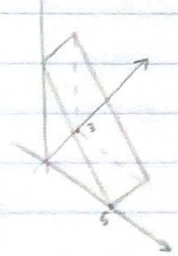
HW due 10/25/20

15.1: 9, 15, 21, 23, 25, 31, 33, 35, 37, 41

15.2: 3, 5, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 43, 49

15.1

9. $\iint_R (15-3x)\,dA \quad R=[0,5]\times[0,3]$



$A=\frac{1}{2}(5)(15)=\frac{75}{2}$

$V=lA=(3)\left(\frac{75}{2}\right)=\boxed{\frac{225}{2}}$

15. $\iint_R x^3\,dA \quad R=[-4,4]\times[0,5]$

$=0$ (due to symmetry cancels out)

21. $\int_4^9\int_{-3}^{8} 1\,dx\,dy$

$=x\Big|_{-3}^{8}=8-(-3)=11$

$=11y\Big|_4^9=99-44=\boxed{55}$

23. $\int_{-1}^{1}\int_0^{\pi} x^2\sin y\,dy\,dx$

$=-x^2\cos y\Big|_0^{\pi}=x^2-(-x^2)=2x^2$

$=\frac{2}{3}x^3\Big|_{-1}^{1}=\frac{2}{3}-\left(-\frac{2}{3}\right)=\boxed{\frac{4}{3}}$

25. $\int_2^6\int_1^4 x^2\,dx\,dy$

$=\frac{1}{3}x^3\Big|_1^4=\frac{64}{3}-\frac{1}{3}=\frac{63}{3}=21$

$=21y\Big|_2^6=\boxed{84}$

31. $\int_1^2\int_0^1 \frac{1}{x+y}\,dy\,dx$

$=\ln|x+y|\Big|_0^1=\ln(x+1)-\ln(x)$

$=x\ln(x)-x\Big|_1^2=(2\ln(2)-2)-(\ln(1)-1)$

$=\boxed{2\ln(2)-1}$

33. $\int_0^4\int_0^5 \frac{1}{\sqrt{x+y}}\,dy\,dx$

$=2\sqrt{x+y}\Big|_0^5=2\sqrt{x+5}-2\sqrt{x}$

$=\frac{4}{3}(x+5)^{3/2}-\frac{4}{3}x^{3/2}\Big|_0^4=36-\frac{32}{3}-\frac{20\sqrt{5}}{3}$

$\boxed{\frac{92-20\sqrt{5}}{3}}$

35. $\int_1^2\int_1^3 \frac{\ln(xy)}{y}\,dy\,dx$

$=\int_1^2\left[\frac{t^2}{2}\right]_{\ln x}^{\ln 3x}\,dx$

$=\int_1^2 \frac{1}{2}\left[(\ln 3x)^2-(\ln x)^2\right]dx$

$=x\ln^2(3x)-2x\ln(3x)+2x$

37. $\iint_R \frac{x}{y}\,dA \qquad R=[-2,4]\times[1,3]$

$\int_1^3\int_{-2}^{4}\frac{x}{y}\,dx\,dy$

$=\frac{x^2}{2y}\Big|_{-2}^{4}=\frac{16}{2y}-\frac{4}{2y}=\frac{12}{2y}=\frac{6}{y}$

$6\ln y\Big|_1^3=\boxed{6\ln 3}$

41. $\int_0^{\pi/4}\int_0^2 e^x\sin y\,dx\,dy$

$=e^x\sin y\Big|_0^2=e^2\sin y-\sin y$

$=-e^2\cos y+\cos y\Big|_0^{\pi/4}=\frac{-e^2}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\boxed{\frac{1-e^2}{\sqrt{2}}}$

15.2

3. $\int_0^1\int_0^{1-x^2} xy\,dy\,dx$

$=\frac{1}{2}xy^2\Big|_0^{1-x^2}=\frac{x(1-x^2)^2}{2} \qquad (1-x^2)(1-x^2)=1-2x^2+x^4$

$=\int_0^1\frac{x-2x^3+x^5}{2}\,dx=\frac{1}{2}\left(\frac{1}{2}x^2-\frac{1}{2}x^4+\frac{1}{6}x^6\right)\Big|_0^1$

