15.1

9) $\int_0^3 \int_0^5 (15-3x)\,dx\,dy$

$\int_0^5 15-3x$

$\left[15x - \frac{3x^2}{2}\right]_0^5$

$\int_0^3 15(5) - \frac{3}{2}(25)\;dy$

$\left(\left[15(5) - \frac{3}{2}(25)\right] y\right)_0^3$

$\left(75 - \frac{75}{2}\right)3$

$\frac{75}{2} \times 3 = \frac{225}{2}$

15) $\int_0^5 \int_{-4}^{4} x^3 \, dx \, dy$

$$\left[\frac{x^4}{4}\right]_{-4}^{4}$$

$$\frac{4^4}{4} - \frac{4^4}{4} = 0$$

21) $\int_4^9 \int_{-3}^{8} 1 \, dx \, dy$

$$1(5)(11)$$

$$= 55$$

23) $\int_{-1}^{1} \int_0^{\pi} x^2 \sin y \, dy \, dx$

$$\int_0^{\pi} x^2 \sin y \, dy$$

$$\left[-x^2 \cos y\right]_0^{\pi}$$

$$x^2 + x^2$$
$$= 2x^2$$

$$\int_{-1}^{1} 2x^2$$

$$\left[\frac{2}{3} x^3\right]_{-1}^{1}$$

$$\frac{2}{3} + \frac{2}{3}$$
$$= \frac{4}{3}$$

$$\int \frac{1}{a+x}$$

31) $\int_1^2 \int_0^4 \frac{dy\,dx}{x+y}$

$$x + y = u$$
$$du = a$$

$$\int_0^4 \frac{du}{u}$$

$$\left[\ln(x+y)\right]_0^4$$

$$\int_1^2 \ln(x+4) - \ln x$$

32) $\displaystyle\int_0^4 \int_0^5 \frac{dy\,dx}{\sqrt{x+y}}$

$$\left[2\sqrt{x+y}\right]_0^5$$

$$\int_0^4 2\sqrt{5+x} - 2\sqrt{x}$$

$$\left[\frac{4}{3}(5+x)^{3/2} - \frac{4}{3}(x)^{3/2}\right]_0^4$$

$$\frac{4}{3}(9^{3/2}) - \frac{4}{3}(4)^{3/2}$$

$$= \boxed{36 - \frac{32}{3}}$$

35) $\int_1^2 \int_1^3 \frac{\ln(xy)\,dy\,dx}{y}$

$\ln(xy) = u$

$\frac{1 \cdot \cancel{x}}{\cancel{x}y} = du$

$$\int_1^3 u\,du$$

$$\left[\frac{u^2}{2}\right]_1^3$$

$$\left[\frac{\ln(xy)^2}{2}\right]_1^3$$

$$\int_1^2 \left(\frac{\ln(3x)^2}{2} - \frac{\ln x^2}{2}\right) dx$$

3f) $\int_{1}^{3}\int_{-2}^{4} \frac{x}{y}\, dx\, dy$

$$\left[\frac{x^2}{2y}\right]_{-2}^{4}$$

$$\frac{8}{y} - \frac{2}{y} = \frac{6}{y}$$

$$\left[6 \ln y\right]_{1}^{3}$$

$$\underline{\underline{6 \ln 3}}$$

41) $\int_0^{\pi/4} \int_0^2 \sin y \, e^x \, dx \, dy$

$$\left[\sin y \, e^x \right]_0^2$$

$$\int_0^{\pi/4} (e^2 - 1) \sin y \, dy$$

$$\left[-(e^2 - 1) \cos y \right]_0^{\pi/4}$$

$$(e^2 - 1) \left(\cos \frac{\pi}{4} - 1 \right)$$

15.2

3, 5, 6, 7 → 15.2
Inverse the region
15.1 plane handout

19 check

31

sketch the domain?

