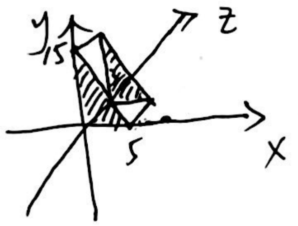
15.1.

9. $\int_0^3\int_0^5 (15-3x)\,dx\,dy$

$= [15x-\frac{3}{2}x^2]_0^5$

$= 75-\frac{75}{2} = \frac{75}{2}$

$\cancel{\int_0^3} 3\times\frac{75}{2} = \frac{225}{2}$



15. $\int_0^5\int_{-4}^4 x^3\,dx\,dy$

$\cancel{2\times[\frac{x^4}{4}]}\ [\frac{x^4}{4}]_{-4}^4$

$=56-56=0$

$\int_0^5 0\,dy = 0$

21. $\int_4^9 11\,dy$

$=(9-4)\times 11 = 55$

23. $\int_0^\pi x^2\sin y\,dy$

$=[-x^2\cos y]_0^\pi$

$=x^2$

$\int_{-1}^1 x^2\,dx$

$=[\frac{x^3}{3}]_{-1}^1$

$=\cancel{\frac{1}{3}-\frac{1}{3}}\ \cancel{2\times1}\ \frac{1}{3}\times 2$

$=\frac{2}{3}$

25. $\int_1^4 x^2\,dx$

$=[\frac{x^3}{3}]_1^4$

$=\cancel{\frac{256}{3}-\frac{1}{3}}\ \frac{64}{3}-\frac{1}{3}$

$=\cancel{\frac{255}{3}}\ 21$

$\cancel{\int_2^6 \frac{255}{3}\,dy}$ $\int_2^6 21\,dy$

$\cancel{=(6-2)\times\frac{255}{3}}$ $=(6-2)\times 21$

$=84$

31. $\int_0^4 \frac{1}{x+y}\,dy$

$=[\ln|x+y|]_0^4$

$=\ln(x+4)-\ln x$

$\cancel{\int_1^2 \ln\frac{x+4}{x}\,dx}$

$\int_1^2 \ln(x+4)-\ln x\,dx$

$=[\ln(x+4)\cdot(x+4)-\ln x\cdot x]_1^2$

$=6\ln 6-2\ln 2-5\ln 5$

33. $\int_0^5 \frac{1}{\sqrt{x+y}}\,dy$

$=[2\sqrt{x+y}]_0^5$

$=2\sqrt{x+5}-2\sqrt{x}$

$2\int_0^4 \sqrt{x+5}-\sqrt{x}\,dx$

$=2\cdot[\frac{2\sqrt{x+5}\cdot(x+5)-2x\sqrt{x}}{3}]_0^4$

$=2\cdot(\frac{2\times3\times9-4}{3}-\frac{2\sqrt{5}\cdot5}{3})$

$=\frac{100}{3}-\frac{20\sqrt{5}}{3}$

$35.\ \int_1^3 \frac{\ln xy}{y}\,dy$

$= \left[\frac{(\ln x + \ln y)^2}{2}\right]_1^3$

$= \frac{(\ln 3x)^2}{2} - \frac{(\ln x)^2}{2}$

$\int_1^2 \left(\frac{(\ln 3x)^2}{2} - \frac{(\ln x)^2}{2}\right) dx$

$= \left[\frac{x\cdot(\ln 3+\ln x)^2 - x(\ln x)^2}{2} - \ln 3\cdot x\right]_1^2$

$= (\ln 3)^2 + (\ln 2)^2 + 2\ln 3\ln 2 - (\ln 2)^2 - 2\ln 3$

$+ \ln 3 - \frac{(\ln 3)^2}{2}$

$= \frac{(\ln 3)^2}{2} - \ln 3 + 2\ln 3\ln 2$

$37.\ \int_1^3 \int_{-2}^4 \frac{x}{y}\,dx\,dy$

$\left[\frac{x^2}{2y}\right]_{-2}^4$

$= \frac{8}{y} - \frac{2}{y} = \frac{6}{y}$

$\int_1^3 \frac{6}{y}\,dy$

$= [6\ln y]_1^3$

$= 6\ln 3$

$41.\ \int_0^{\frac{\pi}{4}} \int_0^2 e^x \sin y\,dx\,dy$

$[e^x \sin y]_0^2$

$= (e^2-1)\sin y$

$\int_0^{\frac{\pi}{4}} (e^2-1)\sin y\,dy$

$= \left[-\cos y\cdot(e^2-1)\right]_0^{\frac{\pi}{4}}$

$= \left(-\frac{\sqrt{2}}{2} + \frac{2}{2}\right)\cdot(e^2-1)$

$= \frac{2-\sqrt{2}}{2}\cdot(e^2-1)$

15.2

$3.\ \int_0^1 \int_0^{1-x^2} xy\,dy\,dx$

$\left[\frac{xy^2}{2}\right]_0^{1-x^2}$

$= \frac{x^5-2x^3+x}{2}$

$\int_0^1 \frac{x^5-2x^3+x}{2}\,dx$

$= \left[\frac{x^6}{12} - \frac{x^4}{4} + \frac{x^2}{4}\right]_0^1$

$= \frac{1}{12} - \frac{1}{4} + \frac{1}{4} = \frac{1}{12}$

$5.\ A.\ \int_0^4 \int_0^{2-\frac{1}{2}x} x^2 y\,dy\,dx$

$\left[\frac{x^2y^2}{2}\right]_0^{2-\frac{1}{2}x} = \frac{\frac{1}{4}x^4 - x^3 + 4x^2}{2}$

