15.1

9. $\iint_R (15-3x)\,dA,\quad R=[0,5]\times[0,3]$

$\int_0^3\int_0^5 (15-3x)\,dx\,dy$

$\int_0^3 15x-\frac{3}{2}x^2\Big|_0^5$

$\int_0^3 \frac{75}{2}\,dy = \frac{225}{2}$

15. $\iint_R x^3\,dA.\quad R=[-4,4]\times[0,5]$

$\int_0^5\int_{-4}^4 x^3\,dx\,dy$

$\int_0^5 \frac{3}{4}x^4\Big|_{-4}^4\,dy$

$\int_0^5 0\,dy = 0$

21. $\int_4^9\int_{-3}^8 1\,dx\,dy$

$\int_4^9 x\Big|_{-3}^8\,dy$

$\int_4^9 11\,dy$

$= 11x\Big|_4^9 = 55$

23. $\int_{-1}^1\int_0^{\pi} x^2\sin \cancel{x}y\,dy\,dx$

$\int_{-1}^1 -x^2\cos y\Big|_0^{\pi}\,dx$

$\int_{-1}^1 2x^2\,dx$

$=\frac{4}{3}$

25. $\int_2^6\int_1^4 x^2\,dx\,dy$

$\int_2^6 \frac{1}{3}x^3\Big|_1^4\,dy$

$=\int_2^6 21\,dy$

$=84$

31. $\int_1^2\int_0^4 \frac{1}{x+y}\,\cancel{dx}\,dy\,dx$

$\int_1^2 \ln|y+x|\Big|_0^4\,dx$

$=\int_1^2 \ln(x+4)-\ln x\,dx$

$=(x+4)\ln(x+4)-x\ln x\Big|_1^2$

$=6\ln 6-5\ln 5-2\ln 2$

33. $\int_0^4\int_0^5 \frac{1}{\sqrt{x+y}}\,dy\,dx$

$=\int_0^4 2\sqrt{x+y}\Big|_0^5\,dx$

$=\int_0^4 2\sqrt{x+5}-2\sqrt{x}\,dx$

$=\frac{4(x+5)^{\frac{3}{2}}}{3}-\frac{4x^{\frac{3}{2}}}{5}\Big|_0^4$

$=-\frac{4}{3}(5\sqrt{5}-19)$

35. $\int_1^2\int_1^3 \frac{\ln(xy)}{y}\,dy\,dx$

$=\int_1^2 \frac{\ln^2(xy)}{2}\Big|_1^3\,dx$

$=\int_1^2 \frac{\ln^2(3x)-\ln^2 x}{2}\,dx$

$=\frac{x(\ln^2(3x)-2\ln(3x)+2)-x(\ln^2 x-2\ln x+2)}{2}\Big|_1^2$

$=\frac{2\ln^2 6-4\ln 6-\ln^2(3)+2\ln(3)-2^2\ln 2+4\ln 2}{2}$

37. $\iint_R \frac{x}{y}\,dA,\ R=[-2,4]\times[1,3]$

$\int_1^3\int_{-2}^4 \frac{x}{y}\,dx\,dy$

$= \int_1^3 \frac{x^2}{2y}\Big|_{-2}^4 dy$

$= \int_1^3 \frac{6}{y}\,dy$

$= 6\ln|y|\Big|_1^3 = 6\ln 3$

41. $\iint_R e^x \sin y\,dA,\ R=[0,2]\times[0,\frac{\pi}{4}]$

$\int_0^{\frac{\pi}{4}}\int_0^2 e^x \sin y\,dx\,dy$

$= \int_0^{\frac{\pi}{4}} e^x \sin y\Big|_0^2 dy$

$= \int_0^{4} (e^2-1)\sin y\,dy$

$= (1-e^2)\cos y\Big|_0^{\frac{\pi}{4}} = (e^2-1)(1-\cancel{\cos 4}\,\frac{\sqrt{2}}{2})$

