10/23/20 . 15.1 Multivariable Calculus HW

15.1 : 9, 15, 21, 23, 25, 31, 33, 35, 37, 41

9) $\iint (15-3x)\,dA$ $R=[0,5]\times[0,3]$.

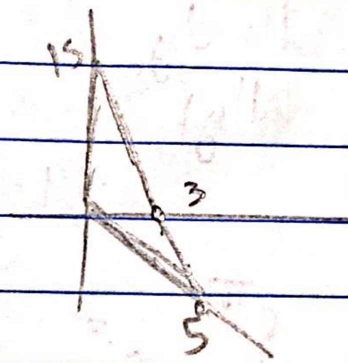
$\int_0^5\int_0^3 (15-3x)\,dy\,dx$

$15y-3xy\Big|_0^3 \to 45-9x$

$\int_0^5 45-9x\,dx$

$45x-\frac{9}{2}x^2\Big|_0^5$

$=112.5$




15) $\iint x^3\,dA$ $R=[-4,4]\times[0,5]$

$\int_{-4}^{4}\int_0^5 x^3\,dy\,dx$

$x^3 y\Big|_0^5$

$\int_{-4}^{4} 125x^3\,dx$

$=\frac{125}{4}x^4\Big|_{-4}^{4}$

$=0$

21) $\int_4^9 \int_{-3}^8 1\, dx\, dy$

$x\Big|_{-3}^{8}$

$8-(-3)=11$

$\int_4^9 11\, dy$

$11y\Big|_4^9$

$99-44=55$

23) $\int_{-1}^{1} \int_0^{\pi} x^2 \sin y \, dy \cdot dx$

$\int_{-1}^{1} x^2\, dx \cdot \int_0^{\pi} \sin y$

$\frac{x^3}{3}\Big|_{-1}^{1} \cdot -\cos y\Big|_0^{\pi}$

$\frac{2}{3}(+1\cdot 1)=0.$

25) $\int_2^6 \int_1^4 x^2\, dx\, dy$

$\frac{x^3}{3}\Big|_1^4 = 21$

$\int_2^6 21\, dy = 21y\Big|_2^6 = 21(4) = 88$

$\ln x$ $\frac{1}{x+y}$ le

31) $\int_1^2 \int_0^4 \frac{dy\,dx}{x+y}$

$\int_1^2 \ln(x+y)\Big|_0^4$

$\int_1^2 \ln(x+4) - \ln(x)\ dx.$

$= (x+4)\ln(x+4) - x - (x\ln x - x)$

$(x+4)\ln(x+4) - x\ln x.$

33) $\int_0^4 \int_0^5 \frac{dy\ dx}{\sqrt{x+y}}$

$2\sqrt{x+y}\Big|_0^5 = 2\sqrt{x+5} - 2\sqrt{x}$

$\int_0^4 (2\sqrt{x+5} - 2\sqrt{x})dx$

$= \frac{4}{3}(x+5)^{\frac{3}{2}} - \frac{4}{3}x^{\frac{3}{2}}\Big|_0^4$

$2.909 - 9.899 = -2.49.$

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35) $\int_1^2 \int_1^3 \frac{\ln(xy)}{y}\, dy\, dx.$

$$\int_1^2 \left. \frac{\ln^2(yy)}{2} \right|_1^3$$

$$\frac{1}{2} \int_1^2 (\ln^2 3x - \ln^2 x)\, dx.$$

$$\frac{1}{6} (6x - 2\ln(3x) + \ln(3x)^2) - \frac{1}{2}(2x - 2x\ln x + \ln^2 x$$

$$= 1.22$$

37) $\iint \frac{x}{y}\, dA \quad R = [-2, 4] \times [1, 3].$

$$\int_{-2}^4 \int_1^3 \frac{x}{y}\, dy\, dx.$$

$$\int_{-2}^4 x\ln(3)\, dx$$

$$\ln 3 \left( \left. \frac{x^2}{2} \right|_{-2}^4 \right) = \ln 3(6)$$

41) $\iint e^x \sin y\, dA, \quad R = [0, 2] \times [0, \frac{\pi}{4}]$

$$\int_0^2 \int_0^{\frac{\pi}{4}} e^x \sin y\, dy\, dx.$$

$$\int_0^2 e^x\, dx \cdot \int_0^{\frac{\pi}{4}} \sin y\, dy$$

$$= (e^2 - 1)\left(1 - \frac{1}{\sqrt{2}}\right)$$

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10/24/20 15.2 → Double Integrals over General Region