$=\boxed{\frac{1}{12}}$

5. $\int_0^4\int_{-\frac{1}{2}x+2}^{2} x^3y\,dy\,dx$

$=\frac{x^3y^2}{2}\Big|_{-\frac{1}{2}x+2}^{2}=\cancel{2x^2}-\frac{x^4}{8}+x^3-\cancel{2x^2} \qquad \left(-\frac{1}{2}x+2\right)\left(-\frac{1}{2}x+2\right)=\frac{1}{4}x^2-2x+4$

$\int_0^4 x^3-\frac{x^4}{8}\,dx=\frac{1}{4}x^4-\frac{x^5}{40}\Big|_0^4=64-6.4=\boxed{57.6}$

6. $\int_0^4\int_{\frac{x}{2}}^{2} x^2y\,dy\,dx$

$=\frac{x^2y^2}{2}\Big|_{\frac{x}{2}}^{2}=2x^2-\frac{x^4}{8}$

$=\frac{2}{3}x^3-\frac{x^5}{40}\Big|_0^4=\frac{128}{3}-25.6=\boxed{17.07}$

7. $\int_{y=0}^{2}\int_{x=y}^{4} x^2y\,dx\,dy$

$=\frac{x^3}{3}y\Big|_y^4=\frac{64y}{3}-\frac{y^4}{3}$

$\int_0^2\frac{64y-y^4}{3}\,dy=\frac{32y^2}{3}-\frac{y^5}{15}\Big|_0^2=\left(\frac{128}{3}-\frac{32}{15}\right)=\boxed{40.53}$

19. $\int_0^1 \int_1^{e^{x^2}} x \, dy \, dx$

$= xy \Big|_1^{e^{x^2}} = xe^{x^2} - x$

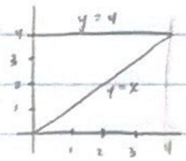
$\int_0^1 xe^{x^2} - x \, dx = \frac{1}{2}e^{x^2} - \frac{1}{2}x^2 \Big|_0^1 = \frac{e}{2} - \frac{1}{2} - \frac{1}{2} = \boxed{\frac{e}{2} - 1}$

21. $\int_0^1 \int_y^{y^2} 2xy \, dx \, dy$

$= x^2 y \Big|_y^{y^2} = y^5 - y^3$

$\frac{1}{6}y^6 - \frac{1}{4}y^4 \Big|_0^1 = \frac{1}{6} - \frac{1}{4} = \boxed{-\frac{1}{12}}$

25.



$\int_0^4 \int_0^4 f(x,y) \, dx \, dy$

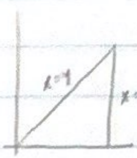
31. $\int_1^e \int_{(\ln(y))^2}^{\ln(y)} (\ln y)^{-1} \, dx \, dy$

$\int_1^e \frac{1}{\ln y} (\ln(y)(1 - \ln(y))) \, dy$

$= \int_1^e 1 - \ln y \, dy = y - (y \ln y - y) \Big|_1^e$

$= (e - e\ln e + e) - (1 - \ln 1 + 1) = 2e - e - 2 = \boxed{e - 2}$

33.

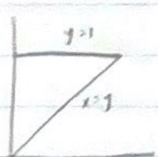


$\int_0^1 \int_0^x \frac{\sin x}{x} \, dy \, dx$

$= \left(\frac{\sin x}{x}\right) y \Big|_0^x = \frac{x \sin x}{x} = \sin x$

$\int_0^1 \sin x \, dx = -\cos x \Big|_0^1 = \boxed{1 - \cos 1}$

35.



$\int_0^1 \int_0^y x e^{y^3} \, dx \, dy$

$= \frac{1}{2}x^2 e^{y^3} \Big|_0^y = \frac{1}{2}y^2 e^{y^3}$

$\frac{1}{6}(e^{y^3}) \Big|_0^1 = \boxed{\frac{e-1}{6}}$

43. $\int_1^2 \int_y^{2y} \frac{\sin y}{y} \, dx \, dy$

$= \frac{x \sin y}{y} \Big|_y^{2y} = 2\sin y - \sin y = \sin y$

$\int_1^2 \sin y \, dy = -\cos y \Big|_1^2 = \boxed{-\cos 2 + \cos 1}$