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Homework due 10.25

15.1 exercise

Q9. $\iint_R (15-3x)\,dA \quad R=[0,5]\times[0,3]$

$= \int_0^3\int_0^5 (15-3x)\,dx\,dy$

$\Rightarrow \int_0^5 (15-3x)\,dx$

$= 15x + (-\frac{3}{2}x^2)\Big|_0^5$

$= \frac{75}{2}$

$\int_0^3\int_0^5 (15-3x)\,dx\,dy$

$= \int_0^3 \frac{75}{2}\,dy$

$= \frac{75}{2}y\Big|_0^3$

$= \frac{225}{2}$

Q15. $\iint_R x^3\,dA$, $R=[-4,4]\times[0,5]$

$\int_0^5\int_{-4}^4 x^3\,dx\,dy$

$\Rightarrow \int_{-4}^4 x^3\,dx$

$= \frac{1}{4}x^4\Big|_{-4}^4$

$= 0$

$\int_0^5 0\,dy$

$= 0$

$\therefore$ the answer $= 0$.

Q21. $\int_4^9\int_{-3}^8 1\,dx\,dy$

$\Rightarrow \int_{-3}^8 1\,dx$

$= x\Big|_{-3}^8$

$= 11$

$\int_4^9 11\,dy$

$= 11y\Big|_4^9$

$= 99-44$

$= 55$

Q23. $\int_{-1}^1\int_0^\pi x^2\sin y\,dy\,dx$

$\Rightarrow \int_0^\pi x^2\sin y\,dy$

$= x^2\cdot\int_0^\pi \sin y\,dy$

$= x^2\cdot -\cos y\Big|_0^\pi$

$= x^2\cdot 1 - x^2\cdot(-1)$

$= 2x^2$

$\int_{-1}^1 2x^2\,dx$

$= \frac{2}{3}x^3\Big|_{-1}^1 = \frac{4}{3}$

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Q25 $\int_2^6 \int_1^4 x^2\,dx\,dy$

$\Rightarrow \int_1^4 x^2\,dx \qquad \int_2^6 21\,dy$

$= \frac{1}{3}x^3\Big|_1^4 \qquad = 21y\Big|_2^6$

$= 21 \qquad = 84$

Q31. $\int_1^2 \int_0^4 \frac{dy\,dx}{x+y}$

$\Rightarrow \int_0^4 \frac{1}{x+y}\,dy$

$= \ln(x+y)\Big|_0^4$

$= \ln(x+4) - \ln x$

$= \ln \frac{x+4}{x}$

$\int_1^2 \ln\frac{x+4}{x}\,dx$

$= 4\ln2 - 5\ln5 + 6\ln3$

≈ 1.317

Q33

$\int_0^4 \int_0^5 \frac{dy\,dx}{\sqrt{x+y}}$

$\Rightarrow \int_0^5 \frac{1}{\sqrt{x+y}}\,dy$

$= -2\sqrt{x} + 2\sqrt{x+5}$

$\int_0^4 -2\sqrt{x} + 2\sqrt{x+5}\,dx$

$= \frac{76}{3} - \frac{20\sqrt5}{3}$

≈ 10.426

Q35. $\int_1^2 \int_1^3 \frac{\ln(xy)\,dy\,dx}{y}$

$\Rightarrow \int_1^3 \frac{\ln(xy)}{y}\,dy \qquad = \int_1^3 \frac{\ln x + \ln y}{y}\,dy$

$u = \ln y$

$\frac{du}{dy} = \frac{1}{y}$

$du = \frac{dy}{y}$

$= \cancel{\ln 3 \ln(xy)}$

$\cancel{\frac{\ln(3)^2}{2} + \ln(3)\ln x}$

$= \int_1^3 \frac{\ln x}{y}\,dy + \int_1^3 \frac{\ln y}{y}\,dy$

$= \ln x \int_1^3 \frac{1}{y}\,dy + \int_1^3 \frac{\ln y}{y}\,dy$

$\cancel{\int_1^2 \frac{\ln(3)^2}{2} + \ln(3)\ln x\,dx}$

$= \ln x \cdot (\ln 3 - \ln 1) + \frac{\ln 3^2}{2}$

$\cancel{= -\ln(3) + \frac{\ln(3)^2}{2} + 2\ln(2)\ln(3)}$

$\int_1^2 \ln 3 \cdot \ln x + \frac{\ln 3^2}{2}\,dx$

$= \ln 3 \int_1^2 \ln x + \frac{\ln 3}{2}\,dx$

$= \frac{1}{2}(\ln 3)\cdot(-2 + \ln 3 + 4\ln 2)$

$= 1.02786$

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Q37. $\iint_R \frac{x}{y}\, dA \quad R=[-2,4]\times[1,3]$