3) $x = 0, 1$

$y = 0, 1$

$y = 1 - x^2$

11) $\int_0^2 \int_0^2 \frac{y}{x} \, dy \, dx$

$\left[\frac{y^2}{2x}\right]_0^2$

$\int_0^2 \frac{2}{x}$

$\left[2 \ln(x)\right]_0^2$

$2 \ln(2)$

19) $$\int_0^1 \int_1^{e^{x^2}} x \, dy \, dx$$

$$\left[xy\right]_1^{e^{x^2}}$$

$$\int_0^1 xe^{x^2} - x \, dx$$

$$\int_0^1 e^u \frac{du}{2} - \int_0^1 x$$

$x^2 = u$

$2x\,dx = du$

$x\,dx = \frac{du}{2}$

$$\left[\frac{e^u}{2}\right]_0^2 - \left[\frac{x^2}{2}\right]_0^1$$

$$\frac{e^2}{2} - \frac{1}{2}$$

$$= \frac{e^2-1}{2}$$

21) $f(x,y) = 2xy$ $\qquad$ $x = y,\ x = y^2$

$$\int_0^1 \int_{y^2}^{y} 2xy \, dx\, dy$$

$x = x$

$y = y^2$

$y^2 - y = 0$

$y = 0,\ y = 1$

$$\left[y x^2 \right]_{y^2}^{y}$$

$$\int_0^1 y^3 - y^5 \, dy$$

$$\left[\frac{y^4}{4} - \frac{y^6}{6} \right]_0^1$$

$$\frac{1}{4} - \frac{1}{6}$$

$$\frac{2}{24} = \frac{1}{12}$$

25) $\int_0^4 \int_x^4 f(x,y)\, dy\, dx$

$y = 4$ right most
$y = x \rightarrow$ left most

$$= \int_0^4 \int_0^y f(x,y)\, dx\, dy$$

31) $\int \frac{1}{\ln(y)}$

$y = e^x$
$y = e^{\sqrt{x}}$

$\int_0^1 \int_{e^x}^{e^{\sqrt{x}}} \frac{1}{\ln(y)}\, dy\, dx$

$e^x = e^{\sqrt{x}}$
$x^2 - x = 0$
$x(x-1) = 0$
$x = 0,\ x = 1$

$e^{0.5} = 1.648$
$e^{\sqrt{0.5}} = 2.028$

Or

$$\int_0^1 \int_{\ln(y)}^{\ln(y)^2} \frac{1}{\ln(y)} \, dx\,dy$$

$x = \ln y$

$x = (\ln y)^2$

$$\left[\frac{x}{\ln(y)} \right]_{(\ln(y))}^{\ln(y)^2}$$

$$\frac{1}{\ln(y)} - 1$$

$$\int_0^1 \frac{-\ln(y) + 1}{\ln(y)}$$

$\ln(y) - (\ln(y))^2 = 0$

$\ln(y)(\ln(y) - 1) = 0$

$\ln(y) =$

$e^y = e^0$

$e^y = e^1$

$y = 0$

$y = 1$

33) $\int_0^1 \int_y^1 \frac{\sin(x)}{x} \, dx \, dy \qquad x = y$

$$\int_0^1 \int_0^x \frac{\sin(x)}{x} \, dy \, dx$$

$$\left[\frac{\sin(x)}{x} y \right]_0^x$$

$$\int_0^1 \sin(x)$$

$$\left[-\cos(x) \right]_0^1$$

$$1 - \cos(1)$$

35) $$\int_0^1 \int_x^1 x e^{y^3} \, dy \, dx$$

$$\int_0^1 \int_0^y x e^{y^3} \, dx \, dy$$

$$\left[\frac{x^2}{2} e^{y^3} \right]_0^y$$

$$\int_0^1 \frac{y^2}{2} e^{y^3} \, dy$$

$$y^3 = u$$

$$3y^2 = du$$

$$y^2 = \frac{du}{3}$$

$$\frac{1}{6} \int_0^1 e^u \, du$$

$$\frac{e}{6} - \frac{1}{6}$$

$$= \frac{e-1}{6}$$

37)

how to sketch domain

$$\int_0^2 \int_0^2 e^{x+y} \, dx \, dy$$

$$\left[e^{x+y} \right]_0^2$$

$$\int_0^2 \left(e^{2+y} - e^{y} \right) dy$$

$$\left[e^{2+y} - e^{y} \right]_0^2$$

$$e^4 - e^2 - e^2 - 1$$

$$= e^4 - 2e^2 - 1$$

43) $y = x$

$$\int_{x/2}^{x} \frac{\sin(y)}{y}\, dy$$

$y = \frac{x}{2}$

$x - \frac{x}{2} = 0$

$2x - x = 0$

$x(2-1) = 0$

$x = 0$

49)