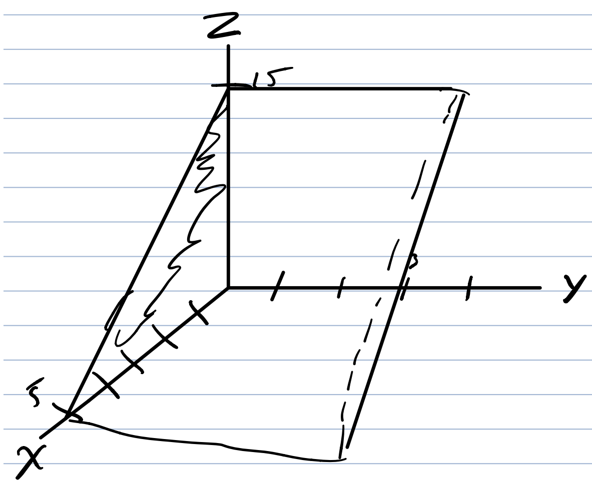
15.1 (9, 15, 21, 23, 25, 31, 33, 35, 37, 41)

9) Evaluate $\iint_R (15-3x)\,dA$ where $R=[0,5]\times[0,3]$ and sketch the corresponding solid region

$$\int_0^3\int_0^5 (15-3x)\,dx\,dy$$

① $\int_0^5 15-3x\,dx = 15x - \frac{3x^2}{2}\Big|_0^5 = 75 - \frac{75}{2} = \frac{75}{2} = 37.5$

② $\int_0^3 37.5\,dy = 37.5\left(y\Big|_0^3\right) = 37.5(3) = 112.5$



15) $\iint_R x^3\,dA,\ R=[-4,4]\times[0,5]$

$$\int_0^5\int_{-4}^4 x^3\,dx\,dy = 0$$

① $\int_{-4}^4 x^3\,dx = \frac{x^4}{4}\Big|_{-4}^4 = 0$

21) $\int_4^9\int_{-3}^8 dx\,dy$

① $\int_{-3}^8 dx = x\Big|_{-3}^8 = 11$ ② $\int_4^9 11\,dy = 11\left(y\Big|_4^9\right) = 55$

23) $\int_{-1}^1\int_0^\pi x^2\sin(y)\,dy\,dx$

① $\int^\pi x^2\sin(y)\,dy = x^2\left(-\cos(y)\Big|^{\pi}\right) = x^2\left[1 - -(1)\right] = 2x^2$

$\int_0$

$$\int_{-1}^{1} 2x^2\,dx = 2\int_0^1 2x^2 = 4\left.\frac{x^3}{3}\right|_0^1 = \frac{4}{3}$$

25) $\int_2^6 \int_1^4 x^2\,dx\,dy$

① $\int_1^4 x^2\,dx = \left.\frac{x^3}{3}\right|_1^4 = \frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 21$

② $\int_2^6 21\,dy = 21\left(y\Big|_2^6\right) = 84$

31) $\int_1^2 \int_0^4 \frac{dy\,dx}{x+y} = \int_1^2 \int_0^4 (x+y)^{-1}\,dy\,dx$

① $\int_0^4 (x+y)^{-1}\,dy$ $\quad u = x+y$, $du = dy$ $\Rightarrow \int_0^4 (u)^{-1}\,du = \ln(u)\Big|_0^4 = \ln(x+y)\Big|_0^4 = \ln(x+4) - \ln(x)$

$\ln\left(\frac{x+4}{x}\right) = \ln(1 + 4/x)$

② $\int_1^2 \ln(1 + 4/x)\,dx = x\ln(1+4/x) + 4\int_1^2 \frac{x}{x^2+4x}\,dx$

$f(x) = \ln(1 + 4/x)$

$f'(x) = \frac{-4x^{-2}}{[1 + 4/x]} = \frac{-4}{x^2[1+4/x]} = \frac{-4}{x^2+4x}dx$ $\quad g(x) = x$, $g'(x) = dx$