$\int_{-2}^{4}\int_{1}^{3} \frac{x}{y}\, dy\, dx$

$\Rightarrow \int_1^3 \frac{x}{y}\, dy$

$= x\int_1^3 \frac{1}{y}\, dy$

$= x(\ln 3 - \ln 1)$

$= \ln 3 \cdot x$

$\int_{-2}^{4}\int_{1}^{3} \frac{x}{y}\, dy\, dx$

$= \int_{-2}^{4} (\ln 3 \cdot x)\, dx$

$= \ln 3 \int_{-2}^{4} x\, dx$

$= \frac{1}{2}\cdot \ln 3 \cdot x^2 \Big|_{-2}^{4}$

$= \frac{\ln 3}{2}\cdot (16-4)$

$= 6\ln 3$

Q41. $\iint_R e^x \sin y\, dA \quad R=[0,2]\times[0,\frac{\pi}{4}]$

$= \int_0^2 \int_0^{\frac{\pi}{4}} e^x \sin y\, \cancel{dA}\, dy\, dx$

$\Rightarrow \int_0^{\frac{\pi}{4}} e^x \cdot \sin y\, dy$

$= e^x \cdot \cancel{8}(-\cos y)\Big|_0^{\frac{\pi}{4}}$

$= e^x \cdot (-\frac{\sqrt{2}}{2} + 1)$

$= \frac{2-\sqrt{2}}{2} e^x$

$\int_0^2 \frac{2-\sqrt{2}}{2} e^x\, dx$

$= \frac{2-\sqrt{2}}{2}\, e^x \Big|_0^2$

$= \frac{2-\sqrt{2}}{2}\cdot (e^2 - e^0)$

$= (e^2-1)\cdot \frac{2-\sqrt{2}}{2}$

Exercise 15.2.

Q3. $0 \le x \le 1 \quad 0 \le y \le 1 - x^2$

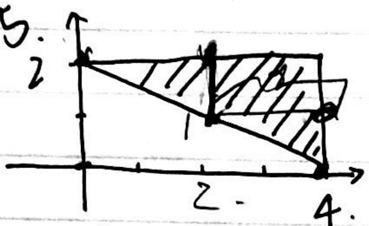
$y = 1 - x^2 \qquad \therefore 0 \le y \le 1$

$x^2 = 1 - y \qquad 0 \le x \le \sqrt{1-y}.$

$x = \sqrt{1-y}$ the horizontal region $0 \le x \le \sqrt{1-y}$, $0 \le y \le 1$

$$\int_0^1 \int_0^{1-x^2} (xy)\,dy\,dx = \frac{1}{12}$$

Q5.



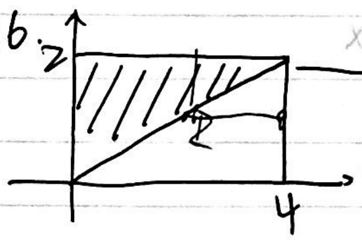
$D = \{(x,y) \mid x = 0..4,\ y = 2 - \frac{1}{2}x..2\}$

line $y = 2 - \frac{1}{2}x \qquad x = 4 - 2y$

$D = \{(x,y) \mid y = 0..2,\ x = 4-2y..4\}$

$$\int_0^2 \int_{4-2y}^4 x^2 y\,dx\,dy = \frac{192}{5}$$

Q6.



$\to y = \frac{1}{2}x$

$D = \{(x,y) \mid x = 0..4,\ y = \frac{1}{2}x..2.\}$

$y = \frac{1}{2}x \quad x = 2y$

$\therefore D = \{(x,y) \mid x = 2y..4,\ y = 0..2\}.$

$$\int_0^2 \int_{2y}^4 (x^2 \cdot y)\,dx\,dy = \frac{256}{15}$$

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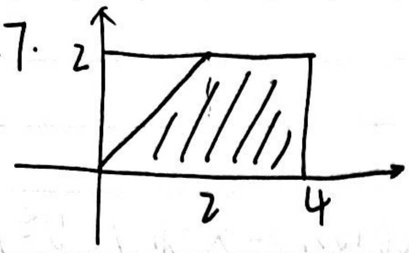
Q7.



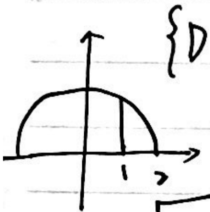
$D = \{(x,y) \mid 0 \le y \le x,\ 0 \le x \le 2\}$

$D = \{(x,y) \mid 0 \le y \le 2,\ 2 \le x \le 4\}$

$\therefore \dfrac{608}{15}$

Q11. $\iint_D \dfrac{y}{x}\, dA$

$\{D(x,y) \mid 1 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2}\}$.



$y = \sqrt{4-x^2}$

$y = \sqrt{4-x^2}$

$y^2 = 4 - x^2$

$x^2 = 4 - y^2$

$x = \sqrt{4-y^2}$

$$\int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x}\, dy\, dx$$

$$= -\frac{3}{4} + 2\ln 2.$$

Q19.