15.2

3. $0\le x\le 1,\ 0\le y\le 1-x^2$

horizon: $0\le y\le 1,\ 0\le x\le \sqrt{1-y}$

$\int_0^1\int_0^{1-x^2} (xy)\,dy\,dx$

5. $y-2=\frac{-2}{4}(x-0)$

$x=4-2y$

$\int_0^2\int_{4-2y}^4 x^2y\,dx\,dy$

$= \int_0^2 y\cdot\frac{x^3}{x}\Big|_{4-2y}^4 dy$

$= \frac{1}{3}\int_0^2 y(64-(4-2y)^3)\,dy$

$= \frac{1}{3}\left[\frac{8}{5}y^5+32y^3-12y^4\right]\Big|_0^2$

$= 38.4$

6. $y=\frac{1}{2}x$

$\int_0^4\int_{\frac{1}{2}x}^2 x^2y\,dy\,dx$

$= \int_0^4 \frac{1}{2}x^2y^2\Big|_{\frac{1}{2}x}^2 dx$

$= \frac{608}{15}$

7. $\int_0^2\int_0^x x^2y\,dy\,dx + \int_2^4\int_0^2 x^2y\,dy\,dx$

$= \int_0^2\int_0^x x^2y\,dy\,dx + 16$

$= \int_0^2 \frac{1}{2}x^2y^2\Big|_0^x\,dy\,dx + 16$

$= \frac{16}{5}+16$

$= 19.2$

11. $\iint_D \frac{y}{x}\,dA$

$\int_1^2\int_0^{\sqrt{4-x^2}} \frac{y}{x}\,dy\,dx$

$= \int_1^2 \frac{1}{2}\frac{y^2}{x}\Big|_0^{\sqrt{4-x^2}} dx$

$= \ln 4 - \frac{3}{4}$

19. $f(x,y)=x,\ 0\le x\le 1,\ 1\le y\le e^{x^2}$

$\int_0^1\int_1^{e^{x^2}} x\,dy\,dx$

$= \int_0^1 xy\Big|_1^{e^{x^2}} dx$

$= \frac{1}{2}x^2(e^{x^2}-1)\Big|_0^1$

$= \frac{1}{2}(e-2)$

21. $f(x,y)=2xy,\ x=y,\ x=y^2$

$$\int_0^1\int_0^{\sqrt{x}} 2xy\,dy\,dx$$

$$= \int_0^1 xy^2\Big|_0^{\sqrt{x}}\,dx$$

$$= \int_0^1 xx\,dx$$

$$= \frac{1}{3}$$

25. $$\int_0^4\int_x^4 f(x,y)\,dy\,dx$$

$$= \int_0^4\int_0^y f(x,y)\,dx\,dy$$

31. $e^x = e^{\sqrt{x}}$

$x=1$

when $x=0$, $y=1$,

when $x=1$, $y=e$

$$\int_1^e\int_{(\ln y)^2}^{\ln y} (\ln y)^{-1}\,dx\,dy$$

$$= \int_1^e (x\ln y^{-1})\Big|_{\ln y^2}^{\ln y}\,dy$$

$$= \int_1^e (1-\ln y)\,dy$$

$$= (y - y\ln y - y)\Big|_1^e$$

$$= e-2$$

33. $$\int_0^1\int_y^1 \frac{\sin x}{x}\,dx\,dy$$

$$= \int_0^1\int_0^x \frac{\sin x}{x}\,dy\,dx$$

$$= \int_0^1 \frac{\sin x\cdot y}{x}\Big|_0^x\,dx$$

$$= 1-\cos 1$$

35. $$\int_0^1\int_{y=x}^1 xe^{y^3}\,dy\,dx$$

$$= \int_0^1\int_0^y xe^{y^3}\,dx\,dy$$

$$= \frac{e-1}{6}$$

37. $$\int_0^2\int_0^2 e^{x+y}\,dy\,dx$$

$$= \int_0^2 e^x(e^2-1)\Big|_0^2\,dx$$

$$= (e^2-1)^2\Big|_0^2$$

$$= (e^2-1)^2$$

43. $y=x$, $y=\frac{x}{2}$

$y=1$, $x=1$, $x=2$

$y=2$, $x=2$, $x=4$

$$\int_1^{4}\int_1^2 \frac{\sin y}{y}\,dy\,dx$$