15.1: 9, 15, 21, 23, 25, 31, 35, 37, 41
15.2: 3, 5, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 43, 49

Chap 15 HW

Orion Kress-Sanfilippo

15.1

9) $\iint_R (15-3x)\,dA \quad R=[0,5]\times[0,3]$

$= \int_0^3\int_0^5 15-3x\,dy\,dx = \int_0^3 75-15x\,dx$

$= 225-15\frac{9}{2} = \frac{450-135}{2} = \boxed{\frac{325}{2}}$

15) $\iint_R x^3\,dA, \quad R=[-4,4]\times[0,5]$

$= \int_0^5\int_{-4}^4 x^3\,dy\,dx = \int_0^5 8x^3\,dx = \boxed{125}$

21) $\int_4^9\int_{-3}^8 1\,dx\,dy = \int_4^9 11\,dy = \boxed{55}$

23) $\int_{-1}^1\int_0^\pi x^2 \sin y\,dy\,dx = \int_{-1}^1 x^2\left(-\cos y\right)\Big|_0^\pi$ $-(-1-1)$ $= +2\left(\frac{1}{3}-\left(-\frac{1}{3}\right)\right)$

$= \boxed{\frac{+4}{3}}$

25) $\int_2^6\int_1^4 x^2\,dx\,dy = \int_2^6 \left(\frac{64}{3}-\frac{1}{3}\right)dy = \boxed{84}$

15.1 Cont

31) $\int_1^2\int_0^4 \frac{1}{x+y}\,dy\,dx = \int_1^2 \ln|x+4| - \ln|x|\,dx$

$= \int_1^2 \ln\left|1+\frac{4}{x}\right| dx = \left(-4\ln\left(\frac{4}{x}\right) + \ln\left(1+\frac{4}{x}\right)\left(1+\frac{4}{x}\right)x\right)\Big|_1^2$

$= \boxed{4\ln(2) - 5\ln(2) + 6\ln 3}$

35) $\int_1^2\int_1^3 \frac{\ln(xy)}{y}\,dy\,dx$ $\quad u = \ln(xy)\Big|_{1=\ln(x)}^{3=\ln(3x)}$ $\quad du = \frac{1}{y}dy$ $= \int_1^2\int_{\ln(x)}^{\ln(3x)} u\,du$

$= \int_1^2 \frac{(\ln(3x))^2}{2} - \frac{(\ln(x))^2}{2} = \frac{1}{2}\int_1^2 \cancel{(\ln(x))^2} \cancel{-(\ln(x))^2} + \ln(3)(\ln(x)) + \ln(3)(\ln(x)) + (\ln(3))^2$

$\left(\frac{\ln 3 + \ln x}{2}\right)^2$

$= \int_1^2 \ln(3)\ln(x) + \frac{(\ln(3))^2}{2}\,dx$

$= \left(x\ln(3)\ln(x) + x\ln(3) + \frac{(\ln(3))^2}{2}x\right)\Big|_1^2$

$= \left(2\ln(3)\ln(2) + 2\ln(3) + \frac{(\ln(3))^2}{\cancel{2}}\cancel{2}\right)$

$- \left(0 + \ln(3) + \frac{(\ln(3))^2}{2}\right)$

$= \boxed{2\ln(3)(\ln(2)) + \ln(3) + \frac{(\ln(3))^2}{2}}$

15.1 Cont

37) $\iint_R \frac{x}{y}\,dA \quad R=[-2,4]\times[1,3]$

$$= \int_1^3\int_{-2}^4 \frac{x}{y}\,dx\,dy = \frac{1}{\cancel{2}}\int_1^3 \frac{\overset{6}{\cancel{12}}}{y}\,dy = \boxed{6\ln(3)}$$

41) $\iint_R e^x \sin y\,dA,\ R=[0,2]\times[0,4]$

$$= \int_0^{\pi/4}\int_0^2 e^x \sin y\,dx\,dy = \int_0^{\pi/4}(e^2-1)\sin y\,dy$$

$$= (e^2-1)\left(\left(-\cos\left(\tfrac{\pi}{4}\right)\right)-(-1)\right) = \boxed{(e^2-1)\left(1-\frac{\sqrt{2}}{2}\right)}$$

15.2

3) $R_1 = \{(x,y) \mid 0 \le x \le 1,\ 0 \le y \le 1-x^2\}$

$R_2 = \{(x,y) \mid 0 \le y \le 1,\ 0 \le x \le \sqrt{1-y}\}$

$$\iint_{R_1} xy\,dy\,dx = \frac{1}{4}\int_0^1 4x\frac{(1-x^2)^{2}}{\cancel{2}}\,dx = -\frac{1}{4}\,\frac{(1-x^2)^3}{3}\Bigg|_0^1$$

