15.1

9. $\iint_R (15-3x)\,dA \quad R=[0,5]\times[0,3]$

$\int_0^5\int_0^3 (15-3x)\,dy\,dx = \int_0^5 15y-3xy\Big|_0^3 = 15(3)-3(3)x = \int_0^5 45-9x$

$\int_0^5 45-9x\,dx = 45x-\frac{9x^2}{2}\Big|_0^5 = 45(5)-\frac{9(5^2)}{2} = 112.5$

15. $\int_{-4}^{4}\int_0^5 x^3\,dy\,dx = \int_{-4}^{4} x^3y\Big|_0^5 = \int_{-4}^{4} 5x^3$

$5\frac{x^4}{4}\Big|^4 = 5\frac{4^4}{4} - 5\frac{4^4}{4} = 0$

21. $\int_4^9\int_{-3}^{8} 1\,dx\,dy$

$x\Big|_{-3}^{8} = 8+3=11 \qquad \int_4^9 11\,dy = 11y\Big|_4^9 = 99-44=55$

23. $\int_{-1}^{1}\int_0^{\pi} x^2\sin y\,dy\,dx$

$\int_0^{\pi} x^2\sin y\,dy = x^2(-\cos y)\Big|_0^{\pi} = x^2(-\cos\pi+\cos(0)) = x^2(\underbrace{-1+1}_{0})$

$\int_{-1}^{1} 0\,dx =$

25. $\int_2^6\int_1^4 x^2\,dx\,dy$

$\int_1^4 x^2\,dx = \frac{x^3}{3}\Big|_1^4 = \frac{4^3}{3}-\frac{1}{3} = \frac{64}{3}-\frac{1}{3} = \frac{63}{3}$

$\int_2^6 \frac{63}{3}\,dy = \frac{63}{3}(y)\Big|_2^6 = \frac{63}{3}(6-2) = 84$

31. $\int_1^2\int_0^4 \frac{dy\,dx}{x+y} = \int_1^2 \ln(x+y)\Big|_0^4 = \ln(x+4)-\ln(x)$

$\int_1^2 \ln(x+4)-\ln(x)\,dx = (x+4)\ln(x+4)-x(\ln x)\Big|_1^2$
$= 6\ln 6 - 2\ln 2 - 5\ln 5$

33. $\int_0^4 \int_0^5 \frac{1}{\sqrt{x+y}}\, dy\, dx$

$\int_0^5 (x+y)^{-1/2}\, dy = (x+y)^{1/2} \cdot 2 \Big|_0^5 = 2(5+x)^{1/2} - x^{1/2}$

$2\int_0^4 (5+x)^{1/2} - x^{1/2} = 2\left((5+x)^{3/2} \cdot \frac{2}{3} - \frac{2x^{3/2}}{3}\right)\Big|_0^4 = \frac{4}{3}(\sqrt{19} - 5\sqrt{5})$

35. $\int_1^2 \int_1^3 \frac{\ln(xy)\, dy\, dx}{y}$

$\int_1^3 \frac{\ln xy}{y}\, dy = \frac{2\ln y \ln(x) + \ln(y^2)}{y} = \frac{2\ln(3)\ln x + \ln(3^2)}{2}$

$\int_1^2 \frac{2\ln(3)\ln x + \ln(3^2)}{2}\, dx = \frac{1}{2}\left(2\ln(3)\, x \ln x + \ln(3^2) \cdot x\right)\Big|_1^2$

$= \frac{(2\ln(3) \cdot 2\ln 2 + \ln(3^2) \cdot 2}{2} - \frac{2\ln 3 \ln 1 + \ln 3^2}{2}$

37. $\int_{-2}^4 \int_1^3 \frac{x}{y}\, dy\, dx$

$\int_1^3 \frac{x}{y}\, dy = x \ln y \Big|_1^3 = x(\ln 3 - \ln 1)$

$\int_{-2}^4 x(\ln 3 - \ln 1)\, dx = \frac{x^2}{2}\Big|_{-2}^4 = \frac{4^2}{2} - \frac{-2^2}{2} = 8 - 2 = 6(\ln 3)$

41. $\int_0^2 \int_0^{\pi/4} e^x \sin y\, dy\, dx$

$\int_0^{\pi/4} e^x \sin y\, dy = e^x(-\cos\frac{\pi}{4} + \cos \pi) = (\frac{-\sqrt{2}}{2} + 1)e^x$

$\int_0^2 e^x (\frac{\sqrt{2}}{2} + 1)\, dx = e^x \Big|_0^2 = e^2 - e^0 = (\frac{-\sqrt{2}}{2} + 1)(e^2 - 1)$

15.2

3. $0 \le x \le 1 \qquad 0 \le y \le 1-x^2$

$$\int_0^1\int_0^{1-x^2} xy\,dy\,dx = x\,\frac{y^2}{2}\Big|_0^{1-x^2} = \frac{x(1-x^2)}{2}$$

$$\int_0^1 \frac{x(1-x^2)}{2}\,dx = \frac{1}{2}\left(\frac{x^2}{2}-\frac{x^4}{4}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{1^2}{2}-\frac{1}{4}\right) = \frac{1}{12}$$

$x = \sqrt{1-y}$

$0 \le x \le \sqrt{1-y} \qquad 0 \le y \le 1$

$$\int_0^1\int_0^{\sqrt{1-y}} xy\,dx\,dy = y\,\frac{x^2}{2}\Big|_0^{\sqrt{1-y}} = \frac{y(1-y)}{2}$$

$$\int_0^1 \frac{y(1-y)}{2} = \frac{1}{2}\left(\frac{y^2}{2}-\frac{y^3}{3}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{1}{2}-\frac{1}{4}\right) = \frac{1}{12}$$