15.2 → 3, 5, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 43, 49

3) $y = 1 - x^2$

$0 \le y \le 1 - x^2$ → Vertical

Horizontal: $0 \le x \le \sqrt{1-y}$.

$$\int_0^1 \int_0^{1-x^2} xy \, dy \, dx = \int_0^1 \frac{xy^2}{2} \Big|_0^{1-x^2} dx = \frac{1}{12}$$

5) $\int_0^4 \int_0^2 x^2 y \, dy \, dx$

$$\frac{x^3}{3}\Big|_0^4 \cdot \frac{y^2}{2}\Big|_0^2$$

$$\frac{64}{3} \cdot 2 = \frac{128}{3}$$

6) $\int_0^4 \int_0^2 x^2 y \, dy \, dx$

$$\frac{x^3}{3}\Big|_0^4 \cdot \frac{y^2}{2}\Big|_0^2 = \frac{128}{3}$$

Similar to previous problem

7) $\int_0^2 \int_y^4 x^2 y \; dx \, dy$

$\int_0^2 \left( \frac{x^3 y}{3} \right) \Big|_y^4 \, dy$

$\int_0^2 \frac{y}{3} (64 - y^3) \, dy$

$\int_0^2 \frac{64y}{3} - \frac{y^4}{3} \, dy$

$\frac{32y^3}{3} - \frac{y^5}{15} \Big|_0^2 = 40.5$

11) $\iint \frac{y}{x} \, dA$

$1 \le x \le 2$

$0 \le y \le \sqrt{4 - x^2}$

$\int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x} \, dy \, dx.$

$\int_1^2 \frac{1}{x} \left( \frac{y^2}{2} \right) \Big|_0^{\sqrt{4-x^2}} dx.$

$\int_1^2 \frac{1}{2} \left( \frac{1}{x} \right) (4 - x^2) \, dx$

$\frac{1}{2} \int_1^2 \frac{4}{x} - x ) dx$

$= \frac{1}{2} (4 \ln(2) - 2 + \frac{1}{2} )$

19) $\int_0^1 \int_1^{e^{x^2}} x \, dy \cdot dx$

$$\frac{x^2}{2}\Big|_1^{e^{x^2}}$$

$$\frac{(e^{x^y}) - e^{x^2}}{2}\Big|_0^1$$

$$\frac{e^1 - e^1}{2} - 0 = 0$$

21) $f(x,y) = 2xy$ , $x = y$, $x = y^2$

$y \to 0$ to $1$

$$\int_0^1 \int_y^{y^2} 2xy \, dx \, dy$$

$$x^2 y\Big|_y^{y^2}$$

$$\int_0^1 y^5 - y^3 \, dy$$

$$\frac{y^6}{6} - \frac{y^4}{4}\Big|_0^1 = \frac{1}{6} - \frac{1}{4}$$

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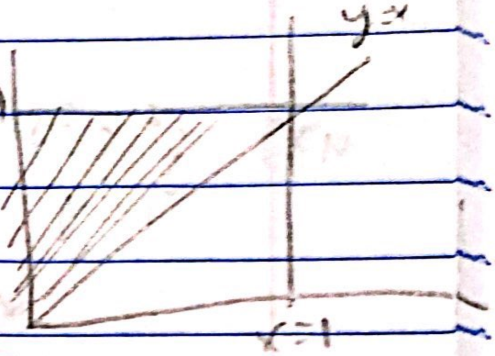
35) $\int_0^1 \int_{y=x}^1 x e^{y^3}\, dy\, dx$