$\int_1^2 \frac{dx}{x+4} \Rightarrow u = x+4$, $du = dx$ $\quad \int_5^6 \frac{du}{u} = \ln(u)\Big|_5^6$

$2\ln(1+4/2) - \ln(1+4) + 4(\ln(6) - \ln(5))$

$2\ln(3) - \ln(5) + 4\ln(6/5)$

$\ln(9) - \ln(5) + \ln\left(\left(\frac{6}{5}\right)^4\right)$

$\ln\left(\frac{9}{5}\right) + \ln\left(\left(\frac{6}{5}\right)^4\right) = \ln\left(\frac{9}{5}\cdot\left(\frac{6}{5}\right)^4\right) \approx 1.317$

33) $\int_0^4 \int_0^5 \frac{dy\,dx}{\sqrt{x+y}}$

$\int_0^5 \frac{dy}{\sqrt{x+y}} = \int_0^5 \left(\frac{1}{x+y}\right)^{1/2} dy \Rightarrow u = x+y$, $du = dy \Rightarrow \int_0^5 \left(\frac{1}{u}\right)^{1/2} du = \int_0^5 u^{-1/2}\,du = \frac{1}{2}u^{1/2}\Big|_{y=0}^{y=5}$

$$\frac{1}{2}(x+y)^{1/2}\Big|_0^5 = \frac{1}{2}(x+5)^{1/2} - \frac{1}{2}x^{1/2}$$

$$\frac{1}{2}\int_0^4\left((x+5)^{1/2} - x^{1/2}\right)dx = \frac{1}{2}\left[\int_0^4 (x+5)^{1/2}dx - \int_0^4 x^{1/2}dx\right] =$$

$$\int_0^4 (x+5)^{1/2}dx = \int_5^9 u^{1/2}du = \frac{2}{3}u^{3/2}\Big|_5^9$$

$u = x+5 \to [5,9]$
$du = dx$

$$\frac{1}{2}\left[\left(\frac{2}{3}u^{3/2}\Big|_5^9\right) - \left(\frac{3}{2}x^{3/2}\Big|_0^4\right)\right] = \frac{1}{2}\left[\frac{2}{3}\left(9^{1.5} - 5^{1.5}\right) - \left(\frac{3}{2}\cdot 4^{1.5}\right)\right] \approx 10.426$$

35) $\int_1^2\int_1^3 \frac{\ln(xy)}{y}\,dy\,dx$

$$\int_1^3 \frac{\ln(xy)dy}{y} = \ln(xy)\ln(y) - \int_1^3 \frac{\ln(y)}{y}dy = \boxed{\ln(xy)\ln(y) - \left(\frac{[\ln(y)]^2}{2}\right)\Big|_1^3}$$

$f(x) = \ln(xy)$ $\quad$ $g(x) = \ln(y)$
$f'(x) = \frac{x}{xy} = \frac{1}{y}$ $\quad$ $g'(x) = \frac{1}{y}dy$

$$\ln(3x)\ln(3) - \frac{(\ln(3))^2}{2}$$

$$\int_1^3 \frac{\ln(y)dy}{y} = \frac{[\ln(y)]^2}{2} - \int_1^3 \frac{\ln(y)}{y}dy$$

$u = \ln(y)$ $\quad$ $v = \ln(y)$
$du = \frac{1}{y}dy$ $\quad$ $dv = 1/y\,dy$

$$\int_1^2\left(\ln(3x)\ln(3) - \frac{\ln^2(3)}{2}\right)dx = \int_1^2 \ln(3)\ln(3x)dx - \int_1^2 \frac{\ln^2(3)}{2}dx$$

$$\int \ln(3x)dx = x\ln(3x) - \int dx = x\ln(3x) - x$$

$u = \ln(3x)$ $\quad$ $v = x$
$du = \frac{3}{3x}dx = \frac{1}{x}dx$ $\quad$ $dv = dx$

$$\ln(3)\left(x\ln(3x) - x\Big|_1^2\right) - \frac{\ln^2(3)}{2}\left(x\Big|_1^2\right) = \ln(3)\Big((2\ln(6) - 2) - (\ln(3) - 1)\Big) - \frac{\ln^2(3)}{2}$$

