Jennifer Gonzalez 15.1

#9 $\int_0^3 \int_0^5 (15-3x)\,dx\,dy \rightarrow \int_0^5 (15-3x)\,dx = 15x - \frac{3}{2}x^2 \Big|_0^5 = 75 - \frac{75}{2} = \frac{75}{2}$

$\int_0^3 \frac{75}{2}\,dy = \frac{75}{2}y \Big|_0^3 = \boxed{\frac{225}{2}}$

#15 $\int_0^5 \int_{-4}^4 x^3\,dx\,dy \longrightarrow \int_{-4}^4 x^3\,dx = 0 \qquad \int_0^5 0\,dy = \boxed{0}$

#21 $\int_4^9 \int_{-3}^8 1\,dx\,dy \rightarrow \int_{-3}^8 1\,dx = x \Big|_{-3}^8 = 8+3 = 11$

$\int_4^9 11\,dy = 11y \Big|_4^9 = 99-44 = \boxed{55}$

#23 $\int_{-1}^1 \int_0^\pi x^2 \sin y\,dy\,dx \longrightarrow \int_0^\pi x^2 \sin y\,dy = x^2 \cos y \Big|_0^\pi = x^2\cos(\pi) - x^2 \cos 0$

$\int_{-1}^1 -2x^2\,dx = -\frac{2}{3}x^3 \Big|_{-1}^1 = -\frac{2}{3} - \frac{2}{3} = \boxed{-\frac{4}{3}}$

#25 $\int_2^6 \int_1^4 x^2\,dx\,dy \longrightarrow \int_1^4 x^2\,dx = \frac{x^3}{3}\Big|_1^4 = \frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 21$

$\int_2^6 21\,dy = 21y \Big|_2^6 = 126 - 42 = 84$

#31 $\int_1^2 \int_0^4 \frac{dy\,dx}{x+y} \longrightarrow \int_0^4 \frac{1}{x+y}\,dy = \ln(x+y) \Big|_0^4 = \ln(x+4) - \ln(x)$

$\int_1^2 \ln(x+4) - \ln(x)\,dx \qquad u = x+4 \quad u' = 1\,du \qquad = \int_5^6 \ln u\,du$

IBP: $u = \ln u \quad v' = 1$
$u' = 1/u \quad v = u$

$\ln(u)u - \int_5^6 \frac{1}{u}\,du = \Big[\ln(u)\cdot u - \ln u\Big]_5^6$

$= (\ln(6)\cdot 6 - \ln(6)) - (\ln(5)\cdot 5 - \ln(5)) = \boxed{5\ln 6 - 4\ln}$

$x^{1/2} = \sqrt[2]{x^1}$ $\quad x^{3/2} = \sqrt[3]{2}$

\#33 $\int_0^4 \int_0^5 \frac{dy\,dx}{\sqrt{x+y}} \rightarrow \int_0^5 \frac{1}{\sqrt{x+y}}\,dy = \int u^{-1/2}\,du = 2u^{1/2} = 2\sqrt{x+y}\Big|_0^3$

$u = x+y \quad u' = du$

$= 2\left(\sqrt{x+5} - \sqrt{x}\right) \rightarrow 2\int_0^4 \sqrt{x+5} - \sqrt{x}\,dx = 2\left(\frac{2(x+5)^{3/2}}{3} - \frac{2x^{3/2}}{3}\right)\Big|_0^4$

$= 2\left[\left(\frac{2(9)^{3/2}}{3} - \frac{2(4)^{3/2}}{3}\right) - \left(\frac{2(5)^{3/2}}{3}\right)\right] = 2\left(\frac{54}{3} - \frac{16}{3} - \frac{2\sqrt[3]{25}}{3}\right) = \frac{38 - 2\sqrt[3]{25}}{3}$

35 $\int_1^2 \int_1^3 \frac{\ln(xy)\,dy\,dx}{y} \longrightarrow \int_1^3 \frac{\ln(xy)\,dy}{y} = \int_1^3 \frac{1}{y} \cdot \ln(xy)\,dy$

IBP:
$u = \ln(xy)$
$u' = \frac{1}{y}$
$v' = \frac{1}{y}$
$v = \ln(y)$

$\ln(xy)\ln y - \int_1^3 \frac{1}{y}\ln y\,dy = \ln xy \ln y -$

\#37 $\int_{-2}^4 \int_1^3 \frac{x}{y}\,dy\,dx = \int_1^3 x \cdot \frac{1}{y}\,dy = x\ln y\Big|_1^3 = x\ln(3)$

$\int_{-2}^4 x\ln(3)\,dx = \frac{x^2\ln 3}{2}\Big|_{-2}^4 = 8\ln 3 - 2\ln 3 = \boxed{6\ln 3}$

\#41 $\int_0^{\pi/4} \int_0^2 e^x \sin y\,dx\,dy \Rightarrow \int_0^2 e^x \sin y\,dx = \sin y\, e^x\Big|_0^2 = \sin y e^2 - \sin y$

$\int_0^{\pi/4} \sin y e^2 - \sin y\,dy = e^2\cos y - \cos y\Big|_0^{\pi/4}$

$= \left[e^2\cos(\pi/4) - \cos(\pi/4)\right] - \left[e^2\cos(0) - \cos(0)\right] = \boxed{\frac{e^2\sqrt{2} - \sqrt{2}}{2} - e^2 + 1}$