$= \frac{1}{8}x^4 - \frac{1}{2}x^3 + 2x^2$

$\int_0^4 \left(\frac{1}{8}x^4 - \frac{1}{2}x^3 + 2x^2\right) dx$

$= \left[\frac{1}{40}x^5 - \frac{1}{8}x^4 + \frac{2}{3}x^3\right]_0^4$

$= \frac{1024}{40} - \frac{256}{8} + \frac{128}{3}$

$= 32 + \frac{385+640}{15}$

$= 237$

6. $\int_0^4 \int_{2-\frac{1}{2}x}^{2} x^2 y \, dy \, dx$

$\left[\frac{x^2y^2}{2}\right]_{2-\frac{1}{2}x}^{2}$

$= 2x^2 - \frac{1}{8}x^4 - x^3 + 2x^2$

$= 4x^2 - \frac{1}{8}x^4 - x^3$

$\int_0^4 (4x^2 - \frac{1}{8}x^4 - x^3) dx$

$= \left[\frac{4}{3}x^3 - \frac{1}{40}x^5 - \frac{x^4}{4}\right]_0^4$

$= \frac{256}{3} - \frac{128}{5} - 54$

$= \frac{86}{15}$

11. $2\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x} \, dy \, dx$

$\left[\frac{y^2}{2x}\right]_0^{\sqrt{4-x^2}}$

$= \frac{4-x^2}{2x}$

$\int_0^2 \frac{4-x^2}{x} dx$

$= \left[4\ln x - \frac{x^2}{2}\right]_0^2$

$= 4\ln 2 - 2$

19. ~~$\int_0^1 \int_1^{e^{x^2}} x \, dy \, dx$~~

~~$= \int_0^1 (e^{x^2} - 1) dx$~~

$\int_1^{e^{x^2}} \int_0^1 x \, dx \, dy$

$= \int_1^{e^{x^2}} \frac{1}{2} dy$

$= \frac{e^{x^2}-1}{2}$

21. $\int_0^1 \int_0^{\sqrt{x}} 2xy \, dy \, dx$

$[xy^2]_0^{\sqrt{x}} = x^2$

$\int_0^1 x^2 dx$

$= \left[\frac{x^3}{3}\right]_0^1$

$= \frac{1}{3}$

25. $D = \{(x,y) \mid 0 \le x \le 4, x \le y \le 4\}$

$D = \{(x,y) \mid 0 \le y \le 4, 0 \le x \le y\}$

$\int_0^4 \int_0^x f(x,y) \, dy \, dx$

31. $\int_1^e \int_{(\ln y)^2}^{\ln y} (\ln y)^{-1} \, dx \, dy$

$(\ln y)^{-1} \cdot (\ln y - (\ln y)^2)$

$= 1 - \ln y$

$\int_1^e (1 - \ln y) dy$

$= [2y - y\ln y]_1^e$

$= 2e - e - 2$

$= e - 2$

33. $\int_0^1 \int_0^x \frac{\sin x}{x} \, dy \, dx$

$= \int_0^1 \sin x \, dx$

$= [\cos x]_0^1$

$= 1 - \cos 1$

35. $\int_0^1\int_0^y xe^{y^3}\,dx\,dy$

$= \left[\frac{e^{y^3}\cdot x^2}{2}\right]_0^y$

$= \frac{y^2e^{y^3}}{2}$

$\int_0^1 \frac{y^2e^{y^3}}{2}\,dy$

$= \left[\frac{e^{y^3}}{6}\right]_0^1$

$= \frac{e-1}{6}$

41. $\begin{cases} 8-x^2-y^2=0 \\ x=y \end{cases}$

$x=y=2.$

$\cancel{\sqrt{2^2-y^2}}$

$2\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}(4-x^2-y^2)\,dy\,dx.$

37. $\int_0^2\int_0^2 e^{x+y}\,dx\,dy - \int_0^1\int_0^1 e^{x+y}\,dx\,dy$

$= \left[e^{y+2}-e^y\right]_0^2 - \left[e^{y+1}-e^y\right]_0^1$

$= e^4-2e^2+1-e^2+2e-1$

$= e^4-3e^2+2e$

43. $\int_0^2\int_0^{2y}\frac{\sin y}{y}\,dx\,dy \cancel{dy\,dx} - \int_0^2\int_0^{y}\frac{\sin y}{y}\,dx\,dy \cancel{dy\,dx}$

$= \int_0^2 2\sin y\,dy - \int_0^2 \sin y\,dy$

$= \int_0^2 \sin y\,dy$

$= \left[-\cos y\right]_0^2$

$= 1-\cos 2.$