15.1 - 15.2 (Oct 25th)

15.1: # 9, 15, 21, 23, 25, 31, 33, 35, 37, 41

15.2: # 3, 5, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 43, 49

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9) volume of solid wedge under the graph of $f(x,y) = 15+3x$

$A = \frac{1}{2}(5)(15) = \frac{75}{2}$

length = 3

$V = lA = 3 * \frac{75}{2} = \frac{225}{2}$

15) let $f(x,y) = x^3$

$f(-x,y) = (-x)^3 = -x^3 = -f(x,y)$

because of symmetry, the negative signed volume cancels out with the positive signed volume. $\iint_R x^3 \, dA = 0$

21) $\int_{-3}^{8} 1 \, dx = 8-(-3) = 11$

$\int_4^9 11 \, dy = 55$

23) $\int_0^{\pi} x^2 \sin(y) \, dy = x^2 \cdot 2$

$\int_{-1}^{1} x^2 \cdot 2 \, dx = \boxed{4/3}$

25) $\int_1^4 x^2 \, dx = 21$

$= \int_2^6 21 \, dy$

$\int_2^6 21 \, dy = 84$

$= 84$

31) $\int_1^2 \int_0^4 \frac{dy\,dx}{x+y}$

$\int_0^4 \frac{1}{x+y} \, dy = \ln(x+4) - \ln(x)$

$= \int_1^2 \ln(x+4) - \ln(x) \, dx$

$= 6\ln 6 - 5\ln 5 - 2\ln 2$

33) $\int_0^4 \int_0^5 \frac{dy\,dx}{\sqrt{x+y}}$

$= \int_0^5 \frac{1}{\sqrt{x+y}} \, dy = 2\sqrt{x+5} - 2\sqrt{x}$

$= \int_0^4 (2\sqrt{x+5} - 2\sqrt{x}) \, dx$

$= \boxed{\frac{76}{3} - \frac{20\sqrt{5}}{3}}$

35) $\int_1^2 \int_1^3 \frac{\ln(xy)}{y} \, dy\,dx$

$\int_1^3 \frac{\ln(xy)}{y} \, dy = -\frac{1}{2} y^2(3) + \ln(3)\ln(3x)$

$= \int_1^2 \left(-\frac{1}{2}\ln^2(3) + \ln(3)\ln(3x)\right) dx$

$= -\frac{3}{2}\ln^2(3) + 2\ln(3) + \ln(6) - \ln(3)$

37) $\iint_R \frac{x}{y} \, dA$; $R: [-2,4] \times [1,3]$

$= \int_1^3 \int_{-2}^{4} \frac{x}{y} \, dx\,dy$

$= \int_1^3 \left[\frac{x^2}{2y}\right]_{-2}^{4} dy$

$= \int_1^3 \frac{6}{y} \, dy$

$= 6\ln(y)\Big|_1^3 = \boxed{6\ln(3)}$

41) $\iint_R e^x \sin y \, dA = \int_0^{\pi/4} \int_0^2 e^x \sin y \, dx \, dy$

$\int_0^{\pi/4} \sin y \, [e^x]_0^2 \, dy$

$\int_0^{\pi/4} \sin y \, [e^2 - e^0] dy$

$= -(e^2-1) [\cos y]_0^{\pi/4}$

$= (1-e^2)\left(\frac{\sqrt{2}}{2} - 1\right)$

15.2: #3, 5, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 41

3) Vertically simple: $0 \le x \le 1$, $0 \le y \le 1 - x^2$

horizontally simple: $0 \le y \le 1$, $0 \le x \le \sqrt{1-y}$

$\int_0^1 \int_0^{1-x^2} xy \, dy \, dx = \frac{1}{12}$

5) $(0,2)$, $(4,0)$

$y - 0 = (x-4)\left(\frac{0-2}{4-0}\right) \rightarrow y = \frac{-1}{2}x + 2$

$0 \le x \le 4$, $\frac{-1}{2}x + 2 \le y \le 2$

$\iint_D x^2 y \, dA = \int_0^4 \int_{-1/2 x+2}^{2} x^2 y \, dy \, dx = \int_0^4 \frac{x^2 y^2}{2} \Big|_{-1/2 x + 2}^{2} dx$

$= \int_0^4 \frac{x^2}{2} \left(2^2 - \left(\frac{-x}{2} + 2\right)^2\right) dx$

$= \int_0^4 \left(x^3 - \frac{x^4}{8}\right) dx = \frac{x^4}{4} - \frac{x^5}{40} \Big]_0^4$

$= 4^3 - \frac{4^5}{40} = 38.4$

6) same as 5 but domain is $0 \le x \le 4$, $\frac{-1}{2}x + 2 \le y \le 2$

$\iint_D x^2 y \, dA$ is also the same, 38.4

7) $0 \le y \le 2$, $y \le x \le 4$

$\iint_D x^2 y \, dA = \int_0^2 \int_y^4 x^2 y \, dx \, dy = \int_0^2 \left[\frac{x^3 y}{3}\right]_y^4 dy$