$\ln(3)(2\ln(6)-2-\ln(3)-1) - \frac{\ln^2(3)}{2} \approx 1.028$

37) $\iint_R \frac{x}{y}\,dA$, $R=[-2,4]\times[1,3]$

$\int_1^3 \int_{-2}^4 \frac{x}{y}\,dx\,dy$

$\int_{-2}^4 \frac{x}{y}\,dx = \frac{1}{y}\int x\,dx = \frac{1}{y}\frac{x^2}{2}\Big|_{-2}^4 = \frac{1}{y}\left(\frac{16}{2}-\frac{4}{2}\right) = \frac{6}{y}$

$\int_1^3 \frac{6}{y}\,dy = 6\int\frac{dy}{y} = 6\ln(y)\Big|_1^3 = 6\ln(3)$

49) $\iint_R e^x \sin y\,dA$, $R=[0,2]\times[0,\pi/4]$

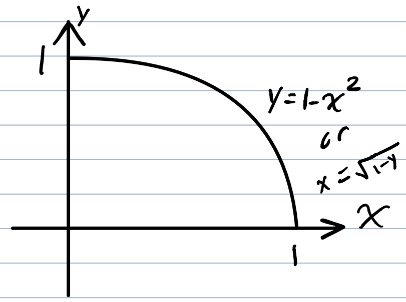
$\int_0^2 \int_0^{\pi/4} e^x \sin y\,dy\,dx$

$\int_0^{\pi/4} e^x \sin(y)\,dy = e^x\left(-\cos(y)\Big|_0^{\pi/4}\right) = e^x\left(-1-\cos(\pi/4)\right)$

$\int_0^2 e^x(-1-\cos(\pi/4))\,dx = (-1-\cos(\pi/4))\left[e^x\Big|_0^2\right] = (-1-\cos(\pi/4))(e^2-1)$

15.2 [3, 5, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 43, 49]

3)



$D_1 = \{(x,y),\ 0 \le x \le 1,\ 0 \le y \le 1-x^2\}$

$D_2 = \{(x,y),\ 0 \le y \le 1,\ 0 \le x \le \sqrt{1-y}\}$

$f(x,y) = xy$

① $\int_0^1 \int_0^{1-x^2} xy\ dy\,dx \Rightarrow {}^* \int_0^{1-x^2} xy\,dy = x \cdot \frac{y^2}{2}\Big|_0^{1-x^2} = \frac{x(1-x^2)^2}{2}$

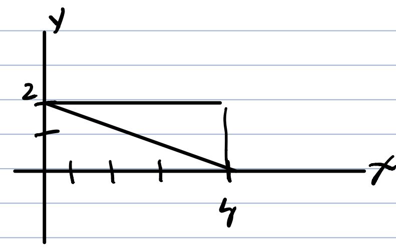
$\int_0^1 \frac{x(1-x^2)^2 dx}{2} \left\{ \begin{array}{l} u = 1-x^2 \to [1,0] \\ du = -2x\,dx \to \frac{du}{-2} = x\,dx \end{array} \right\} \frac{-1}{4}\int_1^0 u^2\,du = \frac{-1}{4} \cdot \frac{u^3}{3}\Big|_1^0 =$

$\frac{-1}{4} \cdot \frac{-1}{3} = \frac{1}{12}$

② $\int_0^1 \int_0^{\sqrt{1-y}} xy\,dx\,dy \Rightarrow \int_0^{\sqrt{1-y}} xy\,dx = y \cdot \frac{x^2}{2}\Big|_0^{\sqrt{1-y}} = \frac{y(1-y)}{2}$

$\int_0^1 \frac{y(1-y)}{2} dy = \int_0^1 \frac{y-y^2}{2} = \frac{1}{2} \cdot \left(\frac{y^2}{2} - \frac{y^3}{3}\Big|_0^1 \right) = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{12}$