5. $0 \le x \le 4 \qquad 0 \le y \le 4-\frac{1}{2}x \qquad y = 4-\frac{1}{2}x$

$y-4 = -\frac{1}{2}x$

$-2y+8 = x$

$$\int_0^4\int_0^{4-\frac{1}{2}x} x^2y\,dy\,dx$$

$$\int_0^{4-\frac{1}{2}x} x^2y\,dy = x^2\,\frac{y^2}{2}\Big|_0^{4-\frac{1}{2}x} = x^2\,\frac{(4-\frac{1}{2}x)^2}{2} = \frac{\frac{x}{4}-x^2+16\cdot x^2}{2}$$

$$\frac{1}{2}\int_0^4 \frac{x}{4}-x^2+16\cdot x^2 = \frac{x^4}{16}-\frac{x^5}{5}+\frac{16x^3}{3}\Big|_0^4 = \frac{4^4}{16}-\frac{4^5}{5}+\frac{16(4^3)}{3} = 38.4$$

6. $0 \le x \le 4 \qquad 0 \le y \le \frac{x}{2}$

$$\int_0^4\int_0^{x/2} x^2y\,dy\,dx = x^2\,\frac{y^2}{2}\Big|_0^{x/2} = x^2\cdot\frac{x}{8} = \frac{x^3}{8}$$

$$\int_0^4 \frac{x^3}{8}\,dx = \frac{x^4}{32}\Big|_0^4 = \frac{4^4}{32} = 8$$

7. $0 \le y \le 2 \qquad 0 \le x \le y \qquad y = 0 + x$

$$\int_0^2\int_0^y x^2y\,dx\,dy = \frac{x^3}{3}\,y\Big|_0^y = \frac{y^4}{3}$$

$$\int_0^2 \frac{y^3}{3} = \frac{y^4}{12}\Big|_0^2 = \frac{2^4}{12} - 0 = \frac{4}{3}$$

11. $0 \le x \le 2 \qquad 0 \le y \le \sqrt{4-x^2}$

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x}\, dy\, dx = \frac{y^2}{2x} \Big|_0^{\sqrt{4-x^2}} = \left(\frac{4-x^2}{2x}\right)$$

$$\int_0^2 \frac{4-x^2}{2x} = 2\ln x - \frac{x^2}{2} \Big|_0^2 = \left(2\ln 2 - \frac{4}{2}\right) = 2(\ln 2 - 1)$$

19. $$\int_2^1 \int_1^{e^{x^2}} x\, dy\, dx = xy \Big|_1^{e^{x^2}} = xe^{x^2} - x$$

$$\int_0^1 xe^{x^2} - x\, dx = 2xe^{x^2} - \frac{x^2}{2}\Big|_0^1 = 2(1)e^1 - \frac{1^2}{2} = 2e - \frac{1}{2}$$

21. $$\int_{1/2}^{\pi/2} \int_1^{2x} 2xy\, dy\, dx = 2x\frac{y^2}{2}\Big|_1^{2x} = \cancel{2}x\frac{4x^2}{\cancel{2}} = 4x^3$$

$$\int_{1/2}^{\pi/2} 4x^3 = x^4 \Big|_{1/2}^{\pi/2} = \left(\frac{\pi}{2}\right)^4 - \frac{1}{16}$$

25. $x \le y \le 4 \qquad 0 \le x \le 4$

$0 \le y \le 4 \qquad y \le x \le 4$

$$\int_0^4 \int_0^y f(x,y)\, dx\, dy$$

31. $f(x,y) = (\ln y)^{-1}$

$1 \le y \le e \qquad \ln^2 y \le x \le \ln y$

$$\int_1^e \int_{\ln^2 y}^{\ln y} (\ln y)^{-1}\, dx\, dy$$

$$\int_1^e \ln^2 y = e - 2$$

33. $0 \le y \le 1 \qquad y \le x \le 1$

$0 \le y \le x \qquad 0 \le x \le 1$

$$\int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dy = \frac{1}{x}(-\cos x + \cos(0)) = \frac{1}{x}(-\cos x + 1)$$

$$\int_0^1 \frac{1}{x}(-\cos x + 1)$$

$1 - \cos 1$

35. $\int_0^1 \int_{y=x}^1 xe^{y^3}\,dy\,dx = \int_0^1 \int_0^y xe^{y^3}\,dx\,dy$

$\frac{x^2}{2}e^{y^3}\Big|_0^y = \frac{y^2}{2}e^{y^3}$

$\int_0^1 \frac{y^2}{2}e^{y^3}dy = \frac{e-1}{6}$

37. $\int_1^2 \int_1^2 e^{x+y}\,dx\,dy = e^{x+y}$

$\int_1^2 e^{x+y}\,dy = e^4 - 3e^2 + 2e$

43. $1 \le y \le 2 \quad y \le x \le 1$

$\int_1^2 \int_y^1 \frac{\sin y}{y}\,dx\,dy = \frac{1}{y}(-\cos 1 + \cos y) =$

$\int_1^2 \frac{1}{y}(-\cos 1 + \cos y) = \cos 1 - \cos 2$

49. $\iint_R (x^2+y^2)^2 - (8-x^2-y^2)^2\,dA$