Calc HW Due
10/25

Rahul Paleja

15.1 - # 9, 15, 21, 23, 25, 31, 33, 35, 37, 41:

(9) $\iint_R (15-3x)\, dA$ where $R=[0,5]\times[0,3]$

Area Of Triangular Face:

$A=\frac{1}{2}\times 5\times 15 = \frac{75}{2}$

Length of triangular wedge = 3

volume = $l \cdot A = 3 \cdot \frac{75}{2} = \boxed{\frac{225}{2}}$

(15) $\iint_R x^3\, dA \quad R=[-4,4]\times[0,5]$

$f(-x,y) = (-x)^3 = -x^3 = -f(x,y)$

Thus, symmetry allows for - volume to cancel out + volume so

$\iint_R x^3\, dA = 0$

(21) $\int_4^9 \int_{-3}^8 1\, dx\, dy \qquad x\Big]_{-3}^8 = 8-(-3)=11$

$\int_4^9 11\, dy \qquad 11y\Big]_4^9 = 99-44 = \boxed{55}$

(23) $\int_{-1}^1 \int_0^\pi x^2 \sin y\, dy\, dx \qquad x^2(-\cos y)\Big]_0^\pi$

$= x^2(-\cos(\pi) - (-\cos(0)))$

$= x^2(2) = 2x^2$

$2\int_{-1}^1 x^2\, dx = \frac{x^3}{3}\Big]_{-1}^1 = 2\left(\frac{1}{3} - \frac{(-1)^3}{3}\right) = \boxed{\frac{4}{3}}$

(25) $\int_2^5 \int_1^4 x^2\,dx\,dy$ $\quad \int_1^4 x^2\,dx = \frac{x^3}{3}\Big]_1^4 = \frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 21$

$\int_2^6 21\,dy = 21\int_2^6 dy = 21\left((y)\Big]_2^6\right)$

$21(6-2) =$

$\boxed{84}$

(31) $\int_1^2 \int_0^4 \frac{dy\,dx}{x+y}$ $\quad : \int_0^4 \frac{1}{x+y}\,dy$ $\qquad u = x+y$, $du = dy$

$= \int_0^4 \frac{1}{u}\,du = \ln(x+y)\Big]_0^4$

$= \ln(x+4) - \ln(x)$

$\int_1^2 (\ln(x+4) - \ln(x))\,dx$ $\qquad u = \ln(x+4)$, $dv = dx$, $du = \frac{1}{x+4}\,dx$, $v = x$

Integration By Parts

$x\ln(x+4) - \int x\left(\frac{1}{x+4}\right)dx$

$= \boxed{6\ln(6) - 2\ln 2 - 5\ln 5}$

$\int \ln(x)\,dx$

$x\ln(x) - \int x \frac{1}{x}dx$

$= x\ln x - x\Big]_1^2$

$= 2\ln(2) - 2$

$-(\ln(1) - 1)$

$= 2\ln(2) - 1$

(33) $\int_0^4 \int_0^5 \frac{dy\,dx}{\sqrt{x+y}}$ $\quad \int_0^5 \frac{1}{\sqrt{x+y}}\,dy$ $\qquad u = x+y$, $du = dy$

$\int_0^5 \frac{1}{\sqrt{u}}\,du = \int_0^5 u^{-1/2}\,du = \frac{u^{1/2}}{1/2} = 2u^{1/2}$

$= 2\sqrt{x+y}\Big]_0^5 = \int_0^4 2\sqrt{x+5} - 2\sqrt{x}\,dx$

$2\int_0^4 \sqrt{x+5}\,dx = 2\left(\frac{2(x+5)^{3/2}}{3}\Big]_0^4\right) - 2\left(\frac{2(x)^{3/2}}{3}\Big]_0^4\right)$

$= 10.426$

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(35) $\int_1^2 \int_1^3 \frac{\ln(xy)}{y}\, dy\, dx$

$\int_1^3 \ln(xy) \cdot \frac{1}{y}\, dy$ $\quad u = \ln(xy)$, $du = \frac{1}{xy} \cdot x\, dy$

