Fayed Raza
10/23/2020

15.1: 9, 15, 21, 23, 25, 31, 33, 35, 37, 41

9. $\int_0^3 \int_0^5 (15 - 3x)\, dx\, dy$

$\int_0^5 15 - 3x\, dx$

$15x - \frac{3x^2}{2} \Big|_0^5$

$15(5) - \frac{3(25)}{2}$

$75 - \frac{75}{2}$

$\frac{150}{2} - \frac{75}{2} = \frac{75}{2}$

$\int_0^3 \frac{75}{2}\, dy$

$\frac{75}{2} y \Big|_0^3$

$\frac{3(75)}{2} - 0$

$\frac{225}{2}$

15. $\iint_R x^3 \, dA$

$\int_0^5 \int_{-4}^{4} x^3 \, dx \, dy$

$\int_{-4}^{4} x^3 \, dx$

$\frac{x^4}{4} \Big|_{-4}^{4}$

$\frac{256}{4} - \frac{256}{4}$

$\int_0^5 0 \, dy$

$0 \Big|_0^5 dy$

$0 - 0$

0

21. $\int_4^9 \int_{-3}^{8} 1 \, dx \, dy$

$\int_{-3}^{8} 1 \, dx$ $\int_4^9 11 \, dy$

$x|_{-3}^{8}$ $11y|_4^9$

$8-(-3)$ $99-44$ (55)

11

23. $\int_{-1}^{1} \int_{0}^{\pi} x^2 \sin y \, dx \, dy$

$\int_0^{\pi} x^2 \sin y \, dy$ $\int_{-1}^{1} 2x^2 \, dx$

$-x^2 \cos y|_0^{\pi}$ $\frac{2x^3}{3}|_{-1}^{1}$

$-x^2\cos\pi + x^2\cos 0$ $\frac{2}{3} - \left(-\frac{2}{3}\right)$

$-x^2 + x^2$ $\frac{4}{3}$

$2x^2$

25.

$$\int_2^6 \int_1^4 x^2 \, dx \, dy$$

$$\int_1^4 x^2 \, dx$$

$$\frac{x^3}{3}\Big|_1^4$$

$$\frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 21$$

$$\int_2^6 21 \, dy$$

$$21y\Big|_2^6$$

$$21(6) - 21(2)$$

$$126 - 42 = \boxed{84}$$

$$\begin{array}{r} 0 \\ -42 \\ \hline 84 \end{array}$$

31.