5)



$f(x,y) = x^2 y$

$D1 = \{(x,y),\ 0 \le x \le 4,\ -\frac{x}{2}+2 \le y \le 2\}$

$D2 = \{(x,y),\ 0 \le y \le 2,\ 0 \le x \le 4-2y\}$

$P: (0,2)$ $m = \frac{0-2}{4} = \frac{-1}{2}$ $y - 0 = -1/2(x-4) = -x/2 + 2$

$Q: (4,0)$

$y = -x/2 + 2$

$y - 2 = -\frac{x}{2}$

$-2(y-2) = x$

$x = -2y + 4$

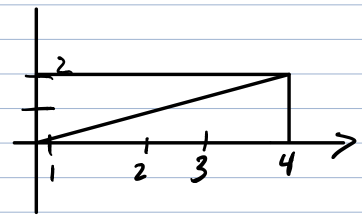
$x = 4 - 2y$

① $\int_0^4 \int_{-\frac{x}{2}+2}^{2} x^2 y\,dy\,dx \Rightarrow \int_{-\frac{x}{2}+2}^{2} x^2 y\,dy = x^2\left(\frac{y^2}{2}\Big|_{-\frac{x}{2}+2}^{2}\right) = x^2\left(\frac{2^2}{2} - \left(\frac{((-x/2)+2)^2}{2}\right)\right),$

$x^2\left(2-\left(\frac{(-x/2)^2-2x+4}{2}\right)\right) = x^2\left(2-\left(\frac{x^2}{8}-x+2\right)\right) = x^2\left(-\frac{x^2}{8}+x\right) =$

$-\frac{x^4}{8}+x^3 = x^3-\frac{x^4}{8} \Rightarrow \int_0^4 x^3-\frac{x^4}{8}\,dx = \frac{x^4}{4}-\frac{x^5}{40}\Big|_0^4$

$\frac{4^4}{4}-\frac{4^5}{40} = \frac{10(4^4)}{40}-\frac{4^5}{40} = 38.4$

6) 

$D=\{(x,y),\ 0\le x\le 4,\ 0\le y\le \tfrac{1}{2}x\}$

$f(x,y)=x^2y$

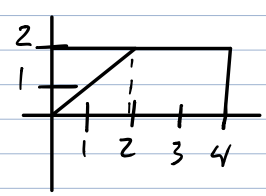
$\int_0^4\int_0^{1/2x} x^2y\,dy\,dx \Rightarrow \int_0^{1/2x} x^2y\,dy \Rightarrow x^2\cdot\frac{y^2}{2}\Big|_0^{1/2x}$

$= x^2\cdot\frac{1/4\,x^2}{2}$

$= \frac{x^4}{8}$

$\int_0^4 \frac{x^4}{8}\,dx = \frac{1}{8}\,\frac{x^5}{5}\Big|_0^4 = \frac{4^5}{40} = 25.6$

?

7) 

$f(x,y)=x^2y$

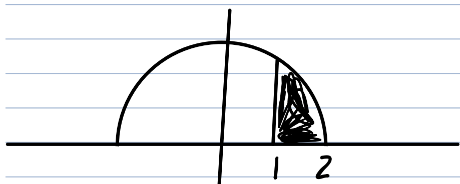
$D=\{(x,y)\mid 0\le y\le 2 \quad y\le x\le 4\}$

$\int_0^2\int_y^4 x^2y\,dx\,dy \Rightarrow \int_y^4 x^2y\,dx = \frac{x^3}{3}y\Big|_y^4 \Rightarrow \int_0^2 \frac{4^3y}{3}-\frac{y^4}{3} = \frac{4^3y^2}{6}-\frac{y^5}{15}\Big|_0^2$

$\frac{4^3\cdot 2^2}{6}-\frac{2^5}{15}\approx 40.5\overline{3}$

11) Evaluate $\iint_D \frac{y}{x}\,dA$ where D is the shaded part of the semicircle of radius 2 in

