15.1 Homework

9) $\iint_R (15-3x)\,dA \quad R=[0,5]\times[0,3]$

$\int_0^3\int_0^5 (15-3x)\,dx\,dy$

$\int_0^5 (15-3x)\,dx = \left[15x - \frac{3x^2}{2}\right]_0^5$

$\int_0^5 (15-3x)\,dx = \frac{75}{2}$

$\int_0^3 \left(\frac{75}{2}\right)dy = \left[\frac{75}{2}y\right]_0^3$

$\int_0^3 \left(\frac{75}{2}\right)dy = \frac{225}{2}$

$\boxed{\int_0^3\int_0^5 (15-3x)\,dx\,dy = \frac{225}{2}}$

15) $\iint_R x^3\,dA\,,\ R=[-4,4]\times[0,5]$

$f(x,y) = x^3$

$f(-x,y) = (-x)^3 = -x^3 = -f(x,y)$

Because of symmetry the neg signed vol cancels with the pos signed vol

$\boxed{\int_0^5\int_{-4}^4 x^3\,dx\,dy = 0}$

21) $\int_4^9\int_{-3}^8 1\,dx\,dy$

$\int_{-3}^8 1\,dx = [x]_{-3}^8 = 11$

$\int_4^9 11\,dy = [11y]_4^9 = 55$

$\boxed{\int_4^9\int_{-3}^8 1\,dx\,dy = 55}$

23) $\int_{-1}^1\int_0^\pi x^2\sin y\,dy\,dx$

$\int_0^\pi x^2\sin y\,dy = \left[-x^2\cos y\right]_0^\pi$

$\left[-x^2\cos y\right]_0^\pi = -x^2(\cos\pi) + x^2(\cos 0)$

$\int_{-1}^1 2x^2\,dx = \left[\frac{2x^3}{3}\right]_{-1}^1 = \frac{2}{3}+\frac{2}{3}$

$\boxed{\int_{-1}^1\int_0^\pi x^2\sin y\,dy\,dx = \frac{4}{3}}$

25) $\int_2^6\int_1^4 x^2\,dx\,dy$

$\int_1^4 x^2\,dx = \left[\frac{x^3}{3}\right]_1^4 = \frac{64}{3} - \frac{1}{3}$

$\int_2^6 21\,dy = [21x]_2^6 = 84$

$\boxed{\int_2^6\int_1^4 x^2\,dx\,dy = 84}$

31) $\int_1^2\int_0^4 \frac{1}{x+y}\,dx\,dy$

$\int_0^4 \frac{1}{x+y}\,dx = \left[\ln|x+y|\right]_0^4$

$\left[\ln|x+y|\right]_0^4 = \ln|4+y| - \ln|y|$

$\int_1^2 (\ln|4+y| - \ln|y|)\,dy = \left[(y+4)\ln(y+4) + y\ln(y) - 2y\right]_1^2$

$\left[(y+4)\ln(y+4) + y\ln(y) - 2y\right]_1^2 = 6\ln 6 - 5\ln 5 - 2\ln 2$

$\boxed{\int_1^2\int_0^4 \frac{1}{x+y}\,dx\,dy = 6\ln 6 - 5\ln 5 - 2\ln 2}$

33) $\int_0^4\int_0^5 \frac{1}{\sqrt{x+y}}\,dy\,dx$

$\int_0^5 \frac{1}{\sqrt{x+y}}\,dy = \left[2\sqrt{x+y}\right]_0^5 = 2\sqrt{x+5} - 2\sqrt{x}$

$\int_0^4 (2\sqrt{5+x} - 2\sqrt{x})\,dx = \left[\frac{4((x+5)^{3/2} - x^{3/2})}{3}\right]_0^4$

$\boxed{\int_0^4\int_0^5 \frac{1}{\sqrt{x+y}}\,dy\,dx = -\frac{4\sqrt[3]{5} - 76}{3}}$

35) $\int_1^2\int_1^3 \frac{\ln(xy)}{y}\,dy\,dx$

$\int_1^3 \frac{\ln(xy)}{y}\,dy = \left[\frac{\ln^2(xy)}{2}\right]_1^3$

$\left[\frac{\ln^2(xy)}{2}\right]_1^3 = \frac{\ln^2(3x) - \ln^2(x)}{2}$

$\int_1^2 \frac{\ln^2(3x) - \ln^2(x)}{2}\,dx = \left[\frac{x[\ln^2(3x) - 2\ln(3x) - \ln^2(x) + 2\ln(x)]}{2}\right]_1^2$

$\boxed{\int_1^2\int_1^3 \frac{\ln(xy)}{y}\,dy\,dx = \frac{2\ln^2 6 - 4\ln 6 - \ln^2 3 + 2\ln 3 - 2\ln^2 2 + 4\ln 2}{2}}$

37) $\iint_R \frac{x}{y}\,dA\,,\ R=[-2,4]\times[1,3]$

$\int_1^3\int_{-2}^4 \frac{x}{y}\,dx\,dy$

$\int_{-2}^4 \frac{x}{y}\,dx = \left[\frac{x^2}{2y}\right]_{-2}^4 = \frac{6}{y}$

$\int_1^3 \frac{6}{y}\,dy = \left[6\ln|y|\right]_1^3$

$\boxed{\int_1^3\int_{-2}^4 \frac{x}{y}\,dx\,dy = 6\ln(3)}$

41) $\iint_R e^x\sin y\,dA\,,\ R=[0,2]\times[0,\frac{\pi}{4}]$

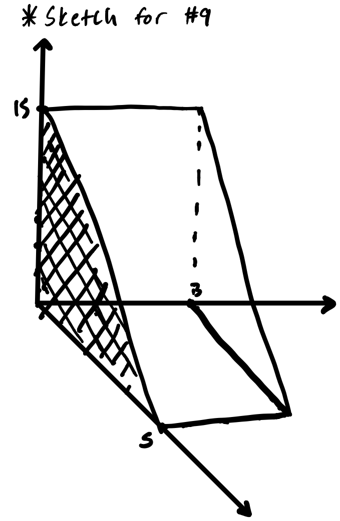
$\int_0^{\frac{\pi}{4}}\int_0^2 e^x\sin y\,dx\,dy$