$$\int_1^2 \int_0^4 \frac{dy\,dx}{x+y}$$

$$\int_0^4 \frac{1}{x+y}\,dx$$

$$\ln|x+y|\Big|_0^4$$

$$\ln(4+x) - \ln(x)\,dx$$

$$(x+4)\ln(4+x) - (x+4) - x\ln(x) + x\Big|_1^2$$

$$6\ln(6) - 6 - 2\ln(2) + \ln(1)$$

$$-(5)\ln(5) + (5)$$

$$6\ln(6) - 5\ln(5) - \ln(2)$$

33

$$\int_0^4 \int_0^5 (x+y)^{-\frac{1}{2}}\,dy\,dx$$

$$\int_0^5 (x+y)^{-\frac{1}{2}}\,dx$$

$$2(x+y)^{\frac{1}{2}}\Big|_0^5$$

$$\int_0^4 2(x+5)^{\frac{1}{2}} - 2(x)^{\frac{1}{2}}\,dx$$

$$\frac{4}{3}(x+5)^{\frac{3}{2}} - \frac{4}{3}(x)^{\frac{3}{2}}\Big|_0^4$$

$$36 - \frac{32}{3} - \frac{20\sqrt{5}}{3}$$

35

$$\int_1^2 \int_1^3 \frac{\ln(xy)}{y}\,dy\,dx$$

$$\int_1^3 \frac{\ln(xy)}{y}\,dy$$

$$\int_0^2 \ln(3x)\cdot\ln(x)\,dx$$

$$\ln(6) - \ln(1)$$

$$\ln(6)$$

3. $\int_1^3 \int_{-2}^{4} \frac{x}{y}\, dx\, dy$

$\int_{-2}^{4} \frac{x}{y}\, dx$

$\frac{1}{2y} x^2 \Big|_{-2}^{4}$

$\frac{16}{2y} - \frac{4}{2y} = \frac{12}{2y} = \frac{6}{y}$

$\int_1^3 \frac{6}{y}\, dy$

$6 \ln y \Big|_1^3$

$6 \ln(3) - 6 \ln(1)$

$6 \ln(3)$

4)

$$\int_0^{\frac{\pi}{4}} \int_0^2 e^x \sin y \, dx \, dy$$

$$e^x \sin y \Big|_0^2$$

$$\int_0^{\frac{\pi}{4}} e^2 \sin y - \sin y \, dy$$

$$-e^2 \cos y + \cos y \Big|_0^{\frac{\pi}{4}}$$

$$-e^2 \cos\frac{\pi}{4} + \cos\frac{\pi}{4}$$

$$e^2 \cos(0) - \cos(0)$$

$$-e^2 \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$e^2 - 1$$

$$\frac{-e^2\sqrt{2} + \sqrt{2}}{2} + e^2 - 1$$

+(1,0)

15, 2, 3, 3, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 43, 49

3. Domain

Vertically $0 \le x \le 1$, $0 \le y \le 1 - x^2$

Horizontally $0 \le y \le 1$, $0 \le x \le \sqrt{1-y}$

$$\int_0^1 \int_0^{1-x^2} (xy)\,dy\,dx$$

$y = 1 + x^2$

$y - 1 = -x^2$

$\sqrt{1-y} = x$

$$\int_0^{1+x^2} (xy)\,dy$$

$$\frac{xy^2}{2}\Big|_0^{1-x^2}$$

$$\frac{x(1-x^2)^2}{2} = 0$$

$(1-x^2)(1-x^2)$

$1 - 2x^2 + x^4$

$$\int_0^1 \frac{x(1-x^2)^2}{2}\,dx$$

$$\frac{1}{2}\int_0^1 x - 2x^3 + x^5$$

$$\frac{1}{2}\left(\frac{x^6}{6} - \frac{x^4}{2} + \frac{x^2}{2}\right)\Big|_0^1$$

$\frac{1}{2}(\frac{1}{12}) = \frac{1}{12}$ $\quad \frac{1}{6} - \frac{1}{2} + \frac{1}{2} = \frac{1}{6}$

$\frac{1}{12}$

5

$y = x+2$

$y = x+2$

$$\int_0^4 \int_{x+2}^{2} x^2 y \, dy \, dx$$

$$\frac{x^2 y^2}{2} \Big|_{x+2}^{2}$$

$$2x^2 - \frac{(x^2)(x+2)^2}{2}$$

$$\int_0^4 2x^2 - \frac{x^4}{2} - 2x^3 - 2x^2 \, dx$$

$$-\frac{x^5}{10} - \frac{2x^4}{4} \Big|_0^4$$

$$0 - \left(\frac{1024}{10} - \frac{16}{2}\right) = \frac{-947}{10}$$

$$\left(\frac{128}{3}\right) = \frac{128}{3}$$

6

$$\int_0^4 \int_{\frac{y}{2}}^{2} x^2 y \, dx \, dy$$

$$\int_{\frac{y}{2}}^{2} x^2 y \, dx \qquad \int_0^4 2x^2 - \frac{x^2 y^2}{2} \, dx$$

$$\frac{x^2 y^2}{2} \Big|_{\frac{y}{2}}^{2} \qquad 2x^2 y - \frac{x^2 y^3}{8} \Big|_0^4$$

$$\frac{4x^2}{2} - \frac{x^2 y^2}{8} \qquad 16 - \frac{64}{6} = 8$$

7.