FIG 25



$D = \{(x,y) \mid 1 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2}\}$

$y = \sqrt{4-x^2}$

$\sqrt{y^2-4} = x$

$$\int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x}\, dy\, dx \Rightarrow \int_0^{\sqrt{4-x^2}} \frac{y}{x}\, dy = \frac{y^2}{2x}\Big|_0^{\sqrt{4-x^2}} \Rightarrow \int_1^2 \frac{4-x^2}{2x} dx =$$

$$\int_1^2 \frac{2}{x} dx - \int_1^2 \frac{x}{2} dx = 2\ln(x) - \frac{x^2}{4}\Big|_1^2 = \ln(x^2) - \frac{x^2}{4}\Big|_1^2 = \ln(4) - 1 - (0 - (1/4))$$

$$\ln(4) - 1 + 1/4$$

$$\ln(4) - \frac{3}{4}$$

19) $f(x,y) = x$; $D = \{(x,y) \mid 0 \le x \le 1,\ 1 \le y \le e^{x^2}\}$

$$\int_0^1 \int_1^{e^{x^2}} x\, dy\, dx \Rightarrow \int_1^{e^{x^2}} x\, dy = xy\Big|_1^{e^{x^2}} = xe^{x^2} - x \Rightarrow \int_0^1 xe^{x^2} - x\, dx = \int_0^1 xe^{x^2} dx - \int_0^1 x\, dx$$

$\{$Let $u = x^2 \to [0,1]$

$\{du = 2x\,dx \to \frac{du}{2} = x\,dx$

$$\int_0^1 xe^{x^2} dx - \int_0^1 x\, dx = \int_0^1 \frac{1}{2} e^u du - \int_0^1 x\, dx = \left(\frac{1}{2}\right)\left(e^u\Big|_0^1\right) - \left(\frac{x^2}{2}\Big|_0^1\right) = \frac{1}{2}(e-1) - \left(\frac{1}{2}\right)$$

$$= \frac{e}{2} - \frac{1}{2} - \frac{1}{2} = \frac{e}{2} - 1 = \frac{e-2}{2}$$

21) $F(x,y) = 2xy$ bounded by $x = y$ and $x = y^2$?

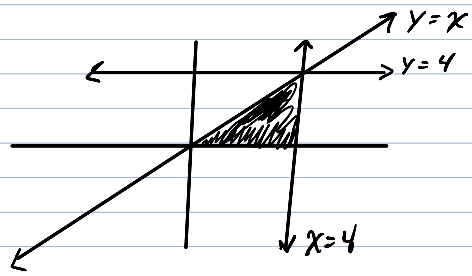
$D = \{(x,y) \mid 0 \le y \le 1 \quad y^2 \le x \le y\}$

$$\int_0^1 \int_{y^2}^{y} 2xy\, dx\, dy = x^2 \cdot y\Big|_{y^2}^{y} = -(y^5 - y^3) \Rightarrow \int_0^1 -(y^5 - y^3\, dy) = -\left(\frac{y^6}{6} - \frac{y^4}{4}\Big|_0^1\right) = -\left(\frac{1}{6} - \frac{1}{4}\right) = -\left(\frac{2}{12} - \frac{3}{12}\right)$$

$$= -\left(\frac{-1}{12}\right) = \frac{1}{12}$$

25) $$\int_0^4 \int_x^4 f(x,y)\, dy\, dx = \int_0^4 \int_0^y f(x,y)\, dx\, dy$$

$D = \{(x,y) \mid 0 \le x \le 4 \quad x \le y \le 4\}$ or



$D_2 = \{(x,y) \mid 0 \le y \le 4 \quad 0 \le x \le y\}$

?
31) $f(x,y)=(\ln(y))^{-1}$ over D: bounded by $y=e^x$ and $y=e^{\sqrt{x}}$

$e^x = e^{\sqrt{x}}$
$x = \sqrt{x}$
$0 \le x \le 1$
$1 \le y \le e$

$D = \{(x,y) \mid 1 \le y \le e \ , (\ln(y))^2 \le x \le \ln(y)\}$

$$\int_1^e \int_{(\ln(y))^2}^{\ln(y)} \ln(y)^{-1}\,dx\,dy \Rightarrow \int_{(\ln(y))^2}^{\ln(y)} (\ln(y))^{-1}dx = x\Big|_{(\ln(y))^2}^{\ln(y)} \cdot (\ln(y))^{-1}$$