$u = 1-x^2 \Big|_{0=1}^{1=0}$

$du = -2x\,dx$

$$\boxed{= \frac{+1}{12}}$$

$$\iint_{R_2} xy\,dx\,dy = \int_0^1 (1-y)y\,dy = \boxed{\frac{1}{12}}$$

15.2 Cont

5) $R = \{(x,y) \mid 0 \le x \le 4, \ \frac{-x}{2}+2 \le y \le 2\}$

$$\int_0^4 \int_{\frac{-x}{2}+2}^{2} x^2 y \, dy\, dx = \int_0^4 x^2 \left(\frac{4}{2} - \frac{\left(\frac{-x}{2}+2\right)^2}{2}\right) dx$$

$$= \int_0^4 x^2 \left(\cancel{2} - \frac{\left(\frac{x^2}{\cancel{4}8} - \cancel{2}x + \cancel{4}^{2}\right)}{\cancel{2}}\right) = \int_0^4 x^2 \left(\frac{-x^2}{8} + x\right) dx$$

$$= \int_0^4 \frac{-x^4}{8} + x^3 = \frac{-x^5}{40} + \frac{x^4}{4} = 64 - 25.6 = \boxed{38.4}$$

6) $R = \{(x,y) \mid 0 \le x \le 4, \ \frac{x}{2} \le y \le 2\}$

$$\int_0^4 \int_{\frac{x}{2}}^{2} x^2 y \, dy\, dx = \int_0^4 x^2 \left(2 - \frac{\left(\frac{x^2}{4}\right)}{2}\right)$$

$$= \int_0^4 2x^2 - \frac{x^4}{8} = \left(\frac{2}{3}x^3 - \frac{1}{40}x^5\right)\Big|_0^4 = 42.67 - 25.6$$

$$= \boxed{17.1}$$

7) $R = \{(x,y) \mid 0 \le y \le 2, \ y \le x \le 4\}$

$$\int_0^2 \int_y^4 x^2 y \, dx\, dy = \int_0^2 y\left(\frac{64}{3} - \frac{y^3}{3}\right) dy$$

$$= \left(\frac{32}{3}y^2 - \frac{y^5}{15}\right)\Big|_0^2 = 42.67 - 2.13 = \boxed{40.5}$$

15.2 Cont

11) $R = \{(x,y) \mid 1 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2}\}$

$$= \int_1^2\int_0^{\sqrt{4-x^2}} \frac{y}{x}\,dy\,dx = \int_1^2 \frac{1}{x}\left(\frac{\cancel{4}^{2}-x^2}{\cancel{2}}\right)$$

$$= \int_1^2 \frac{2}{x} - \frac{x}{2}\,dx = 2\ln 2 - \left(1 - \frac{1}{4}\right) = \boxed{2\ln 2 - \frac{3}{4}}$$

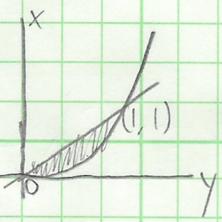
19) $f(x,y) = x \quad 0 \le x \le 1 \quad 1 \le y \le e^{x^2}$

$u = e^{x^2}$
$du = 2xe^{x^2}dx$

$$\int_0^1\int_1^{e^{x^2}} x\,dx = \int_0^1 xe^{x^2} - x\,dx = \frac{1}{2}\int_0^1 2xe^{x^2}dx - \int_0^1 x\,dx$$

$$= \frac{1}{2}\left(e^{x^2}\right)\Big|_0^1 - \frac{1}{2} = \frac{e}{2} - \frac{1}{2} - \frac{1}{2} = \boxed{\frac{e-2}{2}}$$

21) $f(x,y) = 2xy$ top by $x = y$; bot by $x = y^2$;