$\int_0^1 \int_0^y x e^{y^3}\, dx\, dy$

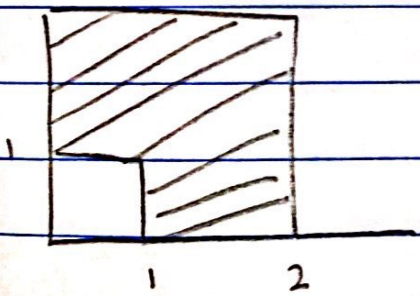
$\int_0^1 \frac{x^2}{2} e^{y^3} \Big|_0^y \, dy$

$\int_0^1 \frac{y^2}{2} \cdot e^{y^3}\, dy$ let $y^3 = u$

$du = 3y^2\, dy$

$\frac{1}{6} \int e^u\, du$

$= \frac{1}{6} e^{y^3} \Big|_0^1 = \frac{e-1}{6}$

37) 


$\int_0^2\int_0^2 e^{x+y}\, dx\, dy - \int_0^1\int_0^1 e^{x+y}\, dx\, dy$

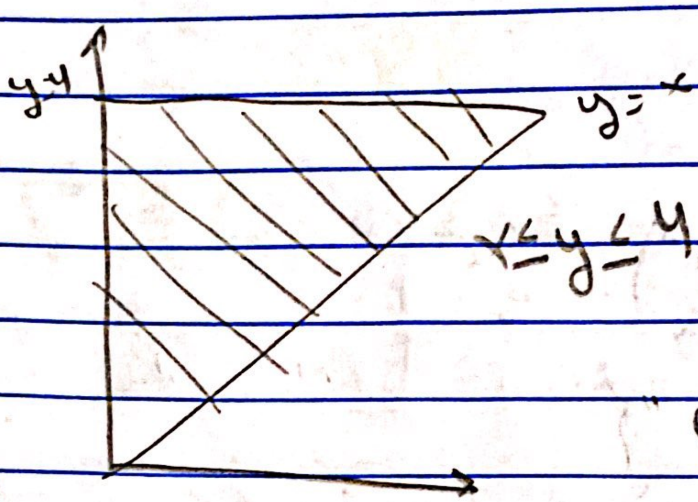
$\int_0^2 (e^{x+y})_0^2\, dy$

$\hookrightarrow e^{y+1} - e^{y} \Big|_0^2 - \left(e^{y+2} - e^{y} \Big|_0^1\right)$

$= 37.878.$

$=$

25) $\int_0^4\int_x^4 f(x,y)\,dy\,dx.$




$\int_0^4\int_0^y f(x,y)\,dy\,dx$

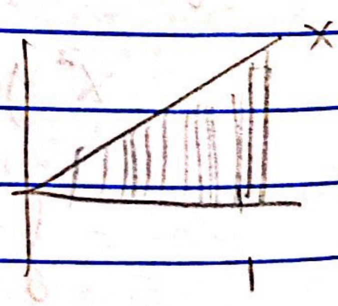
31) $f(x,y)=\ln(y)^{-1}$

$y=e^{x}, \quad y=e^{\sqrt{x}}.$

$x=0, \quad x=1.$

$\int_1^e\int_{\ln^2 y}^{\ln y} (\ln y)^{-1}\,dx\,dy = 0.718.$

33) $\int_0^1\int_y^1 \frac{\sin x}{x}\,dx\,dy.$




$\int_0^1\int_x^1 \frac{\sin x}{x}\,dx\,dy.$

$=0.460$

43) $\int_1^2 \int_y^{2y} \frac{\sin y}{y} \, dx \, dy$

$\frac{x \sin y}{y} \Big|_y^{2y}$

$\int_1^2 (2\sin y - \sin y) \, dy$
$= 0.956$

49) $z = x^2 + y^2$
$z = 8 - x^2 - y^2$

$x^2 + y^2 = 8 - x^2 - y^2$
$2x^2 + 2y^2 = 8$
$x^2 + y^2 = 4$
$y = \pm\sqrt{4 - x^2}$

$\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left( (8 - x^2 - y^2) - (x^2 + y^2) \right) dy \, dx$