$x=\ln(y)$
$x=(\ln(y))^2$

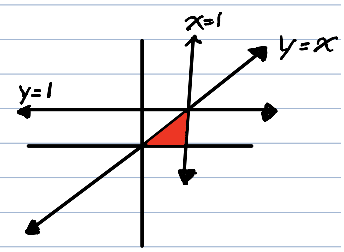
$$=(\ln(y)-(\ln(y))^2)\cdot(\ln(y))^{-1} = \frac{\ln(y)-(\ln(y))^2}{\ln(y)} \Rightarrow \int_1^e (1-\ln(y))dy = \int_1^e dy - \int_1^e \ln(y)dy$$

$$\int_1^e \ln(y)dy = y\ln(y) - \int_1^e dy = y\ln(y)-y\Big|_1^e \rightarrow y\Big|_1^e - \left(y\ln(y)-y\Big|_1^e\right) = (e-1)-([\cancel{e-e}]+(0+1)])$$

$f(x)=\ln(y)$ $\quad g(y)=y$
$f'(x)=\frac{dy}{y}$ $\quad g(x)=dy$

$e-1-1=e-2$

33) $\int_0^1\int_y^1 \frac{\sin(x)}{x}\,dx\,dy$ $\quad D=\{(x,y) \mid 0\le y\le 1 \ , \ y \le x \le 1\}$



$$\int_0^1\int_y^1 \frac{\sin(x)}{x}\,dx\,dy = \int_0^1\int_0^x \frac{\sin(x)}{x}\,dy\,dx$$

$D=\{(x,y) \mid 0\le y\le 1 \ , \ y \le x \le 1\}$

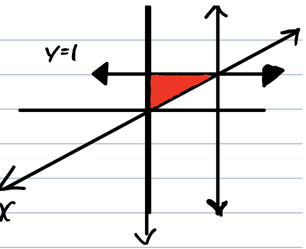
$S=\{(x,y) \mid 0\le x\le 1, \ 0 \le y \le x\}$

$$\int_0^1\int_0^x \frac{\sin(x)}{x}\,dy\,dx \Rightarrow \int_0^x \frac{\sin(x)}{x}dy = y\Big|_0^x \cdot \frac{\sin(x)}{x} = \int_0^1 \sin(x)\,dx = -\cos(x)\Big|_0^1$$

-1
$-\cos(1)-(-\cos(0))$
$1-\cos(1)$

$x=0$ $\quad x=1$

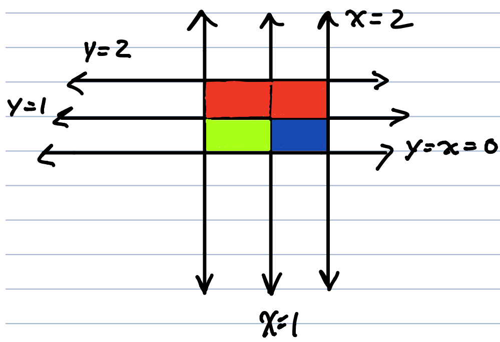
35) $\int_0^1\int_x^1 xe^{y^3}\,dy\,dx$ $\quad D=\{(x,y)\mid 0\le x\le 1,\ x\le y\le 1\}$
$S=\{(x,y)\mid 0\le y\le 1,\ 0\le x\le y\}$



$$\int_0^1\int_0^y xe^{y^3}dx\,dy=\int_0^y xe^{y^3}dx=\frac{x^2}{2}\Big|_0^y e^{y^3}=\frac{y^2}{2}\cdot e^{y^3}=\int_0^1\frac{y^2e^{y^3}}{2}dy \quad \left\{\begin{array}{l} u=y^3\to[0,1]\\ du=3y^2dy\end{array}\right.$$

$$\frac{1}{6}\int_0^1 e^u\,du=\frac{1}{6}\left(e^u\Big|_0^1\right)=\frac{1}{6}(e-1)=\frac{e}{6}-\frac{1}{6}=\frac{e-1}{6}$$