$\int_0^2 e^x\sin y\,dx = \left[e^x\sin y\right]_0^2 = e^2\sin y - \sin y$

$\int_0^{\frac{\pi}{4}} (e^2\sin y - \sin y)\,dy = \left[-e^2\cos y + \cos y\right]_0^{\frac{\pi}{4}}$

$\boxed{\int_0^{\frac{\pi}{4}}\int_0^2 e^x\sin y\,dx\,dy = -\frac{(\sqrt{2}-2)(e^2-1)}{2}}$

* Sketch for #9



(3) Vertically simple region: $x=[0,1]$, $y=[0,1-x^2]$

Horizontally simple region: $x=[0,\sqrt{1-y}]$, $y=[0,1]$

(5) $(0,2)$, $(4,0) \Rightarrow y=-\frac{1}{2}x+2$

$0 \le x \le 4$, $-\frac{x}{2}+2 \le y \le 2$

$$\iint_D x^2 y\, dA = \int_0^4 \int_{-\frac{x}{2}+2}^{2} x^2 y\, dy\, dx$$

$$\int_{-\frac{x}{2}+2}^{2} x^2 y\, dy = \left[\frac{x^2 y^2}{2}\right]_{-\frac{x}{2}+2}^{2}$$

$$\int_{-\frac{x}{2}+2}^{2} x^2 y\, dy = 2x - \frac{x^2(-\frac{x}{2}+2)}{2}$$

$$\boxed{\int_0^4 \left(2x - \frac{x^2(-\frac{x}{2}+2)}{2}\right)dx = 38.4}$$

(6) Same as 5 $\boxed{38.4}$

(7) $y \le x \le 4$, $0 \le y \le 2$

$$\iint_D x^2 y\, dA = \int_0^2 \int_y^4 x^2 y\, dx\, dy$$

$$\boxed{\int_0^2 \int_y^4 x^2 y\, dx\, dy = 40.53}$$

(11) $\iint_D \frac{y}{x}\, dA$, $1 \le x \le 2$, $0 \le y \le \sqrt{4-x^2}$

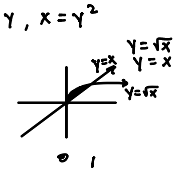
$$\boxed{\int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x}\, dy\, dx = \ln 4 - \frac{3}{4}}$$

(19) $f(x,y)=x$; $0 \le x \le 1$, $1 \le y \le e^{x^2}$

$$\boxed{\int_0^1 \int_1^{e^{x^2}} x\, dy\, dx = \frac{1}{2}(e-2)}$$

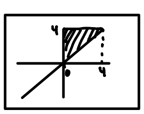
(21) $f(x,y)=2xy$, $x=y$, $x=y^2$

$$\boxed{\int_0^1 \int_y^{y^2} 2xy\, dx\, dy = \frac{1}{12}}$$



(25) $0 \le x \le 4$, $x \le y \le 4$ $y=x$

$y=x$ $x=y$



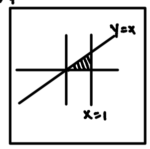
$$\boxed{\int_0^4 \int_0^y f(x,y)\, dx\, dy}$$

(31) $f(x,y)=(\ln y)^{-1}$, $y=e^x$, $y=e^{\sqrt{x}}$

$(0,1)$, $(1,e)$ $x=0$, $x=1$

$$\boxed{\int_0^1 \int_{e^x}^{e^{\sqrt{x}}} (\ln y)^{-1}\, dy\, dx = .718}$$

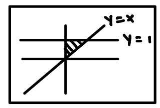
(33) $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$



$$\int_0^1 \int_0^x f(x,y)\, dy\, dx$$

$$\boxed{\int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dx}$$

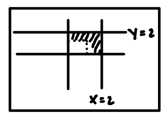
(35) $\int_0^1 \int_x^1 x e^{y^3}\, dy\, dx$



$$\int_0^1 \int_0^x f(x,y)\, dx\, dy$$

$$\boxed{\int_0^1 \int_0^x x e^{y^3}\, dx\, dy}$$

(37) $\iint_D e^{x+y}\, dA$



$$\int_0^2 \int_0^2 e^{x+y}\, dx\, dy - \int_0^1 \int_0^1 e^{x+y}\, dx\, dy$$

40.82 − 2.95249

$$\boxed{\iint_D e^{x+y}\, dA = 37.86751}$$

(43) $f(x,y) = \frac{\sin y}{y}$

$1 \le y \le 2$, $y \le x \le 2y$

$$\boxed{\int_1^2 \int_y^{2y} \frac{\sin y}{y}\, dx\, dy = \cos 1 - \cos 2}$$

(49)

$$\boxed{\int_{-2}^{2} \left(\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left[(8-x^2-y^2)-(x^2+y^2)\right] dy \right) dx}$$