$= \int_1^3 u\, du = \frac{u^2}{2}\Big]_1^3 = \frac{(\ln(xy))^2}{2}\Big]_1^3$

$= \int_1^2 \frac{(\ln(3x))^2}{2} - \frac{(\ln(x))^2}{2}\, dx$

$= \frac{1}{6}\left(6x - 2\ln 3x + (\ln 3x)^2\right]_1^2 - \frac{1}{2}\left(2x - 2x\ln x + (\ln x)^2\right]_1^2$

$= \boxed{1.2214}$

(37) $\iint_R \frac{x}{y}\, dA \quad R = [-2, 4] \times [1, 3]$

$\int_1^3 \int_{-2}^4 \frac{x}{y}\, dx\, dy$

$\frac{1}{y} \int_{-2}^4 x\, dx \quad \frac{1}{y}\left(\frac{x^2}{2}\right]_{-2}^4$

$\frac{1}{y}\left(\frac{16}{2} - \frac{4}{2}\right) = \frac{6}{y}$

$6 \int_1^3 \frac{1}{y}\, dy = 6\left(\ln y\right]_1^3$

$6(\ln(3) - \ln(1))$

$= \boxed{6\ln(3)}$

(41) $\iint_R e^x \sin y \, dA \quad R=[0,2]\times[0,\frac{\pi}{4}]$

$$\int_0^{\pi/4}\int_0^2 e^x \sin y \, dx \, dy$$

$$\sin y \int_0^2 e^x \, dx = \sin y (e^x]_0^2 = \sin y (e^2 - 1)$$

$$= \int_0^{\pi/4} e^2 \sin y - \sin y \, dy$$

$$= -e^2 \cos y]_0^{\pi/4} + \cos y]_0^{\pi/4}$$

$$= -e^2\left(\frac{\sqrt{2}}{2} - \frac{1}{1}\right) + \left(\frac{\sqrt{2}}{2} - 1\right)$$

$$= \boxed{1.8713}$$

Limits: $r^2 = x^2 + y^2$ Rahul Palein

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15.2

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15.2 - #3, 5, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 43, 49:

(3) Vertically: $0 \le x \le 1$, $0 \le y \le 1 - x^2$

Horizontally: $0 \le y \le 1$, $0 \le x \le \sqrt{1-y}$

$$\int_0^1 \int_0^{1-x^2} (xy)\, dy\, dx = \boxed{\frac{1}{12}}$$

(5) $F(x, y) = x^2 y$

$$\int_0^4 \int_{-\frac{1}{2}x+2}^{2} x^2 y\, dy\, dx$$

$$x^2 \int_{-\frac{1}{2}x+2}^{2} y\, dy = \frac{x^2}{1}\left(\frac{2^2}{2} - \frac{(-\frac{1}{2}x+2)^2}{2}\right)$$

$$= \frac{x^2}{1}\left(2 + \frac{1}{2}x^2 - 2\right)$$

$$\int_0^4 x^3 - \frac{x^4}{8}\, dx = \frac{x^4}{4} - \frac{x^5}{40}\Big]_0^4$$

$$= 38.4$$

(6) $38.4 \rightarrow \int_{-\frac{1}{2}x+2}^{2} \int_0^4 x^2 y\, dx\, dy$

(7) $\int_0^2 \int_y^4 x^2 y\, dx\, dy = y\int_y^4 x^2\, dx = \frac{x^3}{3}\Big]_y^4$

$$= y\left(\frac{64}{3} - \frac{y^3}{3}\right)$$

$$\int_0^2 \left(\frac{64}{3}y - \frac{y^4}{3}\right) dy = \frac{64y^2}{6} - \frac{y^5}{15}\Big]_0^2 = \boxed{40.5}$$

(11) $\int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x}\, dy\, dx$

$= \frac{1}{x} \int_0^{\sqrt{4-x^2}} y\, dy = \frac{1}{x} \left(\frac{y^2}{2} \right]_0^{\sqrt{4-x^2}}$

$= \frac{1}{x} \left(\frac{4-x^2}{2} \right) = \int_1^2 \frac{4-x^2}{2x}\, dx = \frac{1}{2} \int_1^2 \frac{4-x^2}{x} dx$

$\frac{1}{2} \left(4\ln x - \frac{x^3}{3} \right]_1^2 = \frac{1}{2} \left(\left(4\ln(2) - \frac{8}{3}\right) - \left(4\ln(1) - \frac{1}{3}\right) \right)$