37) $D=\{(x,y): 0\le x\le 2,\ 0\le y\le 2,\ x>1 \text{ or } y>1\}$



$D=\{(x,y)\mid 0\le x\le 1,\ 0\le y\le 1\}$

$D=\{(x,y)\mid 0\le x\le 2,\ 1\le y\le 2\}$

$D=\{(x,y)\mid 1\le x\le 2,\ 0\le y\le 1\}$

$$\iint_D e^{x+y}dA=\int_0^2\int_1^2 e^{x+y}dy\,dx+\int_1^2\int_0^1 e^{x+y}dy\,dx=e^4\cancel{-e^3}\cancel{-e^2}+e\cancel{+e^3}-\cancel{2e^2}+e$$
$$=e^4-3e^2+2e$$

$$\int_0^2\int_1^2 e^{x+y}dy\,dx\Rightarrow\int_1^2 e^{x+y}dy\Rightarrow e^x\cdot e^y\Big|_1^2\Rightarrow\int_0^2\left[e^x(e^2-e)\right]dx\Rightarrow\int_0^2\left(e^{x+2}-e^{x+1}\right)dx$$

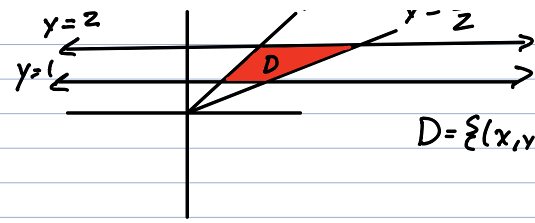
$$e^{x+2}-e^{x+1}\Big|_0^2\Rightarrow e^4-e^3-(e^2-e)\Rightarrow\boxed{e^4-e^3-e^2+e}$$

$$\int_1^2\int_0^1 e^{x+y}dy\,dx=\int_0^1 e^{x+y}dy=e^x e^y\Big|_0^1\Rightarrow\int_1^2\left[e^x(e-1)\right]dx=\int_1^2 e^{x+1}-e^x\,dx$$

$$e^{x+1}-e^x\Big|_1^2=e^3-e^2-(e^2-e)=e^3-e^2-e^2+e=\boxed{e^3-2e^2+e}$$

y=x y=x

43) $f(x,y)=\frac{\sin(y)}{y}$



$D=\{(x,y): 1\le y\le 2, \cancel{y}\le x\le 2y\}$

$\int_1^2\int_y^{2y}\frac{\sin(y)}{y}\,dx\,dy \Rightarrow \int_y^{2y}\frac{\sin(y)}{y}\,dx = x\Big|_y^{2y}\cdot\frac{\sin(y)}{y} = \cancel{y}\cdot\frac{\sin(y)}{\cancel{y}} = \int_1^2 \sin(y)\,dy$

$-\cos(y)\Big|_1^2 = -\cos(2)-(-\cos(1)) = -\cos(2)+\cos(1)$

49) Set up double integral that gives the volume bounded by $z=x^2+y^2$ and $8-x^2-y^2$

$8-x^2-y^2=x^2+y^2$

$8=2x^2+y^2(2)$

$4=x^2+y^2$

$y=\pm\sqrt{4-x^2}$

$y=0;\quad x=\pm 2$

$D=\{(x,y)\ -2\le x\le 2,\ -\sqrt{4-x^2}\le y\le\sqrt{4-x^2}\}$

$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\left((8-x^2-y^2)-(x^2+y^2)\right)dy\,dx$$