$f(x,y) = x$; $0 \le x \le 1,\ 1 \le y \le e^{x^2}$

$$\int_0^1 \int_1^{e^{x^2}} x\, dy\, dx$$

$$= \int_0^1 \left(x \cdot y\Big|_1^{e^{x^2}}\right) dx$$

$$= \int_0^1 (x \cdot e^{x^2} - x)\, dx$$

$$= \frac{1}{2}e^{x^2} - \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}e^1 - \frac{1}{2} - \frac{1}{2}$$

$$= 0.35914$$

Campus

Q21. $f(x,y) = 2xy$, bounded by $x = y$, $x = y^2$

$y = y^2$

$y - y^2 = 0$

$y(1-y) = 0$

$y = 0$ or 1

$\therefore D\{(x,y) : y^2 \le x \le y,\ 0 \le y \le 1$

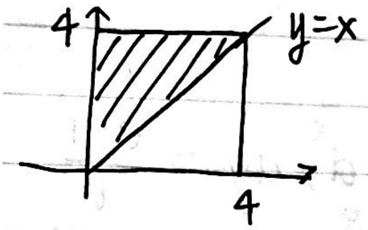
$\int_0^1 \int_{y^2}^{y} 2xy \, dx \, dy$

$= \int_0^1 y \cdot (x^2 \big|_{y^2}^{y}) \, dy$

$= \int_0^1 (y^3 - y^5) \, dy$

$= \frac{1}{4}y^4 - \frac{1}{6}y^6 \big|_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \frac{1}{12}$

Q25. $\int_0^4 \int_x^4 f(x,y) \, dy \, dx$



$0 \le x \le 4$, $x \le y \le 4$

$x = y$

$\therefore 0 \le x \le y$, $0 \le y \le 4$

$\int_0^4 \int_0^y f(x,y) \, dx \, dy$

Q31. $y = e^x$ & $y = e^{\sqrt{x}}$ $\quad f(x,y) = (\ln y)^{-1}$

$e^x = e^{\sqrt{x}}$

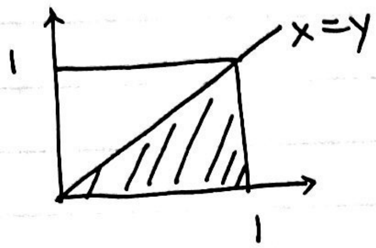
$x = 0, 1,$

$0 \le x \le 1 \quad e^x \le y \le e^{\sqrt{x}}$

$\int_0^1 \int_{e^x}^{e^{\sqrt{x}}} (\ln y)^{-1} \, dy \, dx$

$= -2 + e$

Q33. $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$



$0 \le y \le 1 \quad y \le x \le 1$

$x = y$

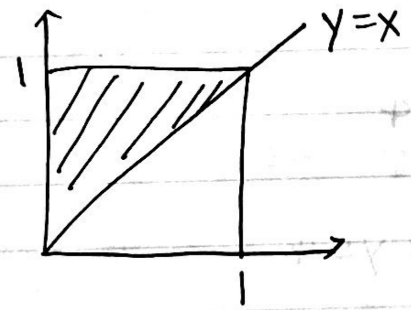
$\therefore\ 0 \le y \le x \ ,\ 0 \le x \le 1$

$\int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dx$

$= 1 - \cos 1 = 0.460$

Q35 $\int_0^1 \int_{y=x}^1 x e^{y^3}\, dy\, dx$

$0 \le x \le 1 \quad y = x \le y \le 1$



$x = y$

$\therefore\ 0 \le x \le y$

$0 \le y \le 1$

$\int_0^1 \int_0^y x e^{y^3}\, dx\, dy = \frac{e-1}{6}$

$= 0.286$

Q37. $0 \le x \le 2 \quad 0 \le y \le 2.$

$\iint_D e^{x+y}\, dA.$

$= \int_0^2 \int_0^2 e^{x+y}\, dx\, dy$

$= \int_0^2 \left(e^{x+y} \Big|_0^2 \right) dy$

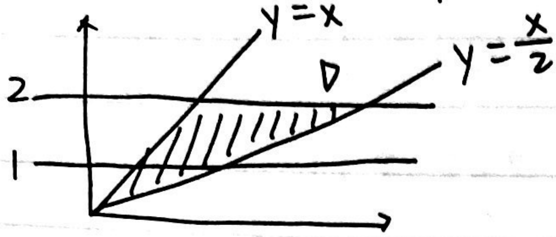
$= \int_0^2 \left(e^{2+y} - e^{0+y} \right) dy$

$= \left(e^{2+2} - e^2 \right) - \left(e^2 - e^0 \right)$

$= e^4 - e^2 - e^2 + 1$

$= e^4 - 2e^2 + 1$

Q43. $f(x,y)=\frac{\sin y}{y}$



$0 \le y \le 2$

$$\iint_D \frac{\sin y}{y}\, dA$$

Q49. $z = x^2+y^2$

$z=8-x^2-y^2$

when $y=0$, $x=\pm 2$.

$x^2+y^2=8-x^2-y^2$

$\therefore \sqrt{4-x^2} \le y \sqrt{4-x}$

$2x^2+2y^2=8$

$-2 \le x \le 2$

$x^2+y^2=4$

$y^2=4-x^2$

$y=\sqrt{4-x^2} \,/\, -\sqrt{4-x^2}$

$$\therefore \int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (8-x^2-y^2)-(x^2+y^2)\,dy\,dx$$

$$=\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} 8-2x^2-2y^2\,dy\,dx$$