$= \boxed{2\ln 2 - 3/4}$

(19) $F(x,y) = x \quad 0 \le x \le 1 \qquad 1 \le y \le e^{x^2}$

$\int_0^1 \int_1^{e^{x^2}} x\, dy\, dx = xy \Big]_1^{e^{x^2}}$

$= \int_0^1 (x e^{x^2} - x)\, dx = \left(\frac{1}{2} e^{x^2} - \frac{x^2}{2} \right]_0^1$

$\left(\frac{1}{2} e^{(1)^2} - \frac{1}{2} \right) - \left(\frac{1}{2} e^0 \right) = \boxed{\frac{e-2}{2}}$

(21) $f(x,y) = 2xy$ bounded by $x=y \quad x=y^2$

$\int_0^1 \int_x^{\sqrt{x}} 2xy\, dy\, dx = xy^2 \Big]_x^{\sqrt{x}}$

$= x(x - x^2) = \int_0^1 x^2 - x^4\, dx$

$= \frac{x^3}{3} - \frac{x^5}{5} \Big]_0^1 = \boxed{1/12}$

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Rahul Patejo

15.2 - #25, 31, 33, 35, 37, 43, 49:

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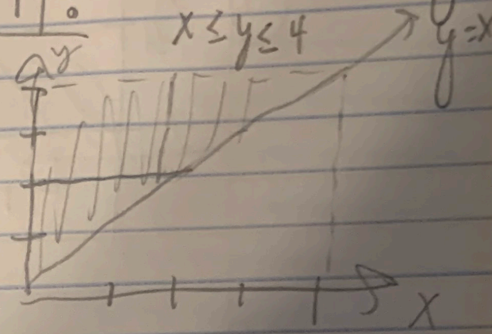
$$\int_0^4 \int_x^4 f(x,y)\, dy\, dx$$

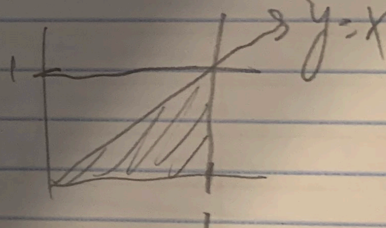
$$= \int_0^4 \int_0^y f(x,y)\, dx\, dy$$



31 $f(x,y) = (\ln y)^{-1}$ $D \Rightarrow y = e^x$ $y = e^{\sqrt{x}}$

$$\int_1^e \int_{\ln^2 y}^{\ln y} (\ln y)^{-1}\, dx\, dy = e - 2$$

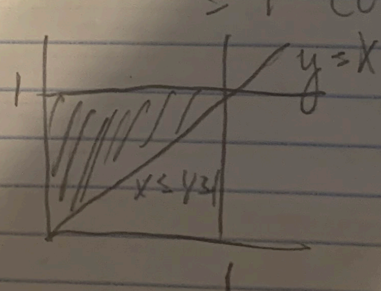
33 $$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$$



$$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$$

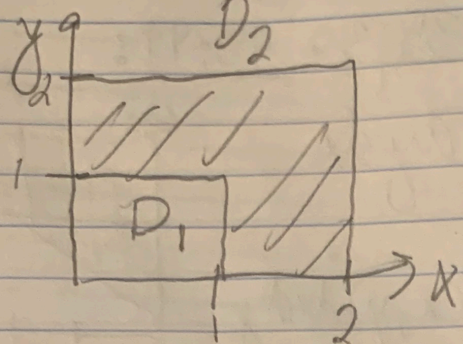
$$= 1 - \cos(1)$$

35



$$\int_0^1 \int_0^y x e^{y^3}\, dx\, dy = \boxed{\frac{e-1}{6}}$$

(37)



$$\iint_D e^{x+y}\,dA = \boxed{e^4-3e^2+2e}$$

(43) $F(x,y)=\frac{\sin y}{y}$ $\iint \frac{\sin y}{y}\,dA = \boxed{\cos(1)-\cos(2)}$

(49) $z=x^2+y^2$ $z=8-x^2-y^2$

$$\int_{-2}^{2}\left(\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left[(8-x^2-y^2)-(x^2+y^2)\right]dy\right]dx$$

$\to$ Put into maple