$$\int_0^4 \int_{\frac{y}{2}}^{2} x^2 y \, dx \, dy$$

$$\frac{x^3}{3} y \Big|_{\frac{y}{2}}^{2}$$

$$\int_0^4 \frac{2x^3}{3} - \frac{x^4}{3} \, dy$$

$$\frac{2x^4}{12} - \frac{x^5}{15} = \frac{(256)}{6} - \frac{1024}{15}$$

11

$$\int_{-2}^{2}\int_{0}^{x}\frac{y}{x}\,dy\,dx$$

$$\int_{-2}^{2}\frac{y^2}{2x}\Big|_0^x\,dx$$

$$\int_{-2}^{2}\frac{x^2}{2x}\,dx$$

$$\int_{-2}^{2}\frac{x}{2}\,dx$$

$$\frac{x^2}{4}\Big|_{-2}^{2}$$

$$\frac{4}{4}-\frac{4}{4}$$

$$1-1$$

$$0$$

19

$$\int_0^1 \int_x^{e^{x^2}} x \, dy \, dx$$

$$xy \Big|_x^{e^{x^2}}$$

$$e^{x^2} x - x$$

$$\int_0^1 e^{x^2} x - x \, dx$$

$$\frac{e^{x^2}}{2} - \frac{x^2}{2} \Big|_0^1$$

$$\frac{e}{2} - \frac{1}{2} - \left(\frac{1}{2}\right) \quad \left(\frac{e}{2} - 1\right) =$$

21.

$$\int_0^1 \int_{x^2}^{x} 2xy \, dy \, dx$$

$$\int_{x^2}^{x} 2xy \, dy$$

$$xy^2 \Big|_{x^2}^{x}$$

$$\int_0^1 x^3 - x^5 \, dx$$

$$\frac{x^4}{4} - \frac{x^6}{6} \Big|_0^1$$

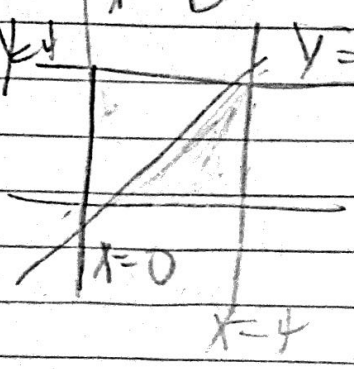
$$\frac{1}{4} - \frac{1}{6} = \frac{3}{12} - \frac{2}{12} = \boxed{\frac{1}{12}}$$

25. $\int_0^4 \int_x^4 f(x,y) \, dy \, dx$

$y = 4$ $\quad x = 4$

$y = x$ $\quad x = 0$

$y = x$ $\quad y = 4$



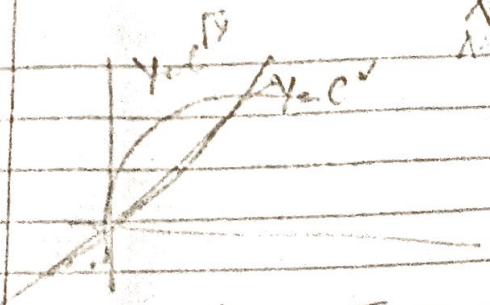
$$\int_0^4 \int_0^y f(x,y) \, dx \, dy$$