$= \int_0^2 \frac{y}{3} (64 - y^3) dy$

$= \int_0^2 \left(\frac{64y}{3} - \frac{y^4}{3}\right) dy$

$= \frac{32y^2}{3} - \frac{y^5}{15} \Big|_0^2$

$= \frac{608}{15} = 40.5$

11) $1 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2}$

$$\iint_D \frac{y}{x}\,dA = \int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x}\,dy\,dx$$

$$= \int_1^2 \frac{1}{x}\left(\frac{y^2}{2}\right)\Big|_0^{\sqrt{4-x^2}} dx$$

$$= \int_1^2 \frac{1}{2}\left(\frac{1}{x}\right)\left(\sqrt{4-x^2}\right)^2 dx = \frac{1}{2}\int_1^2 \frac{4-x^2}{x}\,dx$$

$$= \frac{1}{2}\left[4\ln x - \frac{x^2}{2}\right]_1^2$$

$$= 2\ln 2 - 1 + \frac{1}{4} = \boxed{2\ln 2 - \frac{3}{4}}$$

19) $$\int_0^1 \int_1^{e^{x^2}} x\,dy\,dx$$

$$= \int_0^1 [xy]_1^{e^{x^2}}\,dx$$

$$= \int_0^1 \left(xe^{x^2} - x\right)dx$$

$$= \left[\frac{1}{2}e^{x^2} - \frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{2}e - \frac{1}{2} - \frac{1}{2}$$

$$= \frac{e-2}{2}$$

21) $y^2 - y = 0$

$y = 0,\ y = 1$

$y = 0 \to x = 0,\ y = 1 \to x = 1$

$(0,0)$ $(1,1)$

$$\int_{y=0}^1 \int_{x=y^2}^{y} (2xy)\,dx\,dy$$

$$= \int_{y=0}^1 2y \int_{x=y^2}^{y} x\,dx\,dy$$

$$= \int_{y=0}^1 2y\left[\frac{x^2}{2}\right]_{y^2}^{y} dy = \int_{y=0}^1 2y\,\frac{1}{2}\left[y^2 - (y^2)^2\right]dy$$

$$= \int_0^1 \left(y^3 - y^5\right)dy = \left[\frac{y^4}{4} - \frac{y^6}{6}\right]_0^1$$

$$= \frac{1^4}{4} - \frac{1^6}{6} - \left[\frac{0^4}{4} - \frac{0^6}{6}\right]$$

$$= \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$$

25) $0 \le x \le 4,\ x \le y \le 4$

$\int_0^4 \int_x^4 f(x,y)\,dy\,dx \quad \int_0^4 \int_0^y f(x,y)\,dx\,dy$

31) $y = e^x$ and $y = e^{\sqrt{x}}$

$e^x = e^{\sqrt{x}}$

$x = \sqrt{x}$

$x = 0$ and $x = 1$

$\int \int_D (\ln y)^{-1}\,dA = \int_1^e \int_{(\ln y)^2}^{\ln y} (\ln y)^{-1}\,dx\,dy$

$= \int_1^e \left(x(\ln y)^{-1}\right)\Big|_{(\ln y)^2}^{\ln y} = dy$

$= \int_1^e (1 - \ln y)\,dy$

$= \int_1^e 1\,dy - \int_1^e \ln y\,dy$

$= [y]_1^e - [y\ln y - y]_1^e$

$= e - 1 - (e - e + 1) = e - 2$

33) $0 \le y \le 1,\ y \le x \le 1$

$\int_0^1 \int_0^y \frac{\sin x}{x}\,dx\,dy = \int_0^1 \int_0^x \frac{\sin x}{x}\,dy\,dx$

$= \int_0^1 \left[\frac{\sin x}{x} \cdot y\right]_0^x dx$

$= \int_0^1 (\sin x - 0)\,dx = \int_0^1 \sin x\,dx$

$= -\cos x \Big|_0^1$

$= 1 - \cos 1$

35) $0 \le x \le 1,\quad x \le y \le 1$

$\int_0^1 \int_{y=x}^1 x e^{y^3}\,dy\,dx = \int_0^1 \int_0^y x e^{y^3}\,dx\,dy$

$= \int_0^1 \frac{y^2}{2} \cdot e^{y^3}\,dy$

$= \int \frac{y^2}{2} e^{y^3}\,dy = \int \frac{1}{6} e^t\,dt$

$\frac{1}{6} e^{y^3} \Big|_0^1 = \boxed{\frac{e-1}{6}}$

37) $\iint_{D_1} e^{x+y}\, dA = \iint_{D} e^{x+y}\, dA + \iint_{D_2} e^{x+y}\, dA$

$= \iint_{D_1} e^{x+y}\, dA - \iint_{D_2} e^{x+y}\, dA$

$= \int_0^2 \int_0^2 e^{x+y}\, dx\, dy - \int_0^1 \int_0^1 e^{x+y}\, dx\, dy$

$= \int_0^2 (e^{x+y})_0^2\, dy - \int_0^1 (e^{x+y})_0^1\, dy$

$= [e^{y+2} - e^{y}]_0^2 - [e^{y+1} - e^{y}]_0^1$

$= e^4 - e^2 - (e^2 - 1) - ((e^2 - e^1)) - (e^1 - e^0))$

$= \boxed{e^4 - 3e^2 + 2e}$

41) $y - 4 = (x-3)\left(\frac{3-1}{4-2}\right)$ $\quad (y-2) = (x-5)\left(\frac{4-2}{3-5}\right)$

$\iint_D f(x,y)\, dA = \int_2^4 \int_{y-1}^{7y} \frac{x}{y^2}\, dx\, dy$

$= \int_2^4 \left[\frac{x^2}{2y^2}\right]_{y-1}^{7y} dy$

$= \int_2^4 \left(\frac{(7-y)^2}{2y^2} - \frac{(y-1)^2}{2y^2}\right) dy$

$= \int_2^4 \left(\frac{24}{y^2} - \frac{6}{y}\right) dy = -\frac{24}{y} - 6\ln y \Big|_2^4$

$= -6 - 12\ln 2 + 12 + 6\ln 2$

$= 6 - 6\ln 2$