$R = \{(x,y) \mid 0 \le y \le 1,\ y^2 \le x \le y\}$

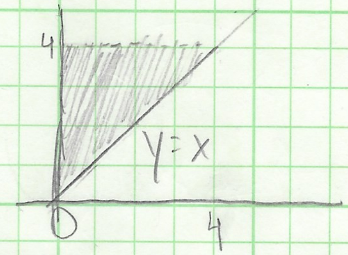
$$\int_0^1\int_{y^2}^{y} 2xy\,dx\,dy =$$

$$\int_0^1 2y\left(\frac{y^2}{2} - \frac{y^4}{2}\right)dy = \int_0^1 y^3 - y^5 = \left(\frac{y^4}{4} - \frac{y^6}{6}\right)\Big|_0^1$$

$$= \boxed{\frac{1}{12}}$$

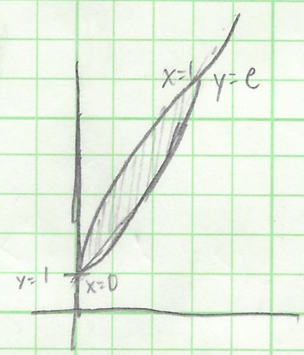
15.2 Cont

25) $\int_0^4\int_x^4 f(x,y)\,dy\,dx$



$=\int_0^4\int_0^y f(x,y)\,dy\,dx$

31) D bounded by: $y=e^{\sqrt{x}}$
$y=e^{x}$



$\ln y=\sqrt{x}$, $\boxed{x=(\ln(y))^2}$ top

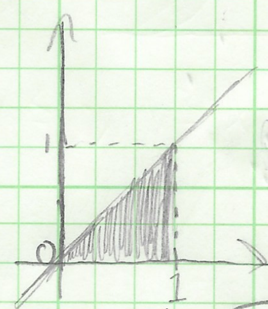
$\boxed{\ln y = x}$ bot

$e^{\sqrt{x}}=e^{x}$

$$\int_1^e\int_{\ln y}^{(\ln y)^2}\frac{1}{\ln(y)}\,dx\,dy=\int_1^e\frac{(\ln(y))^{\cancel{2}}-\cancel{\ln(y)}}{\cancel{\ln y}}$$

$$=\int_1^e \ln y-1\,dy=(y\ln y+\cancel{y}-\cancel{y})\Big|_1^e=\boxed{e}$$

33) $\int_0^1\int_y^1\frac{\sin x}{x}\,dx\,dy$

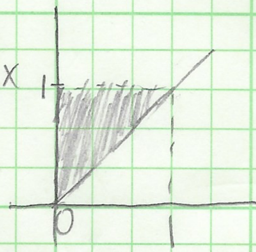


$$=\int_0^1\int_0^x\frac{\sin x}{x}\,dy\,dx=\int_0^1\sin x\,dx=\boxed{1-\cos(1)}$$

Simplification Cancels this x

15.2 Cont

35) $\int_0^1\int_x^1 x e^{y^3}\,dy\,dx$



$= \int_0^1\int_0^y x e^{y^3}\,dx\,dy$

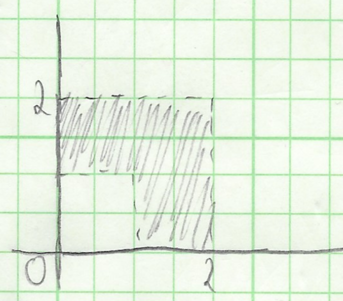
$= \int_0^1 \frac{y^2}{2} e^{y^3}\,dy = \frac{1}{2}\frac{1}{3}\int_0^1 3y^2 e^{y^3}\,dy = \frac{1}{6}\frac{e^{y^6}}{2}$

$\boxed{= \frac{1}{12}(e-1)}$

Simplification removed need for int. by parts, allowed u-sub

37) $\iint_D e^{x+y}\,dA$

D:



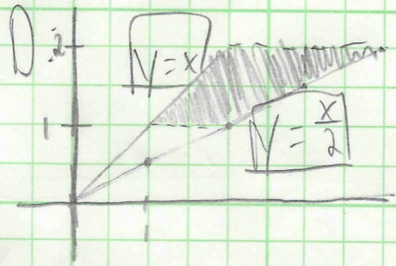
$= \int_0^1\int_1^2 e^{x+y}\,dy\,dx + \int_1^2\int_0^2 e^{x+y}\,dy\,dx$

$= \int_0^1 e^x(e^2-e)\,dx + \int_1^2 e^x(e^2-1)\,dx$

$= (e-1)(e^2-e) + (e^2-e)(e^2-1) = \boxed{(e^2-e)\left((e^2-1)+(e-1)\right)}$

15.2 Cont

43) $\iint_D \frac{\sin y}{y}\, dA$



$$= \int_1^2 \int_y^{2y} \frac{\sin y}{y}\, dx\, dy = \int_1^2 \sin y\, dy = -(\cos(2) - \cos(1))$$

$$= \boxed{\cos(1) - \cos(2)}$$

49) $$\boxed{\int_{-2}^{2} \int_{-\sqrt{-y^2+4}}^{\sqrt{-y^2+4}} 2x^2 + 2y^2 - 8\, dx\, dy}$$