$x = \ln(y)$

$x = \ln^2(y)$

$y = e^x$

$y = e^{\sqrt{x}}$



$e^{x} = e^{\sqrt{x}}$

$x = \sqrt{x}$

$x - \sqrt{x} = 0$

$x^{\frac{1}{2}}(x^{\frac{1}{2}} - 1) = 0$

$x = 0 \quad x = 1$

$0 \le x \le 1$

$e^{x^2} \le y \le e^x$

$1 = \ln(y)$

$0 = \ln(y)$

$$\int_0^1 \int_{e^{\sqrt{x}}}^{e^x} (\ln y)^{-1} \, dy \, dx$$

$$\int_0^e \int_{\ln^2 y}^{\ln y} (\ln x)^{-1} \, dx \, dy$$

x	$\ln y$	$\ln^2 y$

$$\frac{\ln y}{\ln y} - \ln y \, dx$$

$$\int_0^e 1 - \ln y \, dy$$

$$y - y \ln y + y \Big|_1^e \, dx$$

$$2y - y \ln y \Big|_0^1$$

$$2e - e(\ln e)$$

$$2e - e$$

$$-(2) = e - 2$$

33. $\int_0^1 \int_y^1 \frac{\sin x}{x} \, dx \, dy$

$\int_0^1 \int_0^x \frac{\sin x}{x} \, dy \, dx$

$\int_0^x \frac{\sin x}{x} \, dy$

$y \frac{\sin x}{x} \Big|_0^x$

$\int_0^1 \sin x \, dx$

$x = 1$

$x = y$

$y = 1 \quad x$

$y = 0 \quad y = 1$

$x = 1$

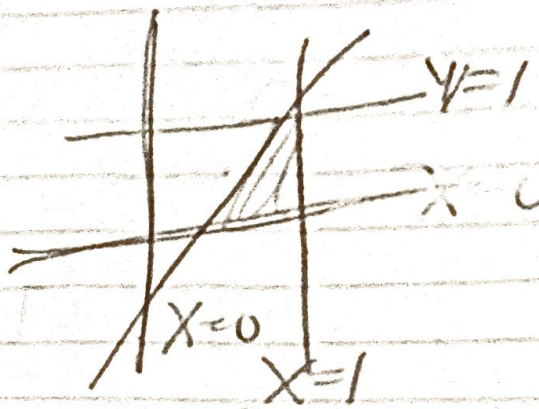
$-\cos x \Big|_0^1$

$-(\cos(1) + \cos(0))$

$1 - \cos(1)$

35 $\int_0^1 \int_{x=y}^{1} x e^{y^3} \, dx \, dy$

$y=1 \quad x=1$
$y=x \quad x=0$



$$\int_0^1 \int_0^y x e^{y^3} \, dx \, dy$$

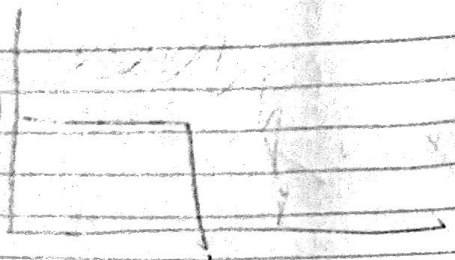
$$\frac{x^2}{2} e^{y^3} \Big|_0^y$$

$$\int_0^1 \frac{x^2 e^{y^3}}{2} \, dy$$

$$\frac{1}{2} \cdot \frac{e^{y^3}}{3} \Big|_0^1$$

$$\frac{1}{2}\left(\frac{e^1}{3} - \frac{1}{3}\right) \quad \boxed{\frac{e-1}{6}}$$

3)



$$\int_0^2 \int_1^2 e^{x+y}\, dx\, dy$$

$$e^{x+y}\Big|_1^2$$

$$\int_0^2 e^{2+y} - e^{1+y}\, dy$$

$$e^{2+y} - e^{1+y}\Big|_0^2$$

$$e^4 - e^3 - e^2 + e^1 - e^1 + e^0$$

45 $\int_1^2 \int_{\frac{x}{2}}^{x} \frac{\sin y}{y}\, dy\, dx$

$\int_1^2 \int_{\frac{x}{2}}^{x} \frac{\sin y}{y}\, dy\, dx$

$\int_1^2 \frac{x \sin x}{2x} - \frac{x \sin x}{x}\, dx = 0.956$

$\frac{x \sin x}{2} \ln|2| - x \sin x \ln|2|$

49. $x^2 + y^2 = 8 - x^2 - y^2$

$2x^2 + 2y^2 = 8$

$x^2 + y^2 = 4$

$x = \pm 2$

$\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left[(8 - x^2 - y^2) - (x^2 + y^2) \right] dy$