2, -2, -1 \rangle$

unit vector $= \langle 2/3, -2/3, -1/3 \rangle$

$D_u f = \langle e^{y-z}, xe^{y-z}, -xe^{y-z} \rangle \langle 2/3, -2/3, -1/3 \rangle$

$= \langle e^5, 3e^5, -3e^5 \rangle \cdot \langle 2/3, -2/3, -1/3 \rangle$

$\boxed{= -e^5/3}$

37. $\langle 2, -4, 4 \rangle \cdot \langle 2, 1, 3 \rangle$

$= 12$, $\boxed{\text{Function is increasing}}$

39.

$\langle y\cos(xy+z), x\cos(xy+z), \cos(xy+z) \rangle \cdot u$

$\cos\theta = \frac{A \cdot B}{|A||B|}$ $\quad \frac{\sqrt{3}}{2} = \frac{\nabla f_p \cdot u}{|\nabla f_p||1|}$

$\nabla f_p = \langle 1, 0, -1 \rangle$

$|\nabla f_p| = \sqrt{2}$ $\quad \boxed{\frac{\sqrt{6}}{2}} = \langle 1, 0, -1 \rangle \cdot u$

41. $\langle 1, 1, -1 \rangle$

$\nabla f = \langle 2x, 2y, -2z \rangle$

$\boxed{\nabla f = \langle 6, 2, -\cancel{4}4 \rangle}$

43. $f_x(a,b,c)(x-a) + f_y(a,b,c)(y-b) +$

$f_z(a,b,c)(z-c) = 0$

DON'T KNOW HOW TO DO

$\left(\frac{1}{2}\right)(x-1) + \left(\frac{2}{9}\right)(y-1) + (-4)(z+2) = 0$

$\frac{x}{2} - \frac{1}{2} + \frac{2}{9}y - \frac{2}{9} - 4z - 8 = 0$

$9x - 9 + 4y - 4 - 72z - 144 = 0$

$9x + 4y - 72z = 157$